Surviving in a Metastable de Sitter Space-Time

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(Sitender Kashyap, Swapnamay Mondal, A.S., Mritunjay Verma)

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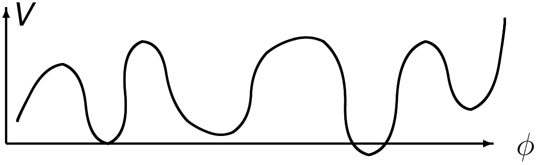
Motivation

String landscape tells us that in string theory we have many Minkowski, AdS and de Sitter vacua.

Bousso, Polchinski; Giddings, Kachru, Polchinski; Kachru, Kallosh, Linde, Trivedi; $\cdots$

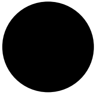


The discovery of accelerated expansion of the universe indicates that we live in a de Sitter vacuum.

Conclusion: Our vacuum is metastable

In a metastable vacuum there is certain probability per unit time per unit volume of producing a microscopic bubble of more stable vacuum.

Kobzarev, Okun, Voloshin; Stone; Frampton; Coleman; Callan, Coleman; Coleman, de Luccia



Once formed, the bubble wall expands with speed aproaching the speed of light and engulfs everything within its reach.

K: probability of producing a bubble per unit time per unit volume.

K can in principle be calculated from the microscopic theory.

But in absence of our knowledge of where we are in the landscape of vacua, we cannot calculate K at present.

The fact that we have survived for 1.4×10^{10} years suggests indirectly that our half-life is at least of order 10^{10} years.

$\Rightarrow$ **The chance of our being engulfed by such a bubble during the next year could be as large as 1 in 10^{10}.**

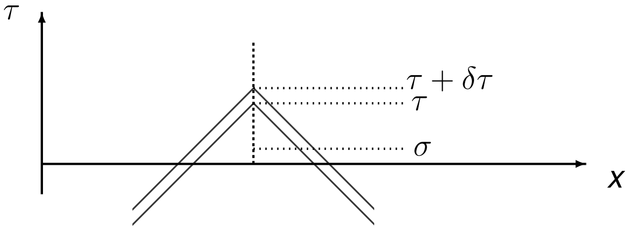
What can we do about this?

Vacuum decay in a metastable FRW space-time

$$\begin{aligned} \mathbf{ds^2} &= -\mathbf{dt^2} + \mathbf{a(t)^2(dx^2 + dy^2 + dz^2)} \\ &= \mathbf{a^2(-d\tau^2 + dx^2 + dy^2 + dz^2)} \end{aligned}$$

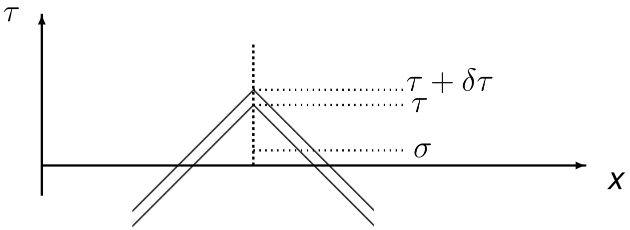
τ**: conformal time,** $\quad \mathbf{d\tau \equiv dt/a(t)}$

Consider a comoving object sitting at the origin.



The probability of the object being hit by a bubble between τ and $\tau + \delta\tau$ is

K $\times$ 4-volume of past light cone between τ and $\tau + \delta\tau$



P(τ): survival probability of an object up to conformal time τ

$$-\delta\tau \frac{\mathbf{d}}{\mathbf{d}\tau}\mathbf{P}(\tau) = \mathbf{P}(\tau) \times \mathbf{K} \times \delta\tau \int_{-\infty}^{\tau} \mathbf{d}\sigma\, \mathbf{a}(\sigma)^{\mathbf{4}}\, \mathbf{4}\pi\, (\tau - \sigma)^{\mathbf{2}}$$

$$\begin{aligned} -\frac{\mathbf{d}}{\mathbf{dt}}\mathbf{P} &= \mathbf{P} \times \mathbf{a}(\tau)^{-\mathbf{1}} \times \mathbf{K} \times \int_{-\infty}^{\tau} \mathbf{d}\sigma\, \mathbf{a}(\sigma)^{\mathbf{4}}\, \mathbf{4}\pi\, (\tau - \sigma)^{\mathbf{2}} \\ &\equiv \mathbf{D}(\mathbf{t})\, \mathbf{P} \end{aligned}$$

D(t): decay rate

Example: de Sitter

$$\mathrm{ds}^2 = -\mathrm{dt}^2 + \mathrm{e}^{2\mathrm{t}}(\mathrm{dx}^2 + \mathrm{dy}^2 + \mathrm{dz}^2) = \tau^{-2}(-\mathrm{d}\tau^2 + \mathrm{dx}^2 + \mathrm{dy}^2 + \mathrm{dz}^2)$$

$$\tau = -\mathrm{e}^{-\mathrm{t}}, \qquad -\infty < \tau < 0$$

Note: We have set the Hubble constant to 1.

In our universe this corresponds to setting unit of distance / time to about

$$1.7 \times 10^{10} \text{ light years / years}$$

– will be used throughout this talk.

de Sitter metric

$$\mathbf{ds^2} = \tau^{-2}(-\mathbf{d}\tau^2 + \mathbf{dx^2} + \mathbf{dy^2} + \mathbf{dz^2})$$

$$\Rightarrow \quad \mathbf{a}(\tau) = \tau^{-1}$$

$$\begin{aligned}
\mathbf{D(t)} &= \mathbf{a}(\tau)^{-1} \times \mathbf{K} \times \int_{-\infty}^{\tau} \mathbf{d}\sigma\, \mathbf{a}(\sigma)^4\, 4\pi\, (\tau - \sigma)^2 \\
&= \tau \times \mathbf{K} \times \int_{-\infty}^{\tau} \mathbf{d}\sigma\, \sigma^{-4} \times 4\pi\, (\tau - \sigma)^2 \\
&= 4\pi\mathbf{K}/3 = 1/\mathbf{T}
\end{aligned}$$

$$\mathbf{T} \equiv \frac{3}{4\pi\mathbf{K}}$$

$$\frac{d}{dt}P(t) = -D(t)P(t) = -\frac{1}{T}P$$

$$P(t) = e^{-t/T} \quad \text{if} \quad P(t=0) = 1$$

Life-expectancy $\equiv$ average time at which it decays

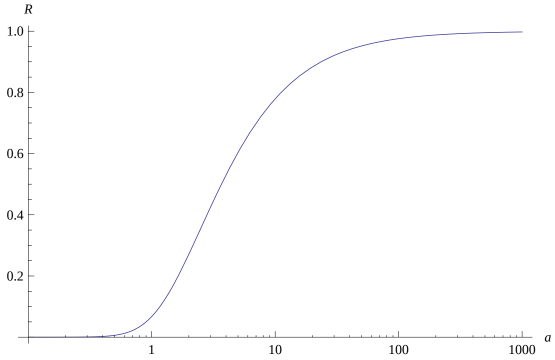
$$\int_0^\infty t\, D(t)\, P(t)\, dt = -\int_0^\infty t\, \frac{d}{dt}P(t) = \int_0^\infty P(t)dt = T$$

We can now repeat the analysis for FRW metric with matter + cosmological constant with the current value

matter density / dark energy density $\simeq .45$

Result: D(t) increases with time.

Decay rate vs. scale factor in FRW space-time ($a_{today} = 1$)



$R(t) \equiv T\,D(t)$: the decay rate normalized to 1 at $t = \infty$.

Today's decay rate = 3.7 times the average rate in the past

Decay rate as $t \to \infty$ = 56 times the average rate in the past

Note: In the absence of cosmological constant the decay rate would continue to grow as t^3

$P(t) \propto \exp[-c\, t^4]$ for some constant c.

In the presence of cosmological constant $P(t) \propto e^{-t/T}$

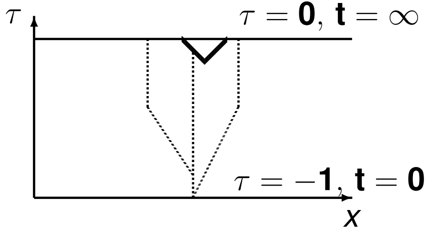
A metastable universe with cosmological constant is a far safer place to live than one without.

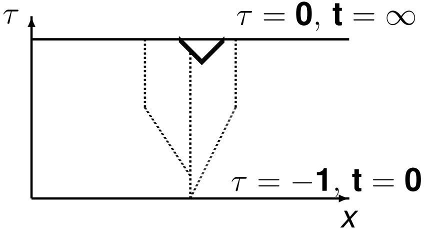
What we have described so far is inevitable, beyond our control.

Can we do something to improve the situation?

Strategy: Spread out in space and let the Hubble expansion pull the different settlements outside each others horizon.

In that case a single bubble may not be able to destroy all the settlements.





Some will live on when others die

– would increase the collective life expectancy – defined as the average time at which the last settlement will undergo vacuum decay.

Take two comoving objects in de Sitter space separated by some distance r.

Assume that both exist at t=0.

$P_1(t), P_2(t)$: survival probability of objects 1 and 2 till time t.

$P_{12}(t_1, t_2)$: joint survival probability of the first object till time t_1 and the second object till time t_2.

The probability that at least one survives till time t is

$$P_1(t) + P_2(t) - P_{12}(t, t)$$

Collective life expectancy

$$-\int_0^\infty t\, dt\, \frac{d}{dt}(P_1(t) + P_2(t) - P_{12}(t,t)) = \int_0^\infty dt\, (P_1(t) + P_2(t) - P_{12}(t,t))$$

Collective life expectancy

$$\int_0^\infty dt\,(P_1(t) + P_2(t) - P_{12}(t,t))$$

If the decays had been independent, then

$$P_1 = P_2 = e^{-t/T}, \quad P_{12} = P_1 P_2 = e^{-2\,t/T}$$

This gives collective life expectancy of

$$\frac{3}{2}T$$

For n objects undergoing independent decay the collective life expectancy is

$$T\left(1 + \frac{1}{2} + \frac{1}{3} + \cdots \frac{1}{n}\right)$$

In practice the decays are not independent and we have to do more detailed study.

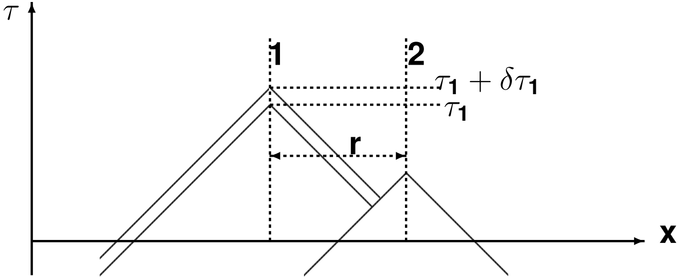
$P_{12}(t_1, t_2)$: joint probability that the first object survives till t_1 and the second object survives till time t_2.

Joint probability that the first object decays between t_1 and $t_1 + dt_1$ and the second object survives till t_2 is

$$-dt_1 \frac{\partial}{\partial t_1} P_{12}(t_1, t_2)$$

$-dt_1 \frac{\partial}{\partial t_1} P_{12}(t_1, t_2)$: probability that the first object decays between t_1 and $t_1 + dt_1$ and the second object survives till t_2.

– given by $P_{12} \times K \times$ the volume of past light-cone of 1 between τ_1 and $\tau_1 + d\tau_1$ outside the past light-cone of 2 at τ_2.



– a simple geometric problem.

Steps:

1. Calculate $-\frac{\partial}{\partial t_1} P_{12}(t_1, t_2)$ by calculating the volume of the shell.

2. Integrate this with boundary condition

$$P_{12}(0, t_2) = P_2(t_2)$$

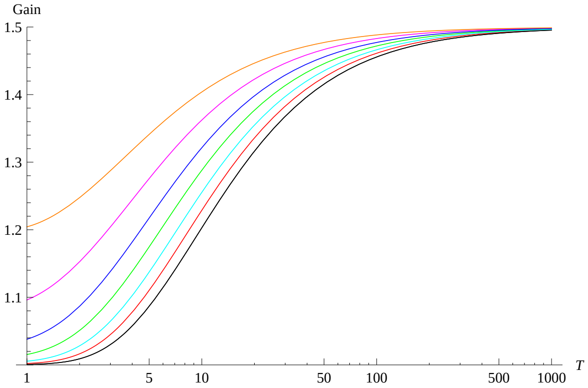
$\Rightarrow P_{12}(t_1, t_2)$ in terms of $P_2(t_2)$.

3. Calculate $P_1(t_1)$ and $P_2(t_2)$ similarly

4. Calculate collective life expectancy from

$$\int_0^\infty dt \, (P_1(t) + P_2(t) - P_{12}(t, t))$$

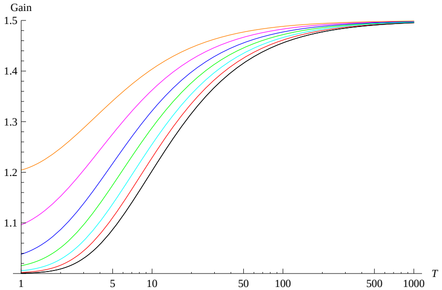
Result:



x-axis: T (inverse decay rate of single object)

y-axis: Gain (ratio of collective life expectancy of two objects to that of a single object)

r= .0003, .001, .003, .01, .03, .1 and .3



r= .0003, .001, .003, .01, .03, .1 and .3

unit: 1.7×10^{10} years / light years

r $= .0003$ corresponds to $\sim 5 \times 10^6$ light years

– minimum distance needed to escape gravitational binding to the local cluster

Making a single copy at this distance we can increase life expectancy by a factor close to 1.5 (1.2) if decay rate is 1% (10%) of the expansion rate.

General Lessons

'The possibility that we are living in a false vacuum has never been a cheering one to contemplate.'

Colemann, De Luccia

But if we are destined to live in a false vacuum, we are far safer in a vacuum with cosmological constant than in one without it.

1. It tames the growth of the decay rate from t^3 to a constant.

2. It gives us an opportunity to fight back and increase our collective life expectancy by spreading out.

Role of string theory

Computation of the inverse decay rate $T = 3/(4\pi K)$ is highly UV sensitive.

A bottom up approach is almost useless.

String theory, being a microscopic theory, has the possibility of calculating this number.

1. Either find our position in the landscape, or

2. develop a statistical argument showing that for a vast majority of the vacua that resemble ours,T lies within a narrow range.

There have been some attempts to develop a statistical argument.

Clifton, Shenker, Sivanandam; Dine, Festuccia, Morisse, van den Broek; · · ·

But it is probably fair to say that we do not yet have robust results based on which we can make our survival plans.

It is not going to be an easy task.

But given its potential effect on the plans for our future, it will be worth making the effort.