

TWISTOR ORIGIN OF THE SUPERSTRING

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BASED ON arXiv: 1503.03080
(see also arXiv: 1105.1147 and 1409.2510)

Spinning String (RNS)

x^m, ψ^m { spacetime vector
worldsheet spinor

Manifest worldsheet susy

$$\delta x^m = \varepsilon \psi^m, \delta \psi^m = \varepsilon \partial x^m$$

	GSO(+)	GSO(-)
NS	✓	✓
R		

Ramond-Ramond backgrounds?

$d=11$ supermembrane?

Multiloop amp's:

2-loop 4-pt. NS (D'Hoker+Phong '05)

Superstring (GS/pure spinor)

x^m, θ^α { spacetime spinor
worldsheet scalar

Manifest spacetime susy

$$\delta x^m = \varepsilon^\alpha \gamma_{\alpha\beta} \theta^\beta, \delta \theta^\alpha = \varepsilon^\alpha$$

	GSO(+)	GSO(-)
NS	✓	
R	✓	

Tachyonic backgrounds?
Noncritical backgrounds?

Multiloop amp's:

2-loop 4-pt (NB+Mafra, '06)

Coefficient of 2-loop 4-pt (Gomez+Mafra, '10)

3-loop 4-pt low energy (Gomez+Mafra, '13)

2-loop 5-pt low energy (Gomez+Mafra+Schlotterer, '15)

Review of Green-Schwarz Superstring

Worldsheet variables : $x^m, \theta^\alpha, \hat{\theta}^{\hat{\alpha}}$

$m = 0 \text{ to } 9, \alpha = 1 \text{ to } 16, \hat{\alpha} = 1 \text{ to } 16$

IIA/B $\Rightarrow \alpha$ and $\hat{\alpha}$ have opposite/same chirality

$$S = \int d^2z [\sqrt{g} g^{ij} (\partial_i x^m + \partial_i \theta^\alpha \gamma^m \theta + \partial_i \hat{\theta}^{\hat{\alpha}} \gamma^m \hat{\theta}) (\partial_j x_m + \partial_j \theta_m \theta + \partial_j \hat{\theta}_m \hat{\theta}) + B]$$

B is WZW term needed for local fermionic K-symmetry

Canonical quantization in conformal gauge $\Rightarrow p_{\theta^\alpha} = \frac{\partial L}{\partial \dot{\theta}^\alpha} = (\partial x^m + \frac{1}{4} \partial \theta \gamma^m \theta) (\theta_m \theta)_\alpha$

$$\Rightarrow d_\alpha = p_\alpha - (\partial x^m + \frac{1}{4} \partial \theta \gamma^m \theta) (\gamma_m \theta)_\alpha = 0$$

$$\{d_\alpha, d_\beta\} = -2 \gamma_{\alpha\beta}^m (\partial x_m + \partial \theta \gamma_m \theta)$$

\Rightarrow 8 first-class and 8 second-class fermionic constraints

First-class constraints generate K-symmetry

Covariant quantization of second-class constraints?

Unsolved problem for 30 years

Review of Pure Spinor superstring

Worldsheet variables : x^m $(\theta^\alpha, p_\alpha)$ $(\hat{\theta}^{\hat{\alpha}}, \hat{p}_{\hat{\alpha}})$ $(\lambda^\alpha, \omega_\alpha)$ $(\hat{\lambda}^{\hat{\alpha}}, \hat{\omega}_{\hat{\alpha}})$

$$\begin{aligned}\lambda \gamma^m \lambda &= 0 \\ \hat{\lambda} \gamma^m \hat{\lambda} &= 0\end{aligned}$$

\Rightarrow

λ^α and $\hat{\lambda}^{\hat{\alpha}}$ are bosonic d:10 "pure spinors"

$\lambda^\alpha \in \frac{SO(10)}{U(5)} \times \mathbb{C} \Rightarrow 11$ indep. complex components

$$S = \int dz \left[\partial x^m \bar{\partial} x_m - p_\alpha \bar{\partial} \theta^\alpha - \hat{p}_{\hat{\alpha}} \bar{\partial} \hat{\theta}^{\hat{\alpha}} + \omega_\alpha \bar{\partial} \lambda^\alpha + \hat{\omega}_{\hat{\alpha}} \bar{\partial} \hat{\lambda}^{\hat{\alpha}} \right]$$

Action is in conformal gauge with $c = +10 - 32 + 22 = 0$

Physical states defined by cohomology at ghost # = 1 of BRST operator

$$Q = \int dz \lambda^\alpha d_\alpha = \int dz \lambda^\alpha (p_\alpha - (\partial x^m + \frac{1}{4} \partial \theta^\alpha \partial) (\gamma_m \theta)_\alpha)$$

Open string superYM state: $V = \lambda^\alpha A_\alpha(x, \theta)$

Closed string supergravity state: $V = \lambda^\alpha \hat{\lambda}^{\hat{\alpha}} A_{\alpha\hat{\alpha}}(x, \theta, \hat{\theta})$

What is worldsheet origin of Q ?

Clues to origin of Q :

- Difficult to obtain constrained bosonic ghost λ^* from gauge-fixing.
- No (b,c) Virasoro ghosts in Q expected from gauge-fixing worldsheet metric.
- Θ^* is a worldsheet scalar with spin-statistics of worldsheet ghost.
- $d=10$ super-YM and sugra eqns. of motion are related to integrability along light-like lines (Witten '86) and pure spinor lines (Howe '91) (Sorokin et al, '89)
- Twistors are useful for understanding K-symmetry
- In even dimension D , pure spinors are higher-dimensional twistors that parametrize $SO(D)/U(D/2)$ and choose a complex structure in \mathbb{R}^D . (Hughston et al, '90)
- As in topological string, pure spinor superstring has equal # of bosonic and fermionic variables with conformal weight 0 and 1.
- Twisted versions of $d=10$ super-YM and supergravity have been related to $\hat{c}=5$ topological theory (Nekrasov '09, Baulieu '11) (Costello + Li, '15)

Proposal: Classical Type IIA/B superstring is obtained from $\hat{c}=5$ topological A/B model by using projective pure spinors $(\lambda^\alpha, \hat{\lambda}^{\hat{\alpha}})$ to choose $\frac{SO(10)}{U(5)}$ complex structure

Topological A/B model: $S_c = \int d^2z (\partial X^\alpha \bar{\partial} \bar{X}^{\bar{\alpha}} + L^\alpha \partial X_\alpha + \bar{L}^{\bar{\alpha}} \bar{\partial} \bar{X}_{\bar{\alpha}})$

Gauge-fixing $L^\alpha = \bar{L}^{\bar{\alpha}} = 0 \Rightarrow$ Faddeev-Popov ghosts $(\psi^\alpha, \bar{\psi}^{\bar{\alpha}})$ and antighosts $(\rho_\alpha, \bar{\rho}_{\bar{\alpha}})$

$$S = S_c + \int d^2z Q (\rho_\alpha L^\alpha + \bar{\rho}_{\bar{\alpha}} \bar{L}^{\bar{\alpha}}) = \int d^2z (\partial X^\alpha \bar{\partial} \bar{X}^{\bar{\alpha}} + \rho_\alpha \bar{\partial} \psi^\alpha + \bar{\rho}_{\bar{\alpha}} \partial \bar{\psi}^{\bar{\alpha}})$$

with BRST operator $Q = \int d^2z \psi^\alpha \partial X_\alpha + \int d^2z \bar{\psi}^{\bar{\alpha}} \bar{\partial} \bar{X}_{\bar{\alpha}}$

Classical Type IIA/B: $S_c = \int d^2z [\partial X^m \bar{\partial} \bar{X}_m + L^\alpha (\partial X^\alpha \lambda)_\alpha + \bar{L}^{\bar{\alpha}} (\bar{\partial} \bar{X}^{\bar{\alpha}} \hat{\lambda})_{\bar{\alpha}} + \omega_\alpha \bar{\nabla} \lambda^\alpha + \hat{\omega}_{\bar{\alpha}} \bar{\nabla} \hat{\lambda}^{\bar{\alpha}} + f_\alpha \nabla \lambda^\alpha + \hat{f}_{\bar{\alpha}} \bar{\nabla} \hat{\lambda}^{\bar{\alpha}}]$
superstring action

Gauge-fixing $L^\alpha = \hat{L}^{\hat{\alpha}} = 0 \Rightarrow$ Faddeev-Popov ghosts $(\theta^\alpha, \hat{\theta}^{\hat{\alpha}})$ and antighosts $(\rho_\alpha, \hat{\rho}_{\hat{\alpha}})$

GS and pure spinor actions are different gauge-fixings of S_c

Pure spinor gauge-fixing \Rightarrow BRST operator $Q = \int dz \lambda^\alpha d_\alpha + \int d\bar{z} \hat{\lambda}^{\hat{\alpha}} \hat{d}_{\hat{\alpha}}$

To simplify discussion, first consider $d=10$ superparticle

Pure Spinor superparticle:

$$S = \int d\tau [P_m \dot{x}^m - p_\alpha \dot{\theta}^\alpha + \omega_\alpha \dot{\lambda}^\alpha]$$

$$\lambda \gamma^m \lambda = 0 \Rightarrow \lambda^\alpha \in \frac{SO(10)}{U(5)} \times \mathbb{C}$$

$$Q = \lambda^\alpha d_\alpha = \lambda^\alpha (p_\alpha - (\not{P}\theta)_\alpha)$$

BRST transf:

$$\delta \theta^\alpha = \lambda^\alpha, \delta x^m = -\delta \theta \gamma^m \theta = -\lambda \gamma^m \theta$$

$$\delta p_\alpha = -(\not{P}\lambda)_\alpha, \delta \lambda^\alpha = 0, \delta \omega_\alpha = d_\alpha$$

BRST cohom. at ghost # = 1

$$\Rightarrow V = \lambda^\alpha A_\alpha(x, \theta), QV = 0, \delta V = Q\Omega$$

$$\Rightarrow A_\alpha(x, \theta) = (\gamma^m \theta)_\alpha a_m(x) + (\theta \gamma^{mn} \rho) \gamma_{mn} \zeta(x)_\alpha + \dots$$

describes onshell $d=10$ super-YM

Green-Schwarz superparticle:

$$S = \int d\tau [P_m (\dot{x}^m + \dot{\theta} \gamma^m \theta) + e P^m P_m]$$

$$\text{Fermionic constraint: } d_\alpha \equiv p_\alpha - (\not{P}\theta)_\alpha = 0$$

Local K-transf:

$$\delta \theta^\alpha = (\not{P}K)^\alpha, \delta x^m = -\delta \theta \gamma^m \theta$$

$$\delta e = 2 K_\alpha \dot{\theta}^\alpha$$

Covariant quantization is complicated
but light-cone analysis implies
spectrum is onshell $d=10$ super-YM.

Twistor-like Superparticle

Replace mass-shell constraint $P^2 = 0$ with twistor constraint $(\not{P}\lambda)_\alpha = 0$

$$S_c = \int d\tau [P_m \dot{x}^m + \omega_\alpha \nabla^\alpha \lambda^\alpha + L^\alpha (\not{P}\lambda)_\alpha]$$

λ^α is a projective pure spinor and parametrizes $\frac{SO(10)}{U(5)}$

$$\nabla \lambda^\alpha \equiv j^\alpha + A_\tau \lambda^\alpha \quad (\lambda^\alpha \approx \Omega \tilde{\lambda}^\alpha, \omega_\alpha \approx \Omega^{-1} \tilde{\omega}_\alpha, L^\alpha \approx \Omega^{-1} \tilde{L}^\alpha, A_\tau \approx A_\tau + \Omega^{-1} \tilde{\Omega} \dot{\lambda}^\alpha)$$

Gauge transf. generated by $(\not{P}\lambda)_\alpha$: $\delta x^m = -\lambda \gamma^m \theta, \delta \omega_\alpha = (\not{P}\theta)_\alpha, \delta L^\alpha = \nabla \theta^\alpha + \xi \lambda^\alpha$

Gauge-for-gauge transf: $\delta \theta^\alpha = \varphi \lambda^\alpha, \delta \xi = -\nabla \varphi, \delta \omega_\alpha = -\varphi \frac{\partial \chi}{\partial L^\alpha}$

BRST quantization $\Rightarrow (\theta^\alpha, \xi)$ are fermionic ghosts, φ is bosonic ghost-for-ghost
 $\theta^\alpha \approx \Omega^{-1} \Theta^\alpha, \xi \approx \Omega^{-2} \zeta, \varphi \approx \Omega^{-2} \psi$

Gauge-fixed action: $S = S_c + \int d\tau Q X$ X is gauge-fixing fermion

$$Q x^m = -\lambda \gamma^m \theta, Q \theta^\alpha = \varphi \lambda^\alpha, Q \zeta = -\nabla \varphi, Q L^\alpha = \nabla \theta^\alpha + \xi \lambda^\alpha, Q \omega_\alpha = (\not{P}\theta)_\alpha - \varphi \frac{\partial \chi}{\partial L^\alpha}$$

Gauge-fixing to pure spinor superparticle:

Choose $\chi = p_\alpha L^\alpha + \beta \xi$

p_α, β are antighosts

$$Q p_\alpha = M_\alpha, Q \beta = N, Q M_\alpha = QN = 0$$

$$\Rightarrow S = S_c + \int dz Q \chi = \int dz [P_m \dot{x}^m + \omega_\alpha \nabla^\alpha \lambda^\alpha - p_\alpha (\nabla^\alpha \theta^\alpha + \xi^\alpha \lambda^\alpha) \\ + \beta \nabla^\alpha \varphi + L^\alpha (M_\alpha + (\not{P} \lambda)_\alpha) + N \xi^\alpha]$$

Use scale symmetry to fix $\varphi = 1$ (assume $\varphi(z) \neq 0$)

$\varphi \simeq \Omega^{-2} \psi$ and carries ghost # = 2 $\Rightarrow \varphi^{s/2} \mathcal{O}_{g,s}$ is scale-invariant with ghost # = $g+s$

\Rightarrow ghost # is shifted by scale weight

$\Rightarrow (\theta^\alpha, p_\alpha)$ carry ghost # = (0, 0) and $(\lambda^\alpha, \omega_\alpha)$ carry ghost # = (1, -1)

After solving auxiliary equations of motion ($A_\alpha = 0, \beta = \frac{1}{2}(\omega_\alpha + p_\alpha \theta^\alpha), M_\alpha = -(\not{P} \lambda)_\alpha$),

$$S = \int dz [P_m \dot{x}^m + \omega_\alpha \dot{\lambda}^\alpha - p_\alpha \dot{\theta}^\alpha] \text{ with}$$

BRST transf's generated by $Q = \lambda^\alpha (p_\alpha - (\not{P} \theta)_\alpha)$

Gauge-fixing to Green-Schwarz superparticle:

Choose $\chi = \dot{\varphi}^* L^*(\not{P}\theta)_* + \beta \xi - \omega_* \nabla (\dot{\varphi}^* \theta^*)$

$$\Rightarrow Q\omega_* = (\not{P}\theta)_* - \dot{\varphi} \frac{\partial \chi}{\partial L^*} = 0$$

$$\Rightarrow S = S_c + \int d\tau Q\chi = \int d\tau [P_m \dot{x}^m + L^*(\not{P}\lambda)_* + \dot{\varphi}^* (\nabla \theta^* + \xi \lambda^*) (\not{P}\theta)_* + L^*(\not{P}\lambda)_* + \beta \nabla \dot{\varphi} + N \xi]$$

Use scale symmetry to fix $\varphi = 1 \Rightarrow \theta^*$ has ghost # = 0

$$S = \int d\tau [P_m (\dot{x}^m + \dot{\theta} \gamma^m \theta) + 2 L^*(\not{P}\lambda)_*]$$

Eq. of motion $(\not{P}\lambda)_* = 0 \Rightarrow P^2 = 0, \lambda^* = (\not{P}K)^* \text{ for some } K_\alpha$

BRST transf's : $Q\theta^\alpha = \lambda^\alpha = (\not{P}K)^\alpha, Qx^m = -\delta\theta \gamma^m \theta, QL^\alpha = \dot{\theta}^\alpha$
 (replaces $\delta e = 2K_\alpha \dot{\theta}^\alpha$)

Noether charge Q for these transf's is zero

\Rightarrow local K-symmetry survives after this gauge-fixing

Twistor-like Type II Superstring

Replace Virasoro constraints $(P + \partial_\sigma X)^2 = (P - \partial_\sigma X)^2 = 0$
 with twistor-like constraints $(P^m + \partial_\sigma X^m)(\gamma_m \lambda)_\alpha = (P^m - \partial_\sigma X^m)(\gamma_m \hat{\lambda})_\alpha = 0$

Also need constraints $\nabla_\sigma \lambda^\alpha = \nabla_\sigma \hat{\lambda}^\alpha = 0$ to close algebra since

$$[(P^m + \partial_\sigma X^m)(\gamma_m \lambda)_\alpha, (P^n + \partial_\sigma X^n)(\gamma_n \lambda)_\beta] = (\gamma^m \partial_\sigma \lambda)_{[\alpha} (\gamma_m \lambda)_{\beta]}$$

$$\text{In first-order form, } S_c = \int d\tau d\sigma [P_m \dot{X}^m + \omega_\alpha \nabla_\tau \lambda^\alpha + \hat{\omega}_\alpha \nabla_\tau \hat{\lambda}^\alpha + L^\alpha (P^m + \partial_\sigma X^m)(\gamma_m \lambda)_\alpha + \hat{L}^\alpha (P^m - \partial_\sigma X^m)(\gamma_m \hat{\lambda})_\alpha + K_\alpha \nabla_\sigma \lambda^\alpha + \hat{K}_\alpha \nabla_\sigma \hat{\lambda}^\alpha]$$

Can integrate out P_m to obtain manifestly reparam. inv. form of action

$$S_c = \int d\tau (det e) [\nabla X^m \bar{\nabla} X_m + \omega_\alpha \bar{\nabla} \lambda^\alpha + \hat{\omega}_\alpha \bar{\nabla} \hat{\lambda}^\alpha + (L \gamma^m \lambda) \bar{\nabla} X_m + (\hat{L} \gamma^m \hat{\lambda}) \bar{\nabla} X_m + K_\alpha \bar{\nabla} \lambda^\alpha + \hat{K}_\alpha \bar{\nabla} \hat{\lambda}^\alpha - \frac{1}{2} (L \gamma^m \lambda) (\hat{L} \gamma_m \hat{\lambda})]$$

With appropriate choice of gauge-fixing fermion χ_{ps} and χ_{gs}

$$S = S_c + \int d\tau Q \chi_{ps} = S_{\text{pure spinor}} \text{ with } Q = \int d\tau \lambda^\alpha d_\alpha + \int d\bar{\tau} \hat{\lambda}^\alpha \bar{d}_\alpha$$

$$S = S_c + \int d\tau Q \chi_{gs} = S_{\text{Green-Schwarz}} \text{ where BRST transf} = K\text{-transf. with Noether charge} = 0$$

Twistor-like $d=11$ supermembrane

$d=10$ superparticle generalizes to $d=11$ superparticle for $d=11$ sugra

$$S_c = \int d\tau [P_M \dot{x}^M + \omega_B \nabla_\tau \lambda^B + L^B (\not{P} \lambda)_B]$$

$$\begin{aligned} M &= 0 \text{ to } 10 \\ B &= 1 \text{ to } 32 \\ \lambda \delta^M \lambda &= 0 \end{aligned}$$

$d=11$ pure spinor superparticle useful for $d=11$ sugra computations (Cederwall, '10)

$$V_{\text{sym}} = \lambda^\alpha A_\alpha(x, \theta) \rightarrow V_{\text{sugra}} = \lambda^B \not{D}^C \not{D}^D A_{BCD}(x, \theta)$$

For supermembrane, replace

$d=3$ reparam. constraints with twistor-like constraint

$$C_B = P_M (\gamma^M \lambda)_B + \frac{1}{2} \epsilon^{jkl} \partial_j x_m \partial_k x_n (\gamma^{mn} \lambda) = 0$$

$(\gamma^+ C)_z$ and $(\gamma^- C)_{\bar{z}}$ reduce to $(P^m + \partial_m x^m)(\gamma_m \lambda)_z$ and $(P^m - \partial_m x^m)(\gamma_m \lambda)_{\bar{z}}$

of Type IIA superstring upon double-dimensional reduction where $X'^0 = \sigma_2$.

$$S_c = \int d\tau d\sigma_1 d\sigma_2 [P_M \dot{x}^M + \omega_B \nabla_\tau \lambda^B + L^B C_B + K_B^j \nabla_j \lambda^B + J_M^j \partial_j x_N (\lambda \gamma^{MN} \lambda)]$$

is proposal for twistor-like supermembrane action.

Summary

- Classical Type IIA/B superstring action is obtained from $\hat{c}=5$ topological A/B model by using projective pure spinors $(\lambda^a, \hat{\lambda}^{\bar{a}})$ to choose $\frac{SO(10)}{U(5)}$ complex structure.
- $(\theta^a, \hat{\theta}^{\bar{a}})$ are fermionic ghosts coming from gauge-fixing twistor-like constraints $(\partial X^\lambda)_a = (\bar{\partial} \bar{X}^{\bar{\lambda}})_{\bar{a}} = 0$ which replace Virasoro constraints $\partial x^m \partial x_m = \bar{\partial} \bar{x}^m \bar{\partial} \bar{x}_m = 0$.
- After giving nonzero expectation value to bosonic ghost-for-ghost $\psi = 1 \Rightarrow (\theta^a, \hat{\theta}^{\bar{a}})$ have ghost # = 0 and $(\lambda^a, \hat{\lambda}^{\bar{a}})$ have ghost # = 1
- Pure spinor and GS formalisms are different gauge-fixings of twistor-like action (superparticle, superstring, supermembrane)

Possible Applications

- 1) Find gauge-fixing to RNS string or to twistor string.
- 2) Derive scattering amplitude prescription in pure spinor formalism (including regulator for integration over λ^α).
- 3) Define superstring action in backgrounds that are not d=10 supergravity backgrounds.
- 4) Relate to $\hat{c}=5$ topological model of Costello+Li for twisted Type IIB sugra where susy ghosts are $(\lambda^\alpha, \hat{\lambda}^{\hat{\alpha}})$. Twisting $\Rightarrow (\lambda^\alpha, \hat{\lambda}^{\hat{\alpha}})$ have non-zero expectation value.

Open superstring field theory = Chern-Simons $\langle V Q V + V^3 \rangle$

Closed superstring field theory = BCOV ?