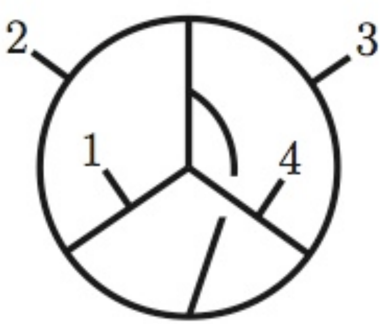
Quantum Inequivalence, Evanescent Operators and Gravity Divergences

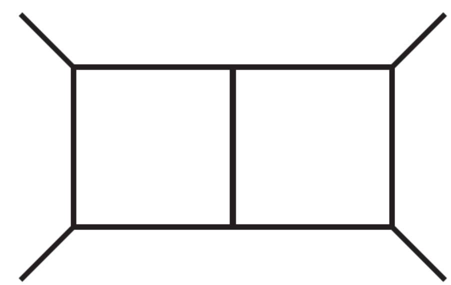
Strings 2015

Bangaluru, June 26

Zvi Bern, UCLA & CERN

ZB, Tristan Dennen, Scott Davies, Volodya Smirnov and Sasha Smirnov, arXiv:1309.2496
ZB, Tristan Dennen, Scott Davies, arXiv:1409.3089
ZB, Clifford Cheung, Huan-Hang Chi, Scott Davies, Lance Dixon, Josh Nohle (to appear)





Outline

1) Basic tools and ideas.

2) Status of supergravity divergences: Nontrivial examples of "enhanced cancellations".

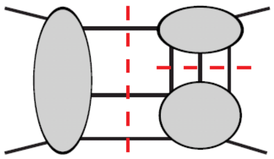
3) Conformal anomaly and question of quantum inequivalence under duality transformations.

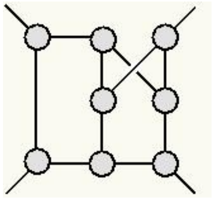
4) Effect of duality transformations on UV properties of gravity theories at two loops. Curious properties.

UV properties of gravity theories subtle and interesting.

Our Basic Tools

We have powerful tools for computing amplitudes and for discovering new structures:





- **Generalized unitarity method.**

  ZB, Dixon, Dunbar, Kosower
  ZB, Carrasco, Johansson , Kosower

- **Duality between color and kinematics. Gravity scattering amplitudes directly from gauge-theory ones. Double copy.**

  ZB, Carrasco and Johansson (BCJ)

- **Advanced loop integration technology.**

  Chetyrkin, Kataev and Tkachov; Laporta; A.V. Smirnov; V. A. Smirnov; Vladimirov; Marcus, Sagnotti; Czakon; etc.

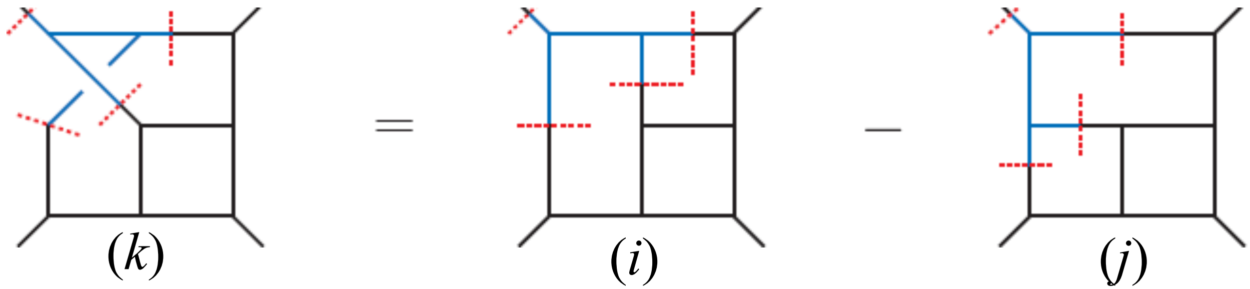
- **I won't explain these tools these but they underlie everything.**
- **Many other tools and advances that I won't discuss here.**

See Tim Adamo, Jake Bourjaily's and Marcus Spradlins' talk

Duality Between Color and Kinematics

ZB, Carrasco, Johansson (BCJ)

Conjecture: kinematic numerators exist with same algebraic properties as color factors



color factor

$$c_k = c_i - c_j$$

$$n_k = n_i - n_j$$

kinematic numerator

$$n_i \sim k_1 \cdot l_1 \, k_3 \cdot l_2 \, \varepsilon_1 \cdot l_3 \, \varepsilon_2 \cdot k_3 \, \varepsilon_3 \cdot l_2 \, \varepsilon_4 \cdot k_3 + \ldots$$

If you have a set of duality satisfying kinematic numerators.

gauge theory ⟶ gravity theory

simply take

color factor ⟶ kinematic numerator $\quad c_i \to n_i$

Gravity loop integrands are trivial to obtain once we have gauge theory in a form where duality holds.

Gravity From Gauge Theory

$$
\begin{array}{lcc}
 & n & \tilde{n} \\
\mathcal{N}=8 \text{ sugra}: & (\mathcal{N}=4 \text{ sYM}) \times & (\mathcal{N}=4 \text{ sYM}) \\
\mathcal{N}=5 \text{ sugra}: & (\mathcal{N}=4 \text{ sYM}) \times & (\mathcal{N}=1 \text{ sYM}) \\
\mathcal{N}=4 \text{ sugra}: & (\mathcal{N}=4 \text{ sYM}) \times & (\mathcal{N}=0 \text{ sYM})
\end{array}
$$

Pure gravity requires extra projectors to remove unwanted states.

Some recent applications of BCJ duality and double copy structure:

- **Construction of nontrivial supergravities.**
  Anastasiou, Bornsten, Duff; Duff, Hughs, Nagy; Johansson and Ochirov; Carrasco, Chiodaroli, Günaydin and Roiban; Chiodaroli, Günaydin, Johansson, Roiban
- **Guidance for constructing string-theory loop amplitudes.**
  Mafra, Schlotterer and Steiberger; Mafra and Schlotterer
- **Recent applications to classical black hole solutions.**
  Monteiro, O'Connell and White
- **Studies of UV properties of $\mathcal{N}=4$ and $\mathcal{N}=5$ supergravity.**
  ZB, Davies, Dennen, Huang; ZB, Davies, Dennen, Smirnov, Smirnov

UV in Gravity

Most people in this audience believe that UV properties of quantum field theories of gravity are well understood, up to "minor" details.

The main purpose of my talk is to try to convince you that the UV structure of gravity is strange and surprising.

1. **Examples of no divergence even with counterterms valid, as far as is known.**
1. **When UV divergences are present in pure (super) gravity, properties are very strange.**

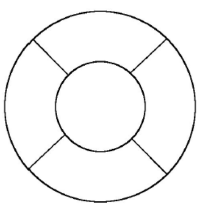
Predictions of Ultraviolet Properties

In recent years a lot of new work on UV properties of supergravity.

ZB, Dixon, Roiban; Green, Russo,Vanhove, Russo; ZB, Carrasco, Dixon, Johansson, Roiban; Bossard, Howe, Stelle; Elvang, Freedman, Kiermaier; Green and Björnsson; Kallosh; Bossard , Hillmann and Nicolai; Ramond and Kallosh; Broedel and Dixon; Elvang, Kiermaier; Beisert, Elvang, Freedman, Kiermaier, Morales, Stieberger; ZB, Davies, Dennen, Huang, Smirnov^2; etc.

Among the various approaches, Björnsson and Green developed a first-quantized formulation based on Berkovits' pure-spinors.

Key point: *all* supersymmetry cancellations exposed.



Identify poorly behaved contribution in individual diagrams.

Poor UV behavior, unless new types of cancellations between diagrams exist that are "not consequences of supersymmetry in any conventional sense".

Bjornsson and Green

- $\mathcal{N} = 8$ **sugra should diverge at 7 loops in** $D = 4$.
- $\mathcal{N} = 8$ **sugra should diverge at 5 loops in** $D = 24/5$.

Maximal Cut Power Counting

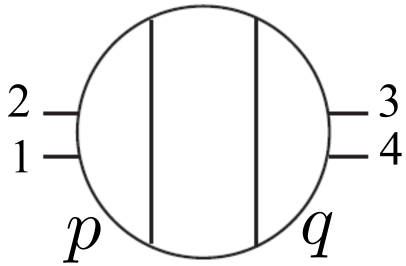
ZB, Davies, Dennen (2014)

Maximal unitarity cuts of diagrams poorly behaved:

already log divergent

$\mathcal{N} = 4$ sugra: pure YM x $\mathcal{N} = 4$ sYM

$\mathcal{N} = 4$ sugra



$$n_i \sim s^3 t A_4^{\text{tree}} (p\cdot q)^2 \, \varepsilon_1 \cdot p \, \varepsilon_2 \cdot p \, \varepsilon_3 \cdot q \, \varepsilon_4 \cdot q + \ldots$$

This diagram is log divergent

$\mathcal{N} = 8$ sugra should diverge at 7 loops in $D = 4$. **Bet with David Gross**

$\mathcal{N} = 8$ sugra should diverge at 5 loops in $D = 24/5$ **Bet with Kelly Stelle**

$\mathcal{N} = 4$ sugra should diverge at 3 loops in $D = 4$
$\mathcal{N} = 5$ sugra should diverge at 4 loops in $D = 4$
Unfortunately no bets

Equivalent to Björnsson and Green approach:
Expose susy, identify poorly behaved terms and count.

All other groups agree this is where divergences should be.

Any UV cancellation beyond this is "enhanced cancellation".

Examples of Enhanced UV Cancellations

Known explicit examples of enhanced cancellations:

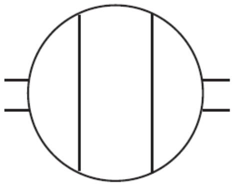
1) $D = 4$, $N = 4$ pure supergravity is UV finite at 3 loops.
ZB, Davies, Dennen, Huang

2) $D = 4$, $N = 5$, supergravity is UV finite at 4 loops.
ZB, Davies, Dennen

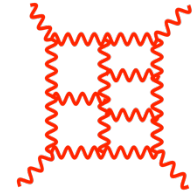
3) Half-maximal pure supergravity in $D = 5$ UV finite at 2 loops.
ZB, Davies, Dennen, Huang

At present no known standard symmetry explanation for any of these.
Bossard, Howe and Stelle; ZB, Davies, Dennen

- **Nonabelian gauge theories do *not* display enhanced cancellations.**
- **The fact the cancellations are "enhanced" means it will be very hard to find a standard symmetry explanation.**



Status of $N = 8$ Supergravity



- $N = 8$ sugra likely will have enhanced UV cancellations at 5 loops and beyond.
- $N = 8$ supergravity is likely UV finite at 7 loops, contrary to predictions from symmetry considerations.
- A general understanding likely will require a much better understanding of BCJ duality between color and kinematics.

Constructing 5 loop $N= 8$ supergravity amplitude has turned out to be a nontrivial challenge.

Key issue: Finding BCJ representations of $N = 4$ sYM amplitudes at 5 loops and beyond. Looks nontrivial.

Recent Progress: 5 loop 4 point $N = 4$ sYM with excellent loop-by-loop power counting. An important step towards $N = 8$ sugra.

ZB, Carrasco, Johansson, Roiban (to appear)

The 4 Loop Divergence of $N = 4$ Supergravity

ZB, Davies, Dennen, Smirnov, Smirnov

$D = 4 - 2\epsilon$

Pure $N = 4$ supergravity is divergent at 4 loops:

$$\mathcal{M}^{4 \text{ loop}}\Big|_{\text{div.}} = \frac{1}{(4\pi)^8} \frac{1}{\epsilon} \frac{1}{144} (1 - 264\zeta_3)\mathcal{K}$$

The kinematic factor $\mathcal{K}$ has a curious property:

The divergence is present in sectors that would vanish if not for an anomaly in the SU(1,1) duality symmetry.

Carrasco, Kallosh, Tseytlin and Roiban

Seems that an anomaly is behind the divergence, but hard to study because of high loop order. Want simpler example.

Pure Einstein Gravity

Standard argument for 1 loop finiteness of pure gravity:

’t Hooft and Veltman (1974)

$\cancel{R^2}$ $\cancel{R_{\mu\nu}^2}$ $\cancel{R_{\mu\nu\rho\sigma}^2}$

Counterterms vanish by equation of motion and can be eliminated by field redefinition.

In *D* = 4 asymptotically flat space, Gauss-Bonnet theorem eliminates Riemann square term.

$$\int d^4x \sqrt{-g}(R^2 - 4R_{\mu\nu}^2 + R_{\mu\nu\rho\sigma}^2) = 32\pi^2\chi$$

Euler characteristic evanescent operator

Pure gravity counterterm with nontrivial topology:

$$\mathcal{L}^{\rm GB} = -\frac{1}{(4\pi)^2}\frac{53}{90\epsilon}(R^2 - 4R_{\mu\nu}^2 + R_{\mu\nu\rho\sigma}^2)$$

Capper and Duff (1974)
Tsao (1977); Critchley (1978)
Gibbons, Hawking, Perry (1978)
Goroff and Sagnotti (1986)
Bornsen and van de Ven (2009)

- **Euler characteristic vanishes in flat space.** ’t Hooft and Veltman (1974)
- **Dimensional regularization makes it subtle.** Capper and Kimber (1980)

This is an enhanced cancellation, but here it is understood.

The Conformal Anomaly

Capper and Duff (1974); Tsao (1977); Critchley (1978); Gibbons, Hawking, Perry (1978); Duff and van Nieuwenhuizen (1980); Siegel (1980); Grisaru, Nielsen, Siegel, Zanon (1984); Goroff and Sagnotti (1986); Bornsen and van de Ven (2009); Etc.

The Gauss-Bonnet counterterm exactly corresponds to conformal anomaly.

$$D = 4 - 2\epsilon$$

$$\mathcal{L}^{\mathrm{GB}} = -\frac{1}{(4\pi)^2}\frac{1}{360\epsilon}\Big(4\cdot 53 + 1 + 91 - 180\Big)(R^2 - 4R_{\mu\nu} + R^2_{\mu\nu\rho\sigma})$$

graviton   scalar   2 form   3 form   Gauss-Bonnet

$$T^{\mu}{}_{\mu} = -\frac{1}{(4\pi)^2}\frac{2}{360}\Big(4\cdot 53 + 1 + 91 - 180\Big)(R^2 - 4R^2_{\mu\nu} + R^2_{\mu\nu\rho\sigma})$$

Referred to as conformal, trace or Weyl anomaly.
Called an "anomaly", but Lagrangian not conformally invariant.

Quantum Inequivalence?

$D = 4 - 2\epsilon$

$D \to 4$

$$T^{\mu}{}_{\mu} = -\frac{1}{(4\pi)^2}\frac{2}{360}\Big(4\cdot 53 + 1 + 91 - 180\Big)(R^2 - 4R^2_{\mu\nu} + R^2_{\mu\nu\rho\sigma})$$

graviton scalar 2 form 3 form Gauss-Bonnet

two form dual to scalar

$$\partial_\mu \phi = \varepsilon_{\mu\nu\rho\sigma} H^{\nu\rho\sigma}$$

three form not dynamical

$$\Lambda = \varepsilon_{\mu\nu\rho\sigma} H^{\mu\nu\rho\sigma} \qquad D = 4$$

- **Quantum *in*equivalence under duality transformations.** Duff and van Nieuwenhuizen (1980)
- **Quantum equivalence under duality. Gauge dependence.** Siegel (1980)
- **Quantum equivalence of UV (ignoring trace anomaly).** Fradkin and Tseytlin (1984)
- **Quantum equivalence of susy 1 loop effective action (with Siegel's argument for higher loops)** Grisaru, Nielsen, Siegel, Zanon (1984)

What is physical significance?

Scattering amplitudes good to look at. Cross sections physical.
One loop not really good enough because anyway evanescent.

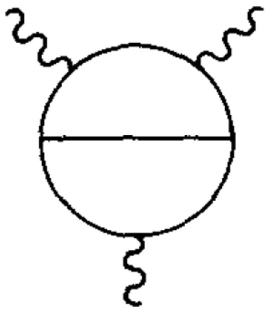
Two Loop Pure gravity

By two loops there is a valid R^3 counterterm and corresponding divergence.

Goroff and Sagnotti (1986); Van de Ven (1992)

Divergence in pure gravity:

$$\mathcal{L}^{R^3} = \frac{209}{2880} \frac{1}{(4\pi)^4} \frac{1}{2\epsilon} R^{\alpha\beta}{}_{\gamma\delta} R^{\gamma\delta}{}_{\rho\sigma} R^{\rho\sigma}{}_{\alpha\beta}$$

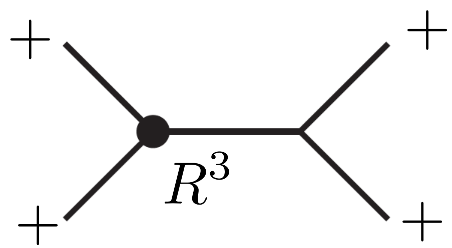


$$D = 4 - 2\epsilon$$

- **The Goroff and Sagnotti result is correct in all details.**
- **On surface nothing weird going on.**

However, a goal of this talk is to show you that UV divergences in pure (super)gravity is subtle and weird, once you probe carefully.

Two Loop Identical Helicity Amplitude

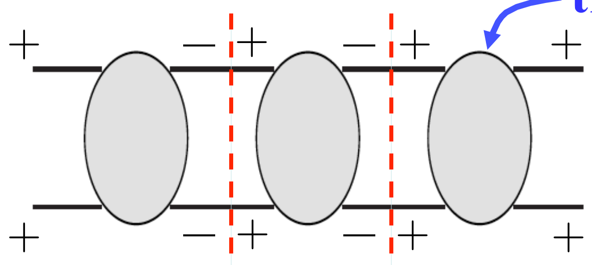


Pure gravity identical helicity amplitude sensitive to Goroff and Sagnotti divergence.

$$\mathcal{M}^{R^3}\Big|_{\text{div.}} = \frac{209}{24\epsilon}\,\mathcal{K} \qquad \mathcal{K} = \left(\frac{\kappa}{2}\right)^6 \frac{i}{(4\pi)^4}\, stu \left(\frac{[12][34]}{\langle 12\rangle\langle 34\rangle}\right)^2$$

Curious feature:

tree amplitude vanishes



- **Integrand vanishes for four-dimensional loop momenta.**
- **Nonvanishing because of ϵ-dimensional loop momenta.**

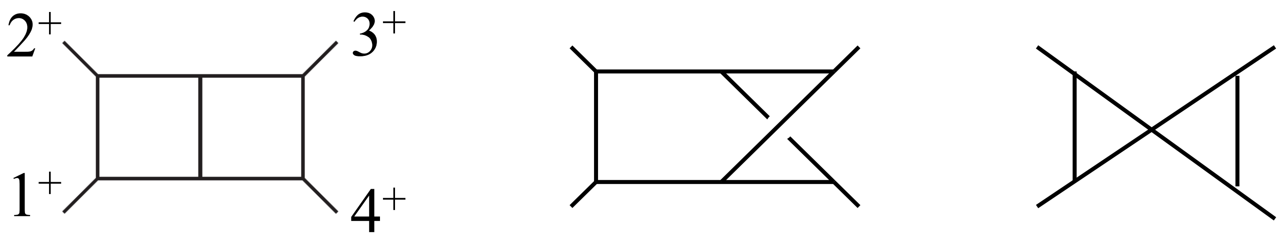
Bardeen and Cangemi pointed out nonvanishing of identical helicity is connected to an anomaly in self-dual sector.

A surprise:

Divergence is *not* generic but appears tied to anomalous behavior.

Two Loop Bare Divergence

ZB, Cheung, Chi, Davies, Dixon and Nohle (to appear)



$$209 = 11 \cdot 19$$

Integrating we obtain:

$$3431 = 47 \cdot 73$$

$$\mathcal{M}_4^{2\text{-loop}}\Big|_{\text{bare div.}} = -\frac{1}{\epsilon}\,\frac{3431}{5400}\,\mathcal{K} \qquad \mathcal{K} = \left(\frac{\kappa}{2}\right)^6 \frac{i}{(4\pi)^4}\, stu \left(\frac{[12][34]}{\langle 12\rangle\langle 34\rangle}\right)^2$$

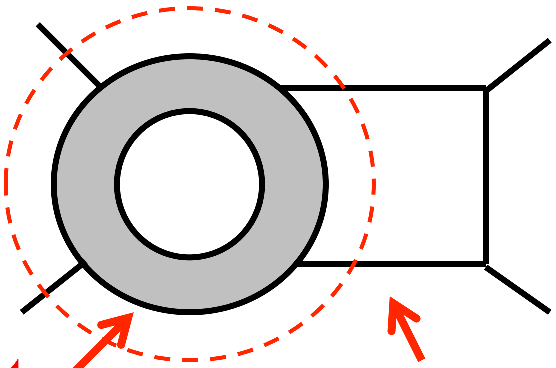
Not the same as the Goroff and Sagnotti result

However, Goroff and Sagnotti subtracted subdivergences integral by integral.

Subdivergences? What subdivergences?
There are no one-loop divergences. Right?

Subdivergences?

The integrand
has subdivergences



Representative diagram.

Gauss-Bonnet
subdivergence

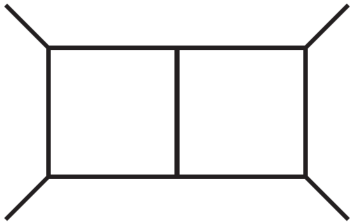
$D = 4$, no subdivergences
$D \neq 4$, subdivergences!

A strange phenomenon: no one loop divergences,
yet there are one-loop subdivergences!

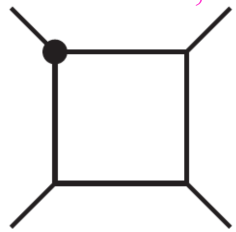
- To match the G&S result we need to subtract subdivergences.
- We use counterterm method.

Two Loop Identical Helicity Divergence

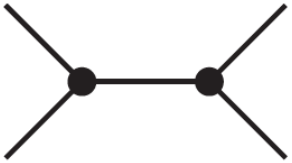
ZB, Cheung, Chi, Davies, Dixon and Nohle (to appear)



2 loop bare



single GB counterterm



double GB counterterm:

$$\mathcal{M}_4^{\text{2-loop}}\Big|_{\text{div.}} = -\frac{1}{\epsilon}\,\frac{3431}{5400}\,\mathcal{K}$$

$$\mathcal{M}_4^{\text{1-loop GB}}\Big|_{\text{div.}} = \frac{1}{\epsilon}\,\frac{689}{675}\,\mathcal{K}$$

$$\mathcal{M}_4^{\text{tree GB}^2}\Big|_{\text{div.}} = \frac{1}{\epsilon}\,\frac{5618}{675}\,\mathcal{K}$$

$$\mathcal{M}_4^{\text{total}}\Big|_{\text{div.}} = \frac{1}{\epsilon}\,\frac{209}{24}\,\mathcal{K}$$

Goroff and Sagnotti divergence reproduced

GB counterterm contributes at two loops even in flat space.

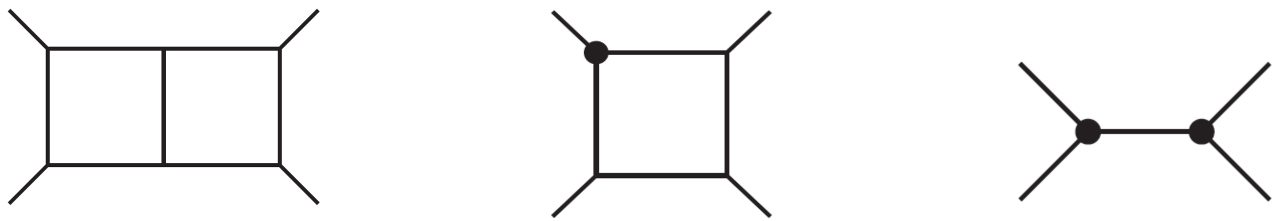
Conformal anomaly plays central role in divergence.

Meaning of Divergence?

What does the divergence mean?

Adding 3 form field offers good way to understand this: $A_{\mu\nu\rho}$

- **On the one hand, no degrees of freedom in *D* = 4, so no change in divergence expected.**
- **On the other hand, the conformal anomaly is affected, so expect change in divergence.**
- **Note that 3 form proposed as way to dynamically neutralize cosmological constant.** Brown and Teitelboim; Bousso and Polchinski



Pure gravity $$\mathcal{M}_4^{\text{total}}\Big|_{\text{div.}} = \frac{1}{\epsilon}\frac{209}{24}\mathcal{K}$$

Gravity + 3 form $$\mathcal{M}_4^{\text{total}}\Big|_{\text{div.}} = \frac{1}{\epsilon}\frac{29}{24}\mathcal{K}$$

Divergence differs!

No extra dynamical degrees of freedom.

Divergences Differ Under Dualities

ZB, Cheung, Chi, Davies, Dixon and Nohle (to appear)

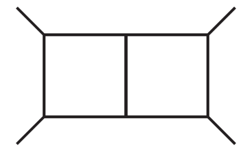
Single scalar or anti-symmetric tensor coupled to gravity.

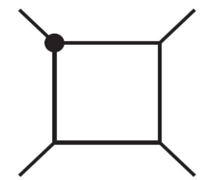
$$\mathcal{L}_{gd} = \left(\frac{2}{\kappa^2}R + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi\right) \qquad \mathcal{L}_{ga} = \left(\frac{2}{\kappa^2}R + \frac{1}{6}H_{\mu\nu\rho}H^{\mu\nu\rho}\right)$$

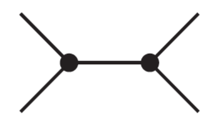
Related by *D* = 4 duality transformation $\partial_\mu\phi = \varepsilon_{\mu\nu\rho\sigma}H^{\nu\rho\sigma}$

Coefficient of $1/\epsilon$

theory	gd	ga
bare divergence	$-\frac{793}{1200}$	$\frac{2027}{1200}$
GB c.t. in 1 loop	$\frac{4\cdot 53+1}{360}\cdot\frac{2\cdot(13-1)}{15}$	$\frac{4\cdot 53+91}{360}\cdot\frac{2\cdot(13-91)}{15}$
GB^2 c.t. in tree	$24\left(\frac{4\cdot 53+1}{360}\right)^2$	$24\left(\frac{4\cdot 53+91}{360}\right)^2$
RHH c.t. in 1 loop	0	$\frac{1}{4}\cdot 20$
total	$\frac{139}{16}$	$\frac{239}{16}$







UV divergences altered by duality transformations

UV result suggests that theories are quantum mechanically *in*equivalent as proposed by Duff and van Nieuwenhuizen.

But wait: what about finite parts?

Scattering Amplitudes

Pure Gravity:

IR singularities subtracted and independent of 3 form

$$\mathcal{M}^{(2)}(1^+,2^+,3^+,4^+) = \mathcal{N}\Big\{\Big(\frac{1}{\epsilon}\frac{209}{24} + \frac{117617}{21600}\Big)stu + \frac{1}{10}stu\log(-s) - \frac{1}{60}s^3\log(-s) + \frac{1}{120}(s^2+t^2+u^2)s\log(-s)^2 + \text{perms}\Big\}$$

Gravity + 3 Form:

$$\mathcal{M}^{(2)}(1^+,2^+,3^+,4^+) = \mathcal{N}\Big\{\Big(\frac{1}{\epsilon}\frac{29}{24} + \frac{411617}{21600}\Big)stu + \frac{1}{10}stu\log(-s) - \frac{1}{60}s^3\log(-s) + \frac{1}{120}(s^2+t^2+u^2)s\log(-s)^2 + \text{perms}\Big\}$$

divergences different.
logarithms identical.

- Value of divergence not physical. Absorb into counterterm.
- 3 form is a Cheshire Cat field: scattering unaffected.

Similar results comparing *gd* to *ga* theories.

These results consistent with quantum equivalence under duality.

Renormalization Scale Dependence

If divergences depend on field representation choice then what is meaningful?

Renormalization scale: log(M^2) controls scaling behavior.

For single antisymmetric tensor field or dilaton or adding 3 forms for two-loop four-graviton amplitude:

$$\mathcal{M}_4^{\text{2-loop}} = -\frac{N_s}{8} \log(M^2)\, \mathcal{K} + \cdots$$

N_s **is number of states in the theory. Vanishes for susy theories.**

Divergences differ under duality but scaling behavior identical.

- **Note the simplicity of the number! No weird prime numbers.**
- **Coefficient of log(M^2) is a better measure of the divergence properties and is the proper quantity to study.**
- **Discrepancy between divergence and scaling behavior might offer clues to interesting UV completions.**

Summary

- Examples of enhanced UV cancellations. UV better than expected based on standard symmetry considerations.
- Reproduced result from Goroff and Sagnotti on 2 loop divergence of pure gravity. Conformal anomaly enters.
- Entire pure gravity divergence has anomaly-like 0/0 behavior, connected to Bardeen & Cangemi's observations.
- UV divergences depend on duality transformations and addition of nondynamical 3 forms. Renormalization scale better to look at.
- Results consistent with quantum equivalence under duality: Logs identical. Log(M^2) identical.

➢ UV properties of supergravity theories are *still* an open problem.

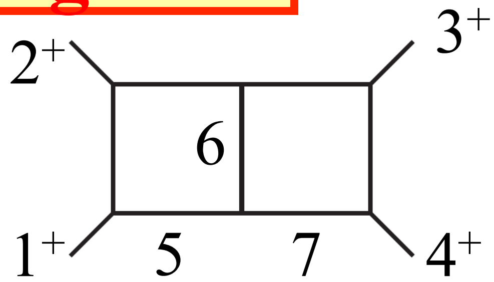
➢ The UV properties of gravity theories are full of subtleties and interesting twists.

➢ Expect many more surprises as we probe gravity theories using modern perturbative tools.

Extra Slides

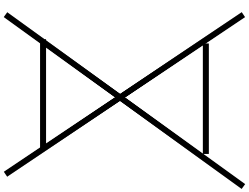
Full Two-Loop Integrand

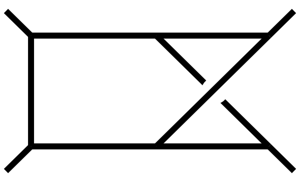
Using spinor helicity very compact:



$$n = \frac{D_s(D_s-3)}{2}(\lambda_p^2\lambda_q^2 + \lambda_p^2\lambda_{p+q}^2 + \lambda_q^2\lambda_{p+q}^2)^2 - \frac{D_s(D_s-6)}{2}\lambda_p^2\lambda_q^2\lambda_{p+q}^2(\lambda_p^2 + \lambda_q^2 + \lambda_{p+q}^2)$$
$$+ 12D_s((\lambda_p \cdot \lambda_q)^2 - \lambda_p^2\lambda_q^2)(\lambda_p^2\lambda_q^2 + \lambda_p^2\lambda_{p+q}^2 + \lambda_q^2\lambda_{p+q}^2) + 144((\lambda_p \cdot \lambda_q)^2 - \lambda_p^2\lambda_q^2)^2\,,$$

Bow-tie and nonplanar contributions similar: $p_i = p_i^{(4)} + \lambda_i$





- **Integrand vanishes for *D* = 4 loop momenta.**
- **Upon integration ultraviolet divergent.**