Geometry of 6D SCFTs

Jonathan J. Heckman

UNC Chapel Hill

Based On

Work with:

D. R. Morrison, T. Rudelius and C. Vafa

as well as:

M. Del Zotto, C. P. Herzog, D. S. Park, A. Tomasiello

See also talks by: C. Vafa, K. Intriligator, J. Park
& T. Rudelius

Why Study 6D SCFTs?

- Nahm: Maximal SCFT dimension is *six*

- Degrees of freedom $\neq$ particles (but it's a QFT!)

- QFT of M5-branes is a 6D SCFT

- Compactification $\Rightarrow$ 5D/4D/3D/2D Theories

Focus: $(1,0)$ SCFTs

Conformal Symmetry: $\mathfrak{so}(6,2)$

Supersymmetry: 8 Q's and 8 S's

R-symmetry: $\mathfrak{su}(2)_{\mathcal{R}}$

Studied since the 1990's

Many groups:

Witten '95; Strominger '95; Ganor and Hanany '96;

Seiberg and Witten '96; Bershadsky and Johansen '96;

Brunner and Karch '96; Blum and Intriligator '97;

Intriligator '97; Hanany Zaffaroni '97;

+ ....

But: Even now, still viewed as "mysterious"...

Results

Punchline #1

Systematically Classify 6D SCFTs

Construction is "Top Down" (via F-theory)

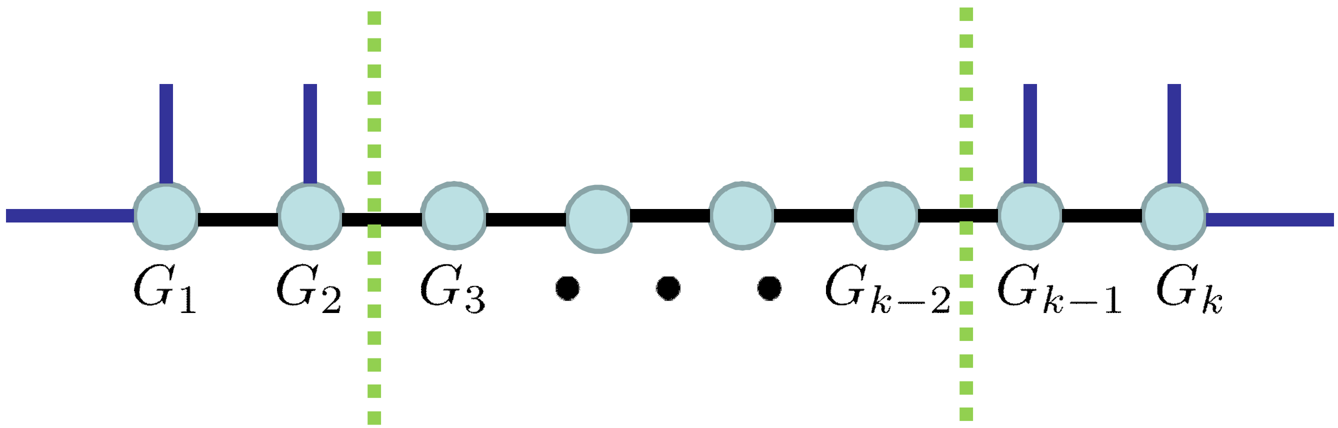
Far stronger than just "alot of examples"

Punchline #2

Nearly all top down conditions

Can be phrased in bottom up terms

Punchline #3



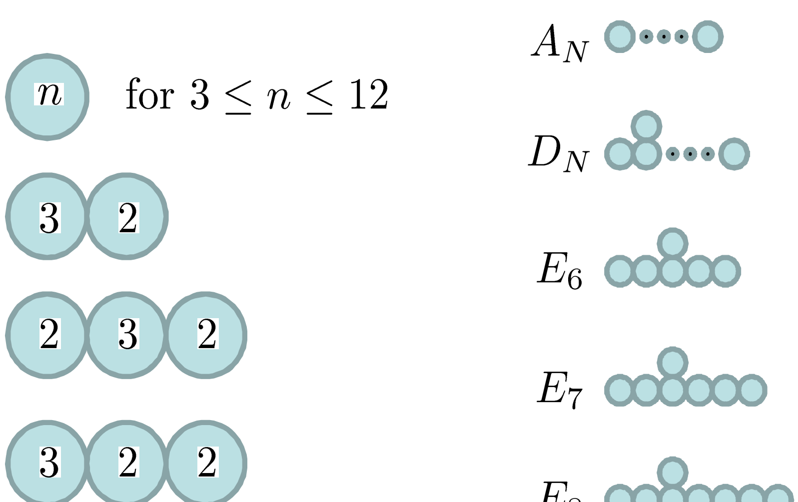
6D SCFTs = Generalized Quivers

Punchline #4

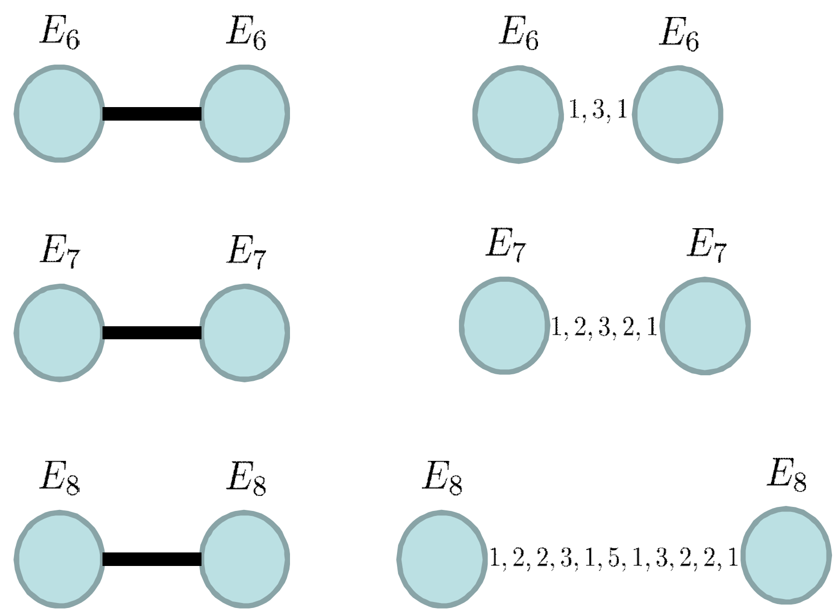
Looks Like Chemistry

“Atoms”

c.f. Morrison and Taylor ’12



“Radicals”



Punchline #5

(See T. Rudelius' Talk for Details)

Homomorphisms $\Gamma_{ADE} \to E_8$

$\Updownarrow$

Specific Class of F-theory 6D SCFTs

Plan of the Talk

- How to build a 6D SCFT
- Classification
- RG Flows

How to Build a 6D SCFT

Example: All $(2,0)$ Theories

Witten '95, Strominger '95

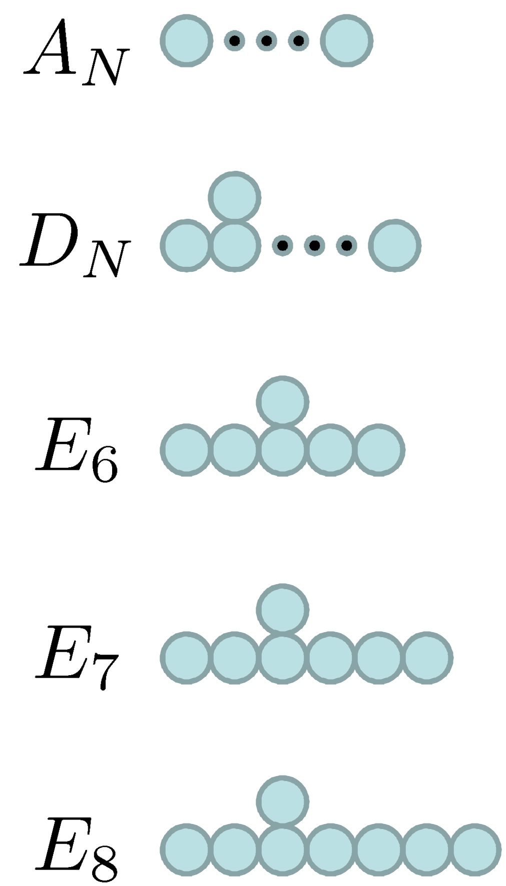
Type IIB on $\mathbb{C}^2/\Gamma_{ADE}$

Resolution Involves:

Bouquet of $\mathbb{CP}^1$'s

$\mathbb{CP}^1_i \cap \mathbb{CP}^1_j = -\mathrm{Dynkin}_{ij}$

Note: $\mathbb{CP}^1_i \cap \mathbb{CP}^1_i = -2$



6D Theories and F-theory

Vafa '96, Vafa Morrison, I/II '96

All known 6D theories have F-theory avatar

IIB: $\mathbb{R}^{5,1} \times B_2$ with pos. dep. coupling $\tau(z_B)$

F-theory on $\mathbb{R}^{5,1} \times CY_3$

$$\begin{array}{ccc} T^2 & \rightarrow & CY_3 \\ & & \downarrow \\ & & B_2 \end{array}$$

Tensionless Strings in F-theory

- Realized by D3-brane on collapsing $\mathbb{CP}^1$

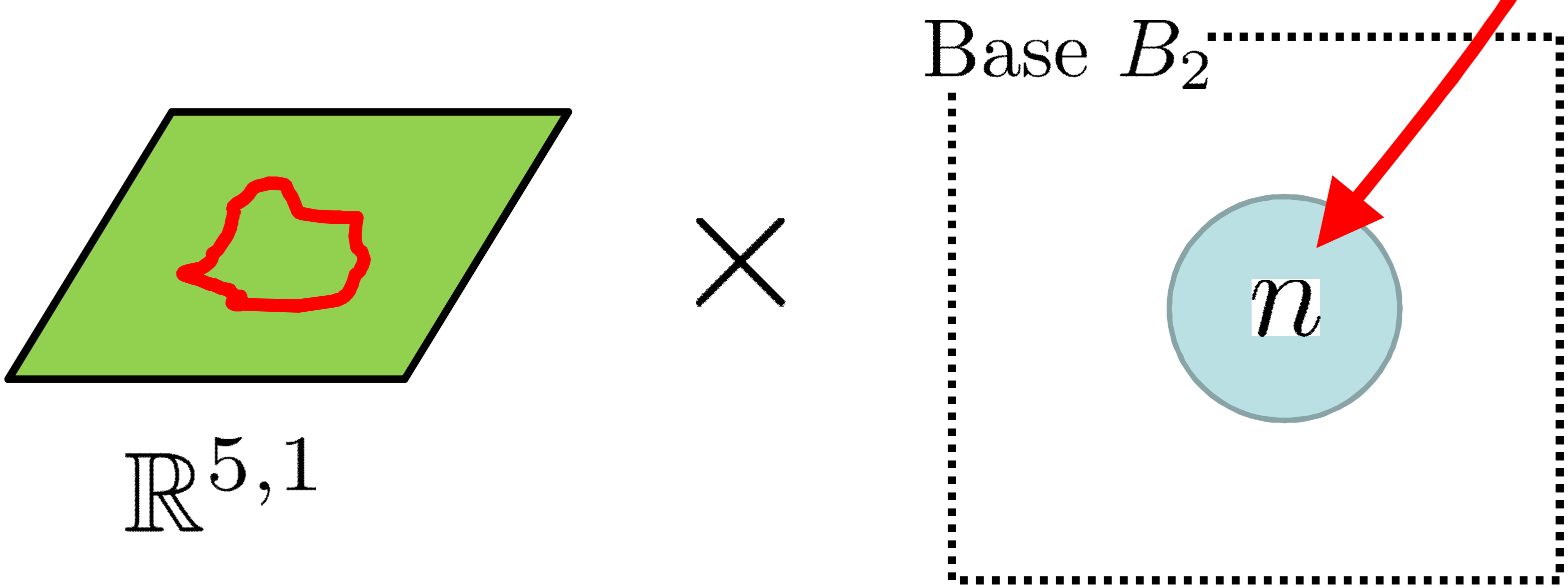
  Tension $= \mathrm{Vol}(\mathbb{CP}^1) \to 0$

	$\mathbb{R}^{5,1}$						$\mathbb{CP}^1$			
	0	1	2	3	4	5	6	7	8	9
D3	$\times$	$\times$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\times$	$\times$	$\bullet$	$\bullet$

Strings from D3 on a $\mathbb{P}^1$

$$-\Sigma \cap \Sigma = \text{String Charge}$$

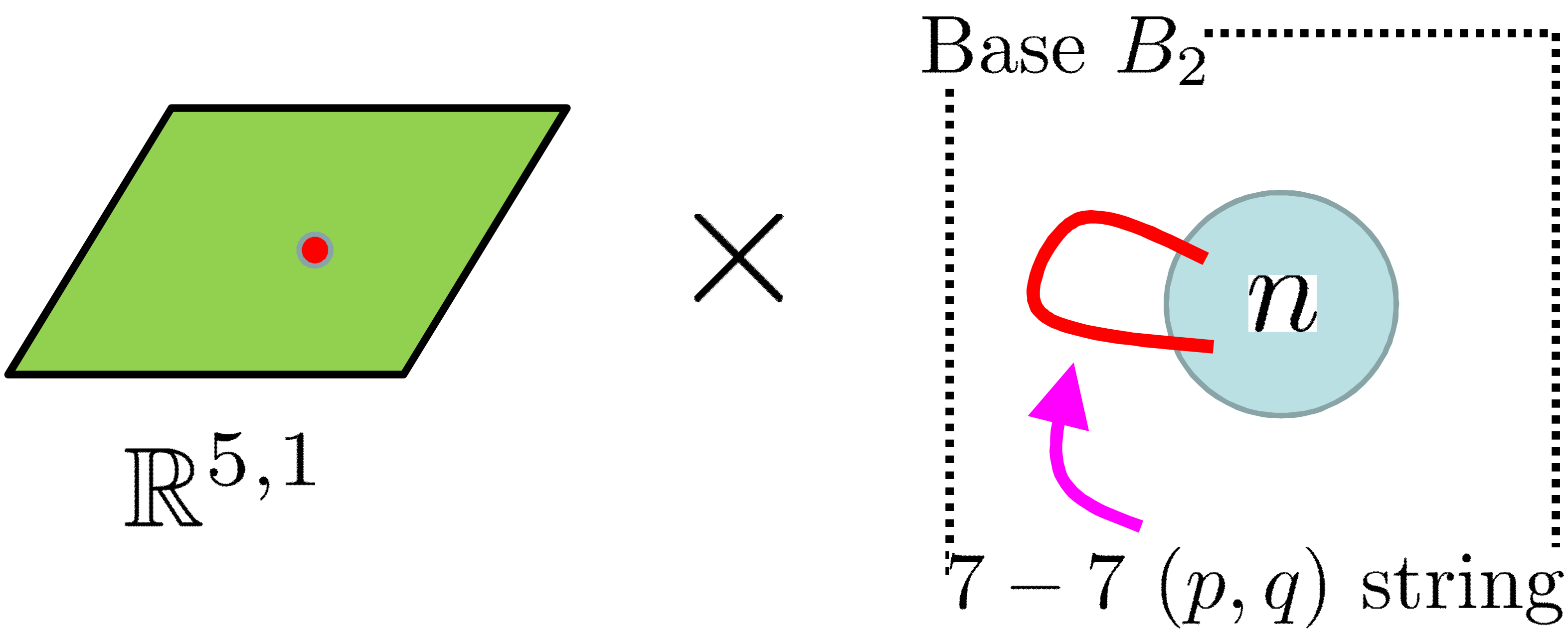
(which must be integer > 0)



Particles from D7's on a $\mathbb{P}^1$

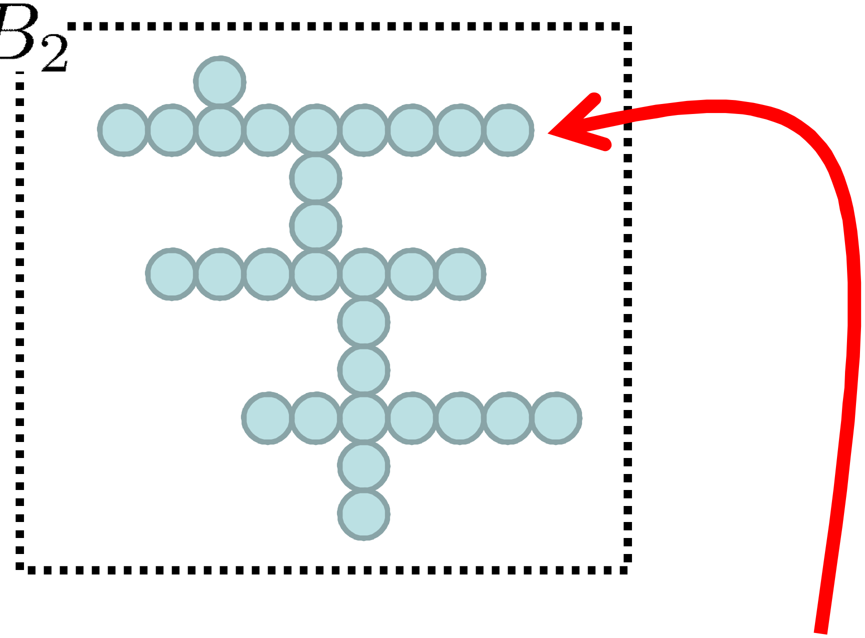
$3 \leq n \leq 12 \Rightarrow$ always have gauge fields

(elliptic fiber is singular: Morrison Taylor '12)



Geometric Picture

Base B_2

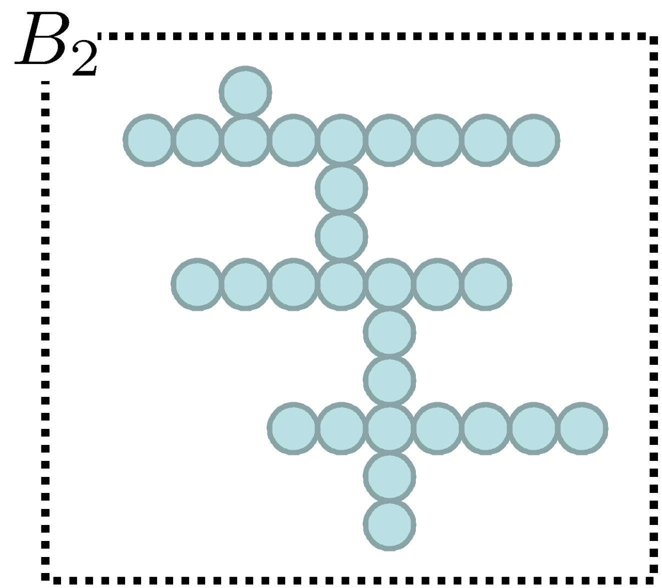


Singularities in base $\Rightarrow$ strings (D3 / $\mathbb{P}^1$)

Singularities in fiber $\Rightarrow$ particles ($7-7'$ strings)

SCFT Limit

Start: A smooth base B_2



End: To get a CFT, sim. contract curves of B_2

Two Deformation Types

Complex Structure Deformation / Higgs Branch

Brane Recombination

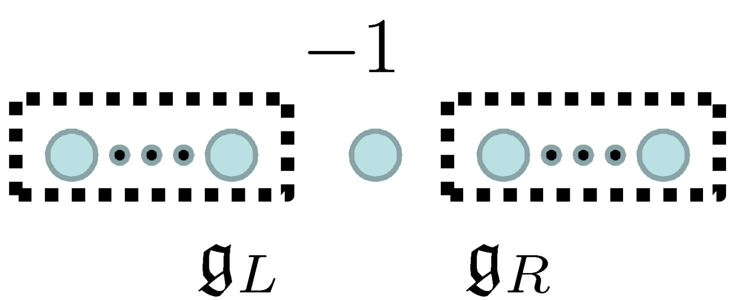
Expand a curve in base to large size

Go to large tension / weak gauge coupling

Building a Base

In base B_2, "gluing" of building blocks:

classified by Morrison and Taylor '12 (see also JJH Morrison Vafa '13)

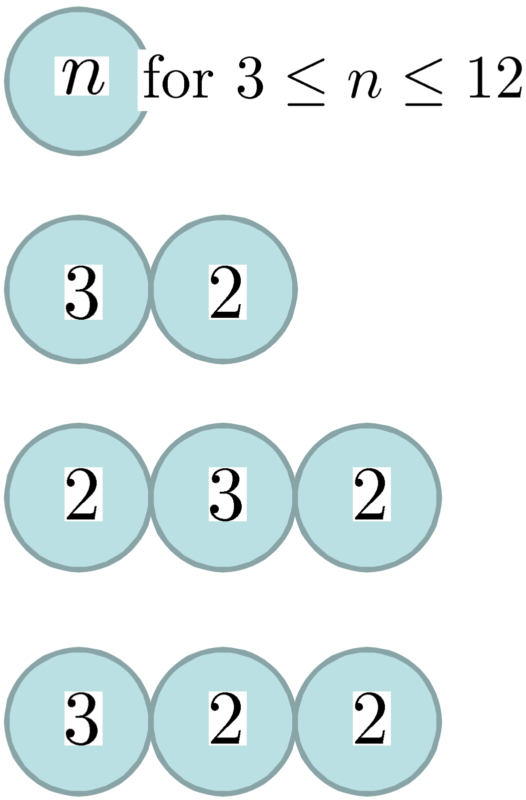


$$\mathfrak{g}_L \times \mathfrak{g}_R \subset \mathfrak{e}_8$$

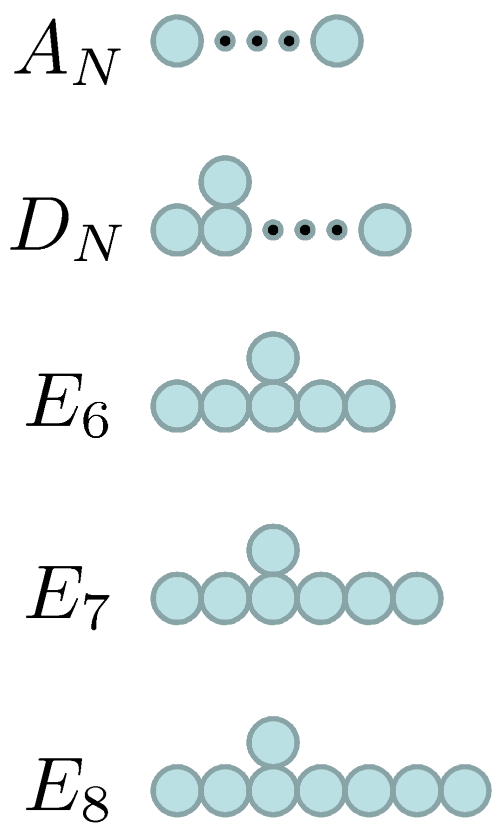
c.f. Morrison and Taylor '12

Building Blocks

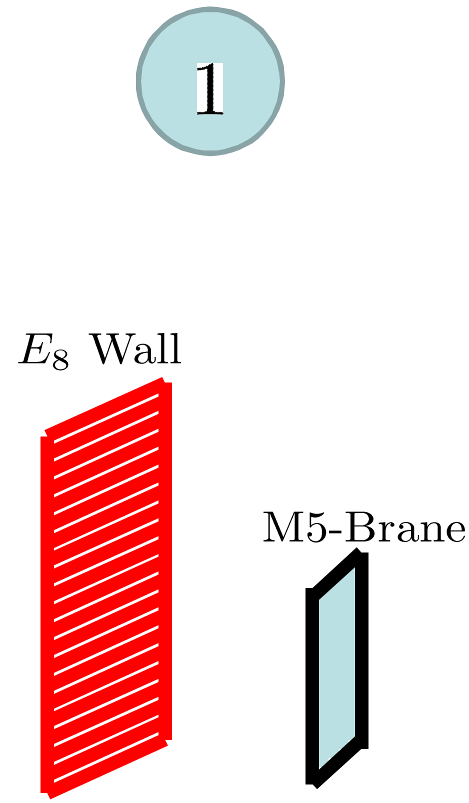
“Non-Higgsable Clusters”



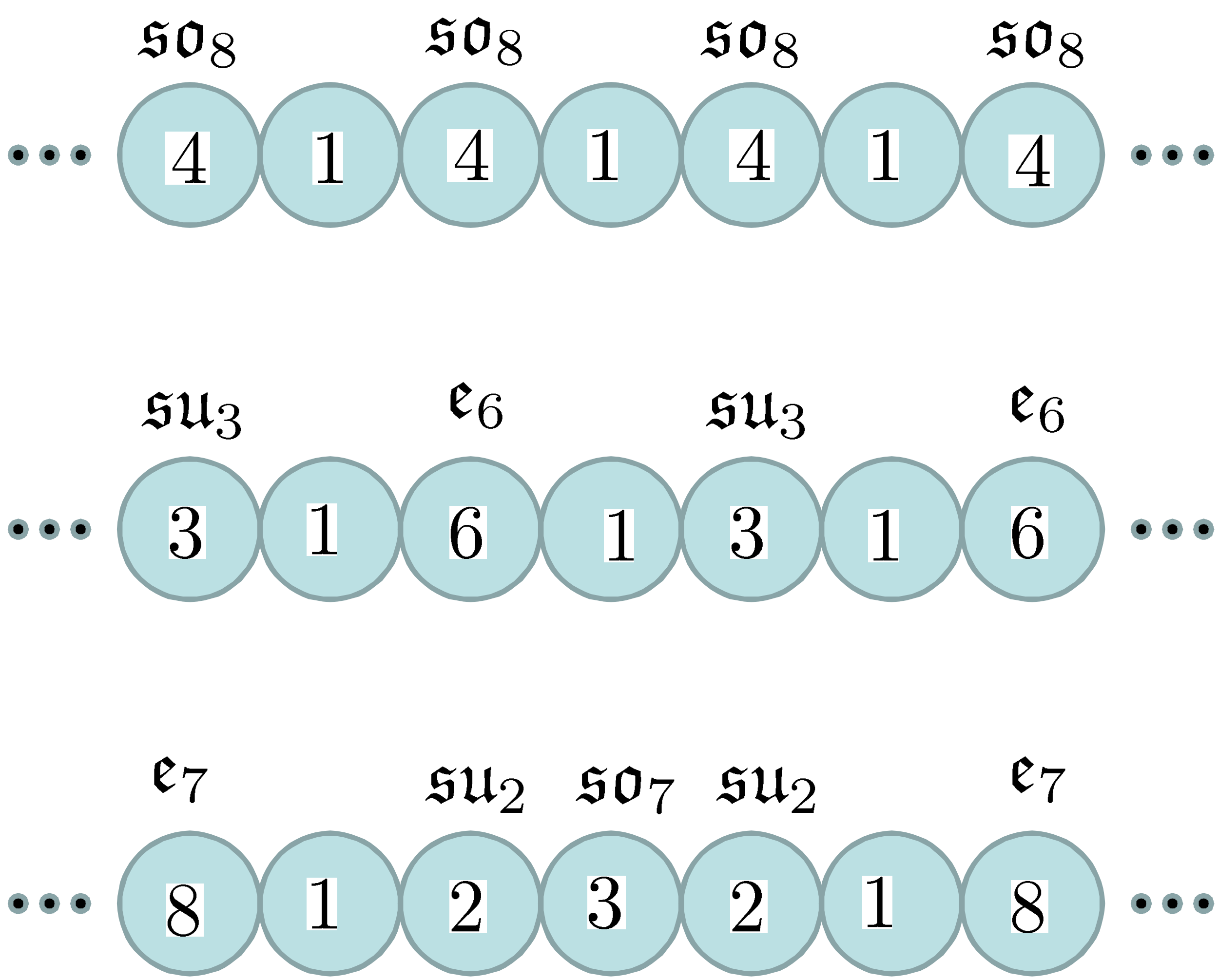
$(2,0)$ Theories



E-String Theory

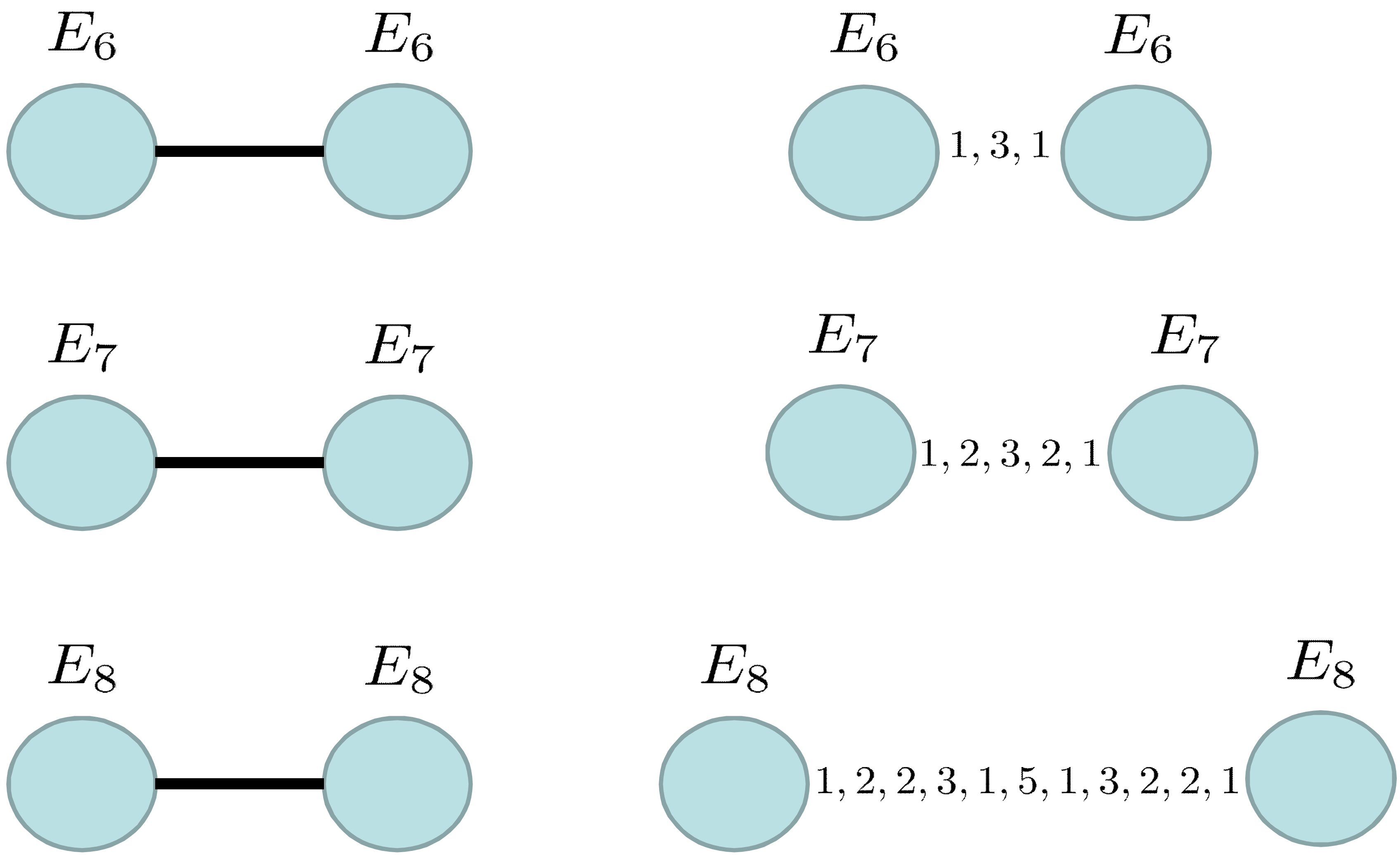


Examples

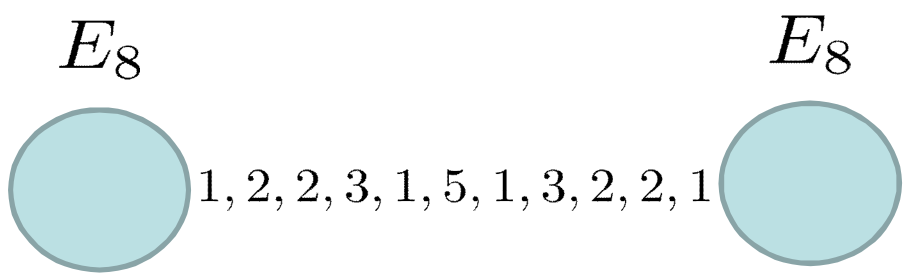


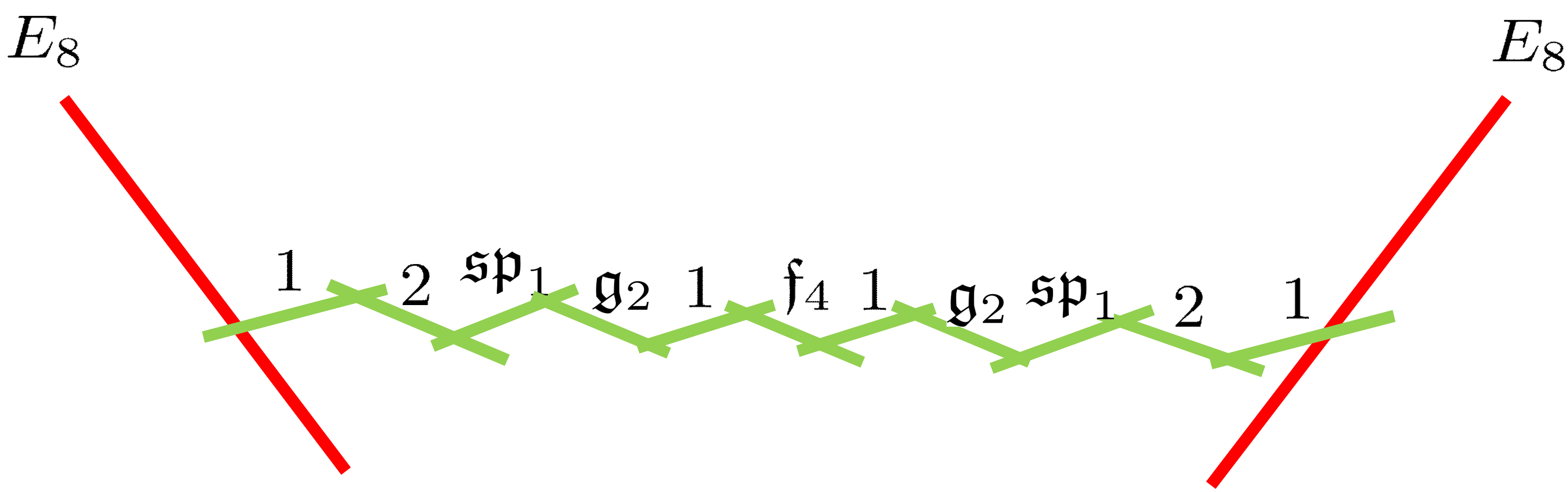
$\mathfrak{so}_8$ $\mathfrak{so}_8$ $\mathfrak{so}_8$ $\mathfrak{so}_8$
4 1 4 1 4 1 4
$\mathfrak{su}_3$ $\mathfrak{e}_6$ $\mathfrak{su}_3$ $\mathfrak{e}_6$
3 1 6 1 3 1 6
$\mathfrak{e}_7$ $\mathfrak{su}_2$ $\mathfrak{so}_7$ $\mathfrak{su}_2$ $\mathfrak{e}_7$
8 1 2 3 2 1 8

E-Type Quivers



The Link is *also* an SCFT!





Classification

Top Down Approach

In 6D, things are quite rigid...

Can we enumerate every possible theory?

¡Yes!

Evidence

I) Systematic geometric classification

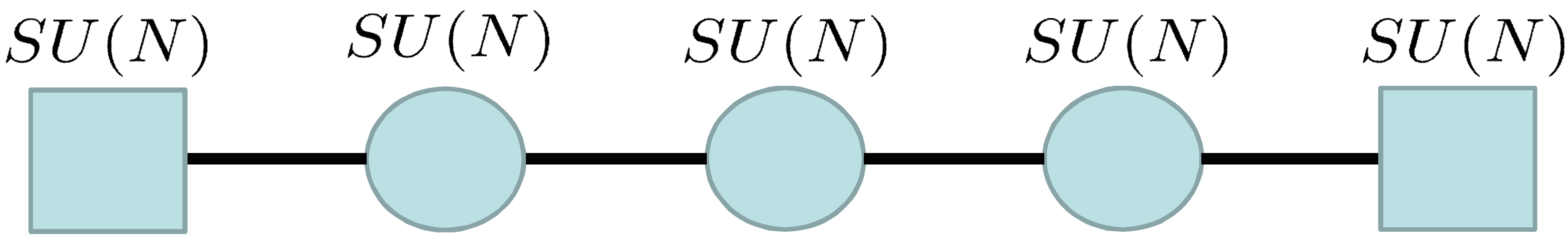
In F-theory, everything is geometric

II) All field / string constructions subsumed

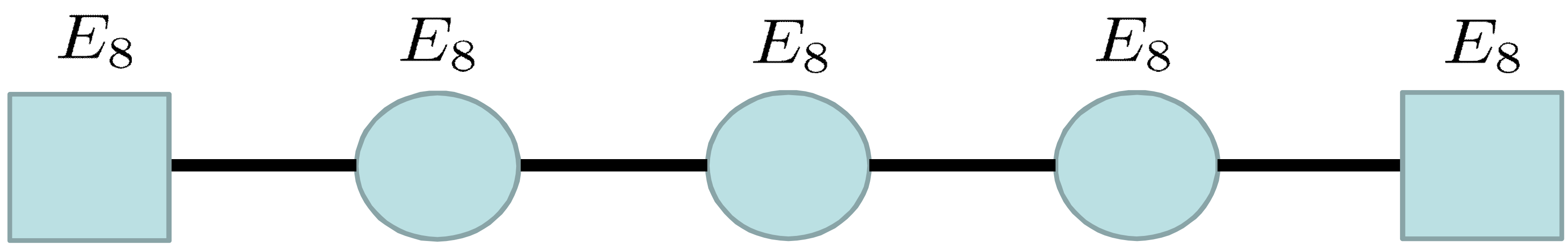
Example: All "classical quivers"

Example: Classification of $\mathrm{Hom}(\Gamma_{ADE}, E_8)$ and small instantons of $\mathbb{C}^2/\Gamma_{ADE}$

Example: Quivers



But Also



Example: Small Instantons

(see T. Rudelius' talk for details)

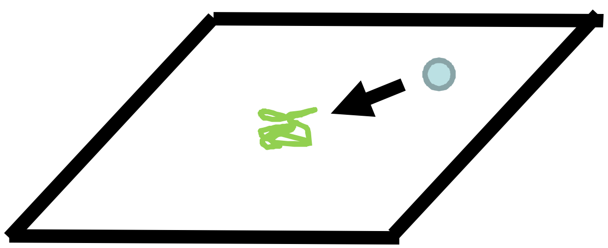
c.f. Aspinwall Morrison '97, Del Zotto JJH Tomasiello Vafa '14

JJH Morrison Rudelius Vafa '15

Heterotic String:

E_8 Small Instantons

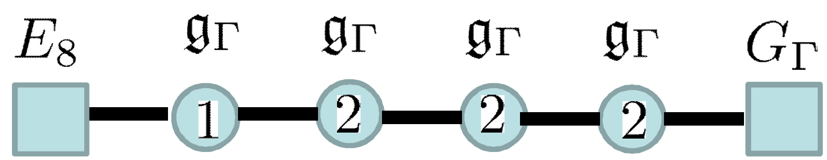
on $\mathbb{C}^2/\Gamma_{ADE}$



F-theory:

Small Instantons

on $\mathbb{C}^2/\Gamma_{ADE}$



Boundary Conditions

$\mathrm{Hom}(\Gamma_{ADE}, E_8)$

Specific CY_3's!!!

Strategy

1) Find all Bases which could support an SCFT

2) Find all Ways to Wrap 7-Branes

Useful Terminology: I / II

Split Up NHCs into two groups: $\mathbf{I}^l = 1, 2, ..., 2$

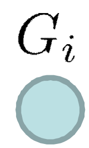
"instantons"

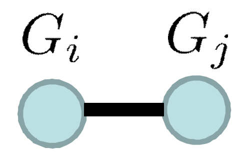
$$
\begin{array}{lcccccccc}
 & & & \frac{1}{2}\mathbf{56} & & \mathbf{I}^3 & \mathbf{I}^2 & \mathbf{I}^1 & \\
 & \mathfrak{so}_8 & \mathfrak{e}_6 & \mathfrak{e}_7 & \mathfrak{e}_7 & \mathfrak{e}_8 & \mathfrak{e}_8 & \mathfrak{e}_8 & \mathfrak{e}_8 \\
\text{DE-type:} & 4, & 6, & 7, & 8, & 9, & 10, & 11, & 12
\end{array}
$$

non-DE-type: 1, 2, 3, 23, 232, 223, 5

Useful Terminology: II / II

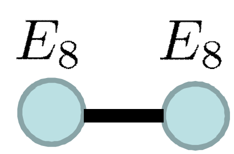
Define a Base Quiver by minimal fiber types:

Nodes: DE-type curves 

Links: Connecting DE-type curves 

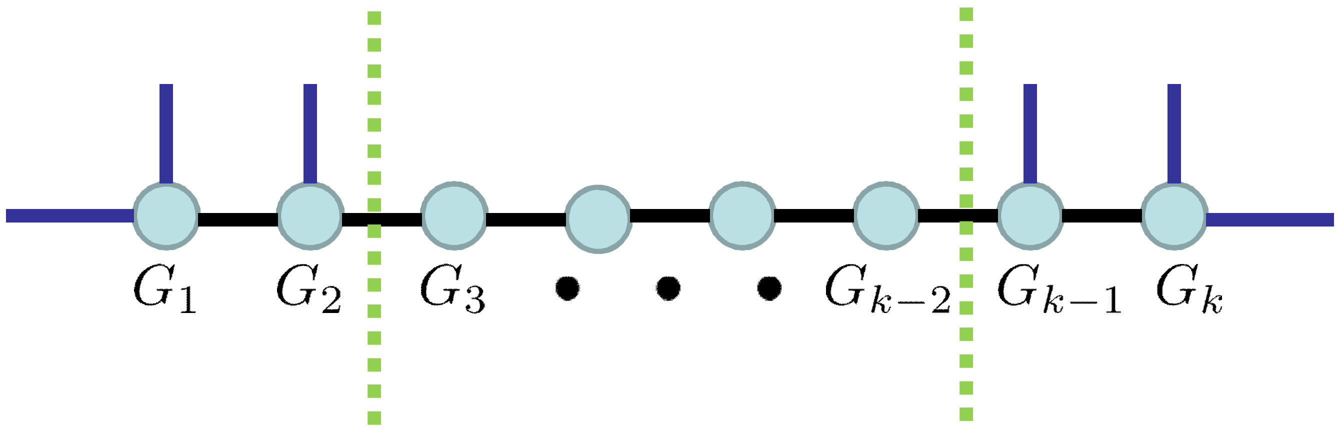
Example:

$$(12), 1, 223, 1, 5, 1, 322, 1, (12)$$



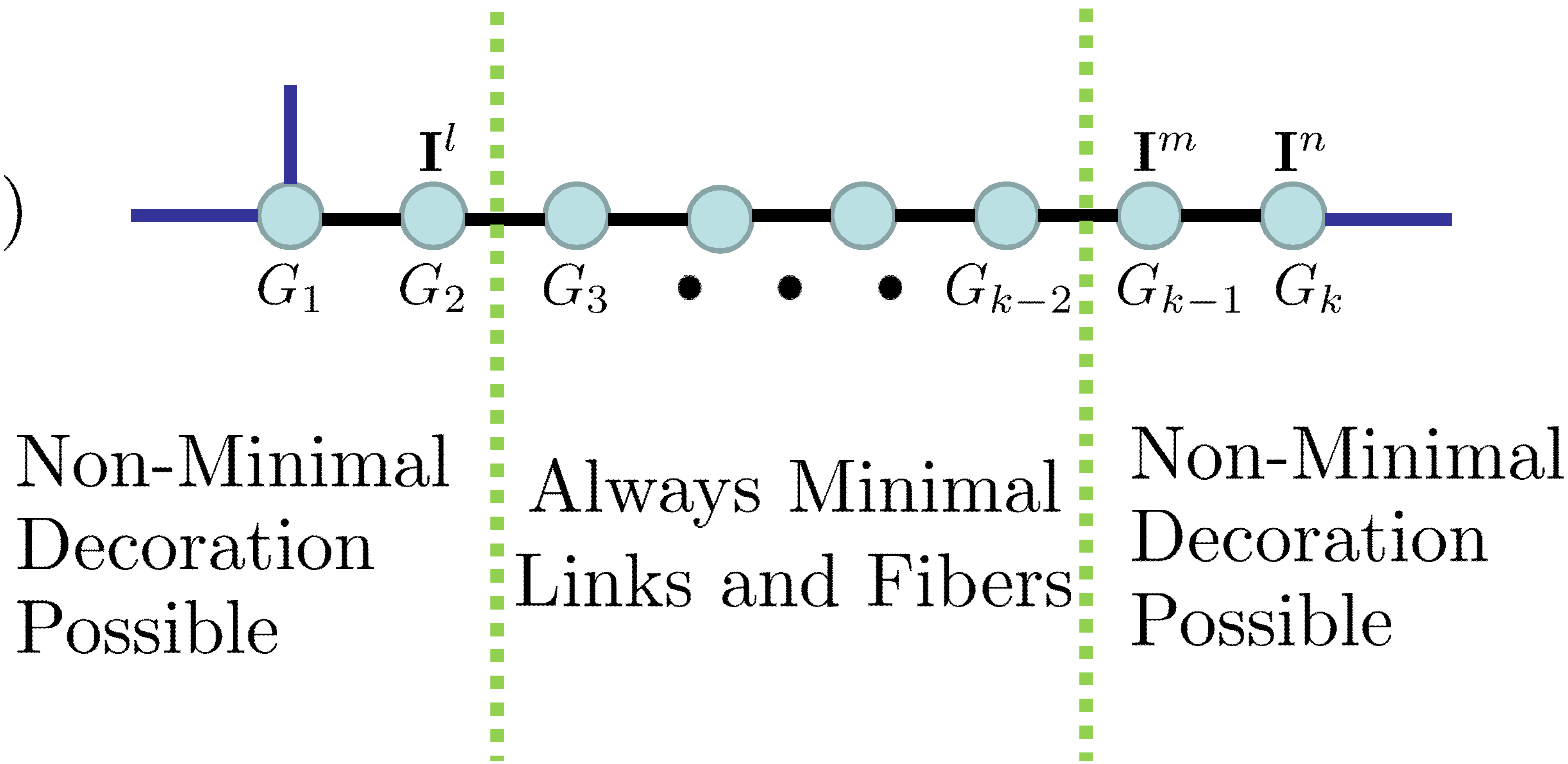
The Big Surprise

The Base Quivers have a *very* simple structure!



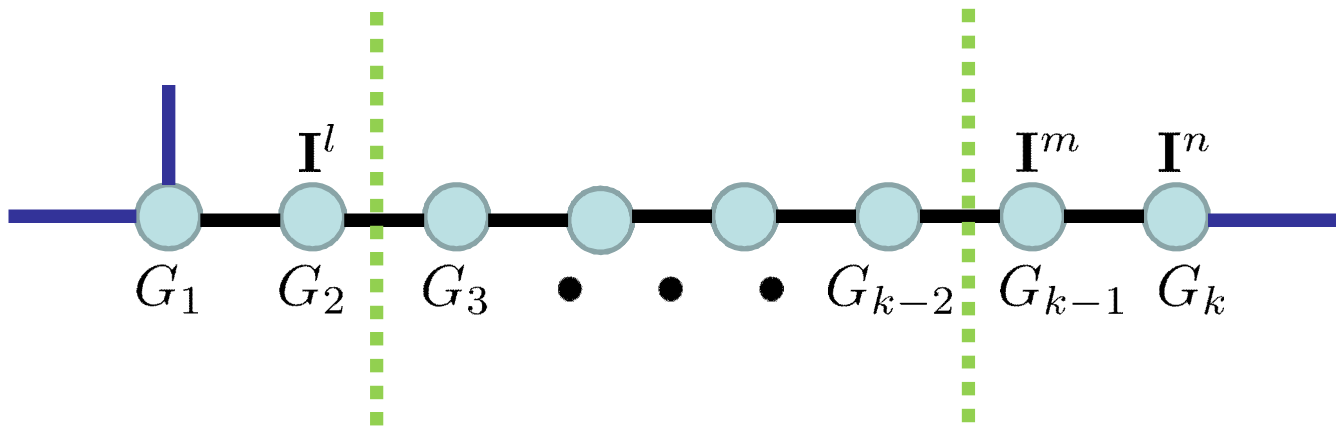
$$G_1 \subseteq G_2 \subseteq \cdots \subseteq G_m \supseteq \cdots \supseteq G_{k-1} \supseteq G_k$$

More Results...

1) 

$$\mathbf{I}^l = 1, 2, ..., 2$$

More Results...

1) 

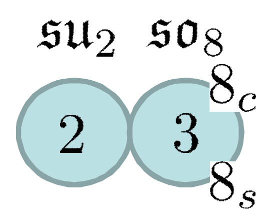
2) Classification of all possible links

3) Classification of all ways to wrap 7-branes

¿Top Down vs Bottom Up?

In almost all cases: 6D field theory constraints *exactly match* F-theory constraints

Example of an outlier:



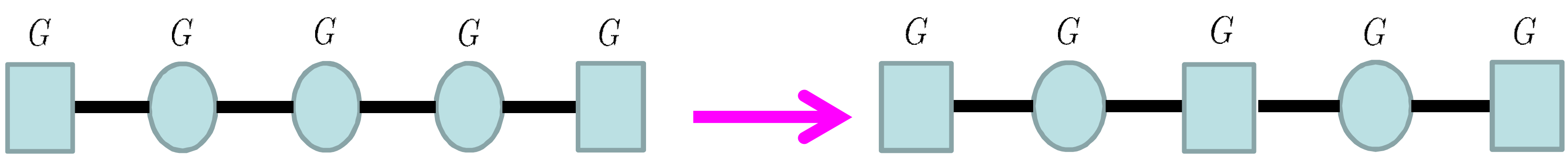
Possible Reason Excluded: $Spin(8)$ vs $Spin(8)/\mathbb{Z}_2$

RG Flows

Geometric Flows

Higgsing / cplx deformations:

Tensor / Kähler deformations:



Relevant vs Irrelevant

$y^2 = x^3 + z^5 + \varepsilon z$ versus $y^2 = x^3 + z^5 + \varepsilon z^{10^{500}}$

Need to check deformation is "normalizable"

Kahler deformation: $\int_{B_2} \delta J \wedge \delta J < \infty$

Complex Deformation: $\int_{CY} \delta\Omega \wedge \overline{\delta\Omega} < \infty$

JJH, Morrison, Rudelius, Vafa '15

Physics Shortcut

Instead, can study change in anomaly polynomial

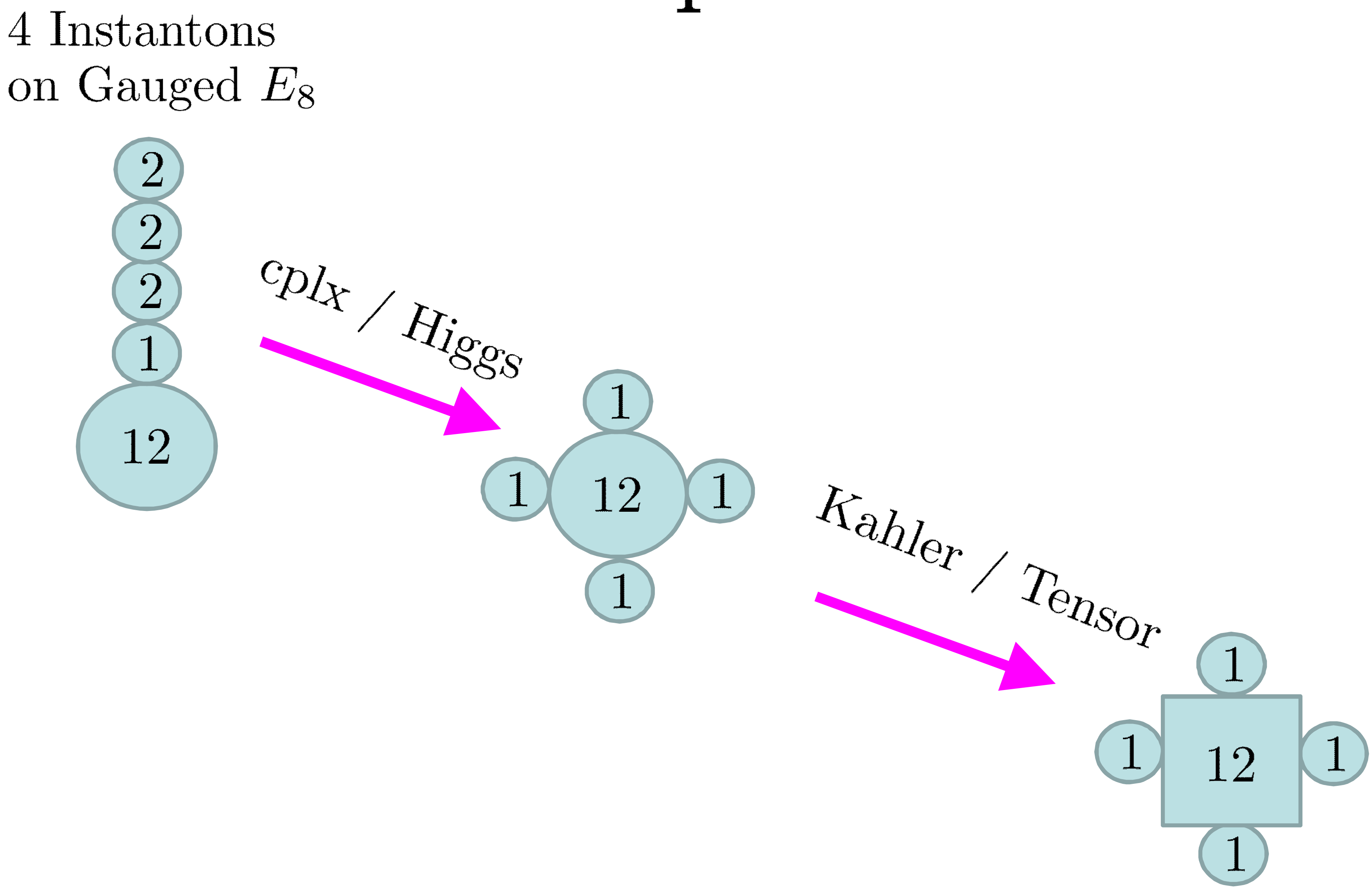
$$\Delta\mathcal{I} = \mathcal{I}_{UV} - \mathcal{I}_{IR}$$

(Activate R-symm and Tangent Bundle field strengths)
(see Ohmori, Shimizu, Tachikawa, Yonekura '14)

$$\mathcal{I}(R,T) = \alpha c_2(R)^2 + \beta c_2(R) p_1(T) + \gamma p_1(T)^2 + \delta p_2(T)$$

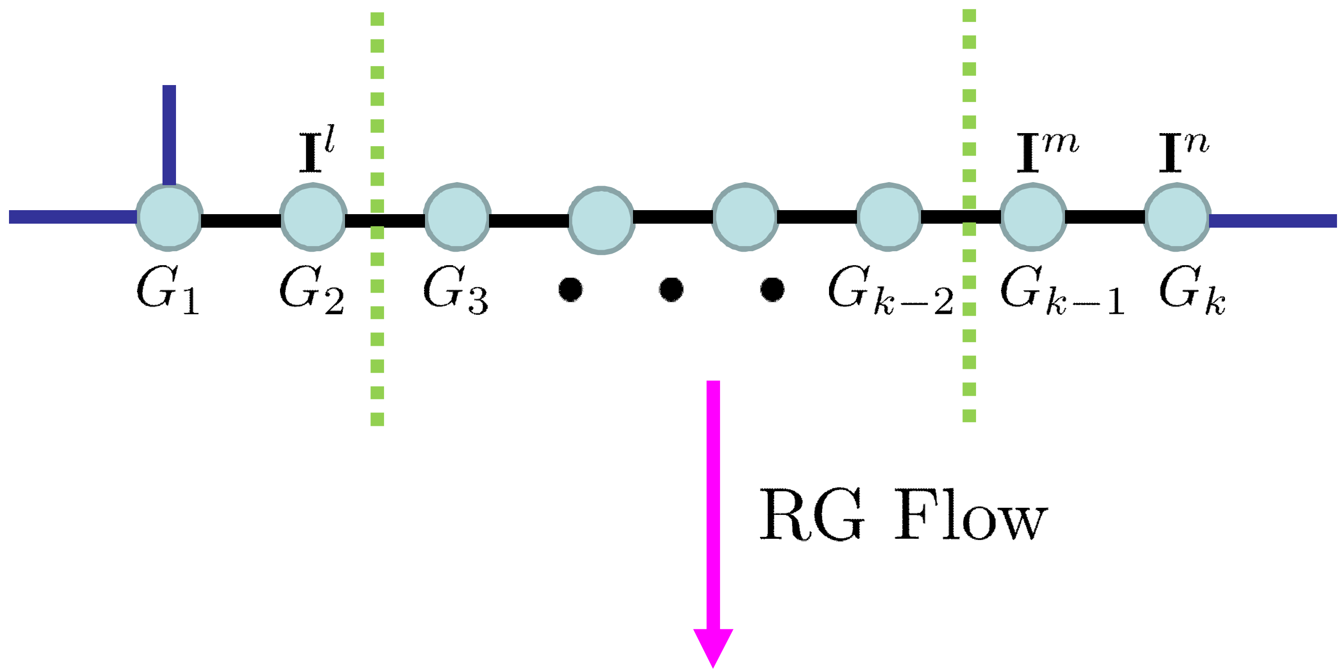
$$+ \text{ Flavor Symmetry Contributions}$$

Example Flow



¿At The Bottom?

UV



IR

$(2,0)$ Theory or a "Rigid" $(1,0)$ Theory

Since We Have a List...

Given a CFT, look for numbers C such that:

$$C_{UV} > C_{IR}$$

Brute Force: Try sweeping over *all* theories

(see also Myers Sinha, '10)

Candidate C-Functions

(Cordova Dumitrescu Intriligator '15; JJH Herzog '15; JJH Rudelius '15)

Linear Combinations: $C = \overrightarrow{m} \cdot \overrightarrow{\alpha}_{\text{anomaly}}$

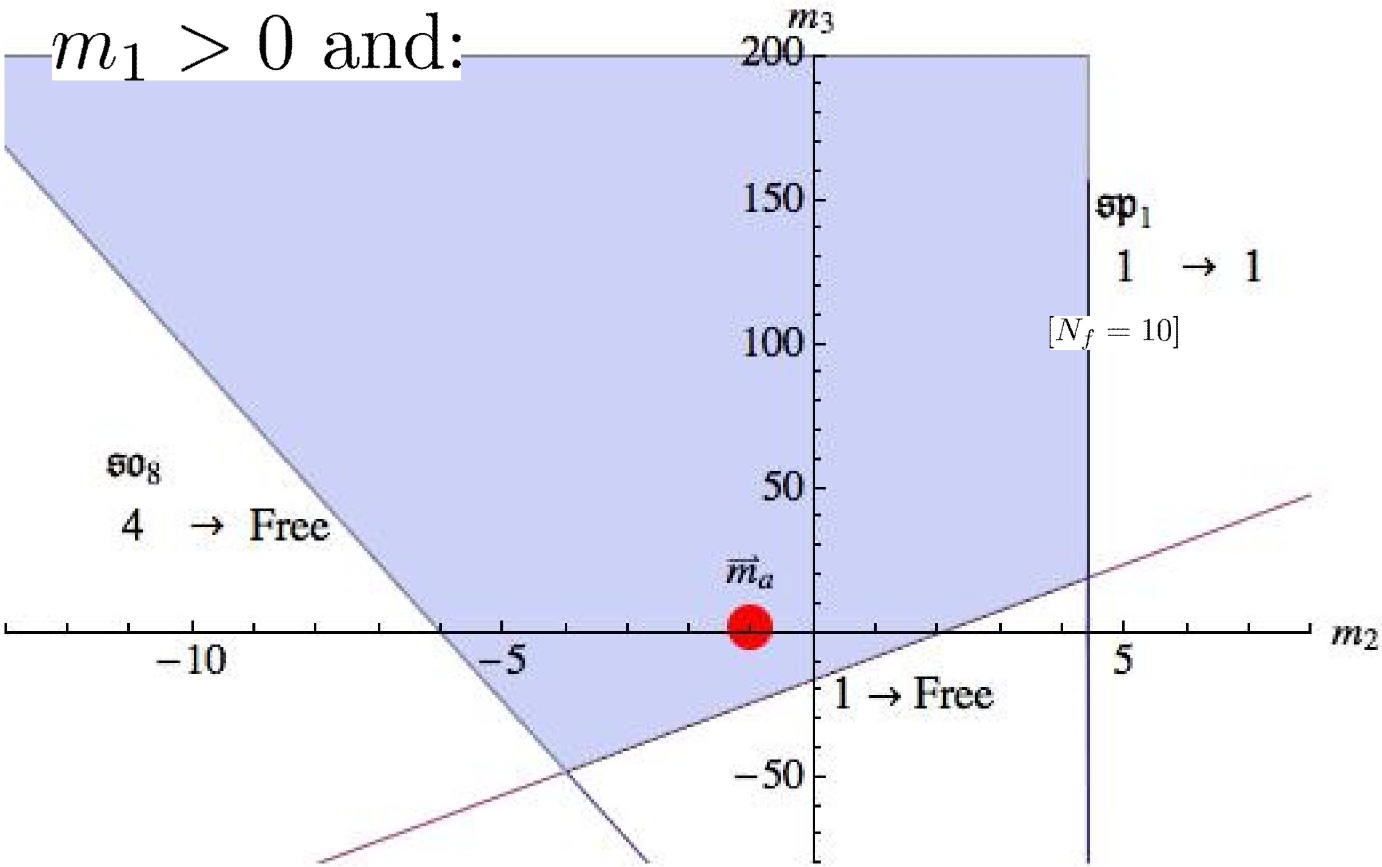
Tightest Bounds from *Simplest* Theories

Big Families of candidate C-Functions

Computer Sweep

(over theories with up to 25 classical gauge groups)

$m_1 > 0$ and:



Conclusions

- 6D SCFTs = Generalized Quivers
- Classification: DONE
- Evidence for C-Theorems
- Next Up: Extract Universal Features