Anomalies, RG flows, and the a-theorem in six- dimensional (1,0) theories.

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based on work in collaboration with

Spectacular Collaborators:

Clay Córdova

Thomas Dumitrescu

Conformal anomaly **a**

Even space-time dim d: $\langle T^\mu_\mu \rangle \sim a E_d + \sum_i c_i I_i$ $\sim (\mathrm{Weyl})^{d/2}$

★ → a

Euler density → E_d

Won't discuss these today. → $c_i I_i$

d=2: "a" called "c" (no 2d c). Counts d.o.f. of QFT **Zamolodchikov**

$$a_{\mathrm{UV}} \geq a_{\mathrm{IR}} \qquad a \geq 0$$

For any unitary theory

d=4: **Cardy. Osborn.** Susy theories: relation to R-symmetry 't Hooft anomalies **Anselmi, Freedman, Grisaru, Johansen**, so a-anomaly is calculable in interacting theories, many checks. Proof of a-thm. via dilaton 4-pt fn. **Komargodski-Schwimmer.**

6d a-theorem?

- Non-susy: dilaton analysis was inconclusive.
- Many examples of 6d interacting SCFTs. (Non-susy examples?) What is their a?
- RG flows? With **Córdova & Dumitrescu, to appear; also, J. Louis & Severin Luest to appear:** 6d SCFTs cannot have susy + Lorentz invt relevant or marginal operator deforms. So deform via vevs, spont. break conf'l symm. What is their $\Delta a?$ is $\Delta a > 0?$

Dilaton analysis

Spontaneous conf'l symm breaking: dilaton has derivative interactions to give Δa anom matching **Schwimmer, Theisen; Komargodski, Schwimmer**

6d case: $$\mathcal{L}_{\rm dilaton} = \frac{1}{2}(\partial\varphi)^2 - b\frac{(\partial\varphi)^4}{\varphi^3} + \Delta a\frac{(\partial\varphi)^6}{\varphi^6}$$ **(schematic; derivative order shown)**

Maxfield, Sethi; Elvang, Freedman, Hung, Kiermaier, Myers, Theisen.

4-dilaton O(p^4) amplitudes, as in 4d, shows b>0 (only free thy has zero b)... but what is b's physical interpretation?
No conclusive restriction on sign of 6-derivative term.

Clue: observed that, for case of (2,0) on Coulomb branch,

$$\Delta a \sim b^2$$

M&S: via (2,0) susy; EFHKMT: some 0 amplitudes then, fits with AdS/CFT

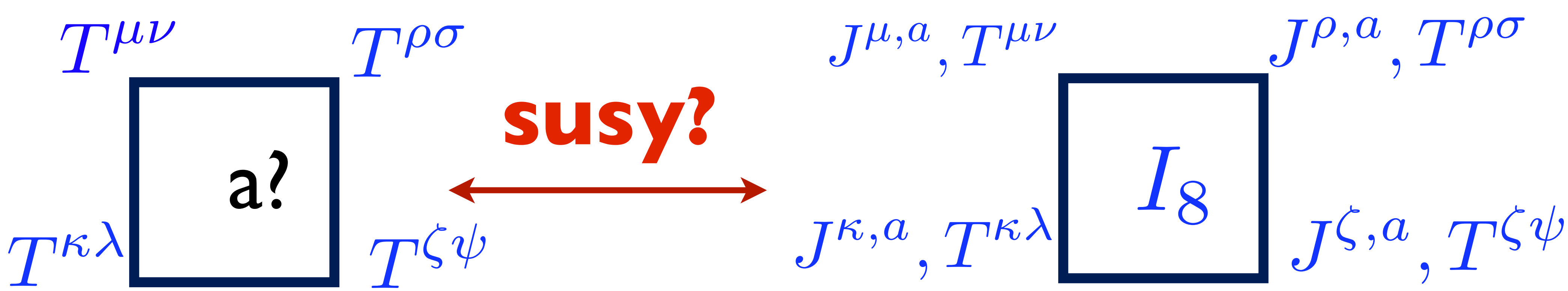
Longstanding hunch

e.g. Harvey Minasian, Moore '98

Supersymmetric multiplet of anomalies: should be able to relate conformal anomaly a to 't Hooft-type anomalies for the superconformal R-symmetry in 6d, as in 2d and 4d.

$T^{\mu\nu} \leftrightarrow J_R^{\mu,a}$ Stress-tensor supermultiplet

$g_{\mu\nu} \leftrightarrow A_{R,\mu}^a$ Source: bkgrd SUGRA supermultiplet



4-point fn with too many indices. Hard to get a (and to compute).

Easier to isolate anomaly term, and enjoys anomaly matching

6d anomaly polynomial

Alvarez-Gaume, Witten

$$\mathcal{I}_{d+2} = \mathcal{I}_{d+2}^{\text{gauge}} + \mathcal{I}_{d+2}^{\text{gravity+global}}$$

Must cancel, restricts G & matter

Analog of 't Hooft anomalies. Matching. Useful.

E.g.:
N=(2,0):

$$\mathcal{I}_g = r_g \mathcal{I}_{u(1)} + \frac{k_g}{24} p_2(F_{SO(5)_R})$$

Duff, Liu, Minasian; Witten; Freed, Harvey, Minasian, Moore

A,D,E group G

Free (2,0) tensor mult

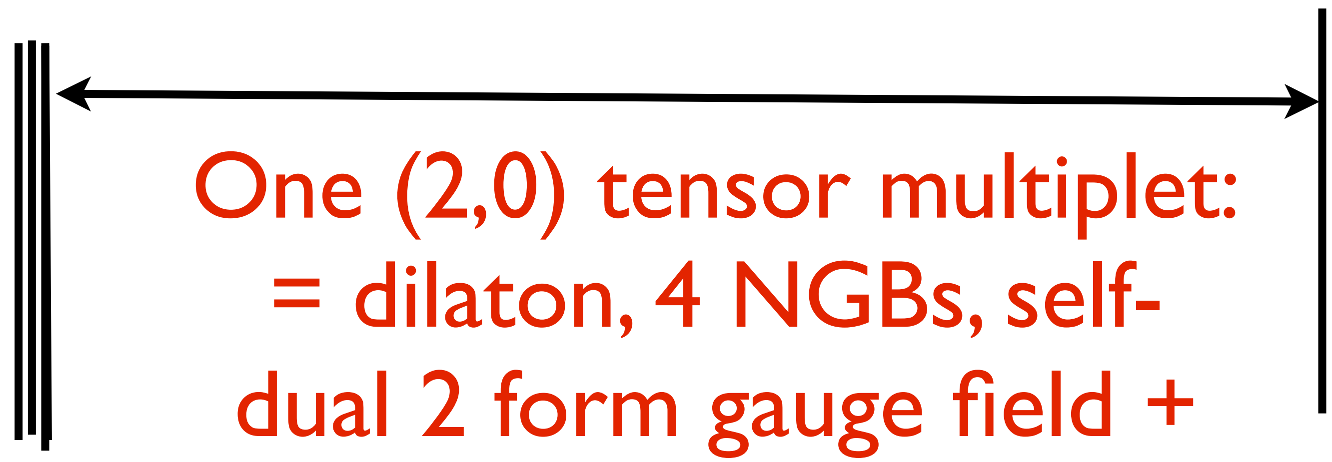
Interaction part

N M5s+inflow: $k_{su(N)} = N^3 - N$ **Harvey, Minasian, Moore**

Other methods: $k_g = h_g^\vee d_g$ **KI; Yi; Ohmori, Shimizu, Tachikawa, Yonekura. See also Ki-Myeong Lee et. al.**

(2,0)'s Coulomb branch

E.g. N M5s, pull one far away:



NGBs: $SO(5)/SO(4) = S^4$ have derivative interactions to match R-symmetry 't Hooft anomalies. Skyrmionic string couples to self-dual 2-form: $\rightarrow \Delta k \sim q^2$ similar to $\Delta a \sim b^2$.
Susy relation? Initial puzzles.

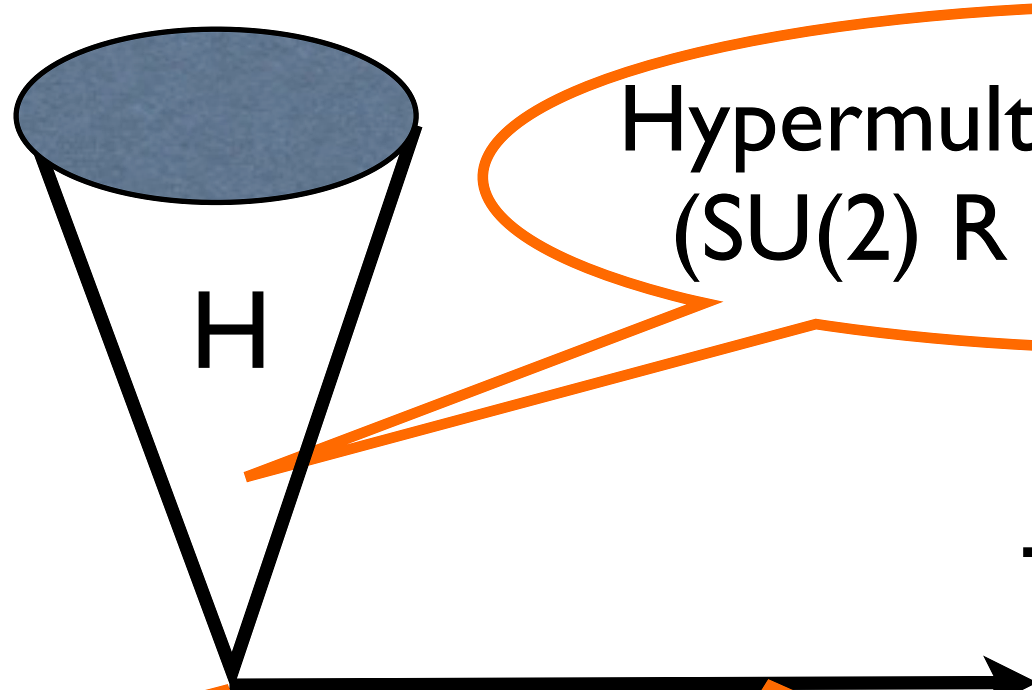
Recently explained and resolved: **Córdova, Dumitrescu, Yin**

$$\rightarrow a_g = \frac{16}{7} h_g^\vee d_g + r_g$$

Relates $\Delta a = \frac{16}{7} \Delta k$ and also gives an independent calc. of $\Delta k_g = \Delta(h_g^\vee d_g)$

We consider (1,0) susy

Deform SCFT by moving on its vacuum manifold:



* Simplifies things, so we will stay on the tensor branch for this talk. We also have some results for Higgs branch, + in progress.

(1,0) 't Hooft anomalies

$$\mathcal{I}_8^{\text{origin}} = \frac{1}{4!}\left(\alpha c_2^2(R) + \beta c_2(R) p_1(T) + \gamma p_1^2(T) + \delta p_2(T)\right)$$

$$c_2(R) \equiv \frac{1}{8\pi^2}\mathrm{tr}(F_{SU(2)_R} \wedge F_{SU(2)_R})$$

$$p_1(T) \equiv \frac{1}{8\pi^2}\mathrm{tr}(R \wedge R)$$

Background gauge fields and metric (~ background SUGRA)

Recently computed for many (1,0) SCFTs

Ohmori, Shimizu, Tachikawa; Ohmori, Shimizu, Tachikawa, Yonekura; Del Zotto, Heckman, Tomasiello, Vafa; Heckman, Morrison, Rudelius, Vafa.

E.g. for theory of N small E8 instantons:

Ohmori, Shimizu, Tachikawa

$$\mathcal{E}_N : \ (\alpha, \beta, \gamma, \delta) = (N(N^2 + 6N + 3), -\frac{N}{2}(6N+5), \frac{7}{8}N, -\frac{N}{2})$$

(Leading N^3 coeff. can be anticipated from Z_2 orbifold of A_{N-1} (2,0) case.)

(1,0) on tensor branch

't Hooft anomaly matching requires

$\Delta \mathcal{I}_8 \equiv \mathcal{I}_8^{\rm origin} - \mathcal{I}_8^{\rm tensor\ branch} \sim X_4 \wedge X_4$ must be a perfect square, match I_8 via X_4 sourcing B:

$\mathcal{L}_{GSWS} = -iB \wedge X_4$ **KI; Ohmori, Shimizu, Tachikawa, Yonekura**

$X_4 \equiv 16\pi^2 (x c_2(R) + y p_1(T))$ for some real coefficients x, y

then get $(\Delta\alpha, \Delta\beta, \Delta\gamma, \Delta\delta) = 1536\pi^3 (x^2, 2xy, y^2, 0)$

Plan: show that (1,0) susy implies that

* $\Delta a = \frac{98304\pi^3}{7} b^2$ (as in (2,0), different normalization conventions)

** $b = \frac{1}{2}(y - x)$ (4 dilaton, 4-derivative coeff.)

Show $\Delta a \sim b^2$:

$$\mathcal{L}_{\text{dilaton}} = \frac{1}{2}(\partial\varphi)^2 - b\frac{(\partial\varphi)^4}{\varphi^3} + \Delta a\frac{(\partial\varphi)^6}{\varphi^6}$$ **(schematic)**

Susy-complete it. On the tensor branch, the dilaton is the scalar of the tensor multiplet, together with 2-form gauge field B + fermions. We show that all the susy interactions on the tensor branch must be "D-terms"

$$\delta\mathcal{L}_D = Q^8(\mathcal{O})$$

Derivative expansion and counting, form operator from the ingredients: $\delta\varphi, \ \psi_\alpha \sim Q_\alpha \sim \sqrt{\partial}, \ \partial^{[\alpha\beta]}, \ H_{(\alpha\beta)} \sim \partial$

Can susy-complete 4-derivative, b-term: $\delta\mathcal{L}_4 \sim bQ^8((\delta\varphi)^4)$

$\Delta a \sim b^2$ continued

$$\mathcal{L}_{\text{dilaton}} = \frac{1}{2}(\partial\varphi)^2 - b\frac{(\partial\varphi)^4}{\varphi^3} + \Delta a\frac{(\partial\varphi)^6}{\varphi^6}$$

yes no!

$$\delta\mathcal{L}_D = Q^8(\mathcal{O})$$

Cannot so susy-complete the 6-derivative term: every candidate op is zero. The 6-derivative term cannot be an independent deformation; instead, it is induced by the susy-completion of the b term, **~ non-renormalization theorem**. Also follows from amplitudes and super-vertices. **Chen, Huang, Wen**

So:

$$\Delta a = \frac{98304\pi^3}{7}b^2$$

Can fix the theory-independent proportionality coeff from (2,0) case

Recall Maxfield Sethi, Elvang et. al.

Proves these flows have $\Delta a > 0$. **Recall b>0 if interacting.**

Now susy-relate anomalies

$$\mathcal{L}_{GSWS} = 16i\pi^2 B \wedge (xc_2(R) + yp_1(T))$$

⟷ susy

$$\mathcal{L}_{R^2} = \langle\varphi\rangle\sqrt{g}\left(\left(y - \frac{x}{4}\right) R^{\mu\nu\rho\lambda}R_{\mu\nu\rho\lambda} + \frac{3}{2}x\, R_{[\mu\nu}{}^{\mu\nu}R_{\rho\sigma]}{}^{\rho\sigma}\right)$$

Follows from **Bergshoeff, Salam, Sezgin '86 (!).**

Implies $b = \frac{1}{2}(y - x)$ recall also b>0. So

$$\Delta a = \frac{24576\pi^3}{7}(x-y)^2 = \frac{16}{7}\Delta(\alpha - \beta + \gamma) > 0$$

Proves a-theorem for tensor branch flows.

Relates conformal anomaly to 't Hooft anomalies.

Generalize

More tensors, e.g. N for N small E8 instantons, just iterate:

$$\rightarrow \Delta a = \frac{24576\pi^3}{7}(\vec{x}-\vec{y})^2 = \frac{16}{7}(\Delta\alpha - \Delta\beta + \Delta\gamma) > 0$$

Theory at origin:

$$\mathcal{I}_8^{\text{origin}} = \frac{1}{4!}\left(\alpha c_2^2(R) + \beta c_2(R) p_1(T) + \gamma p_1^2(T) + \overset{*}{\delta} p_2(T)\right)$$

*constant on vacua space
(no matching mechanism)

Comparing with free hyper or tensor:

$$a^{\text{origin}} = \frac{16}{7}(\alpha - \beta + \gamma) + \frac{6}{7}\delta$$

Determine a for the N small E_8 instanton SCFT

Plug

$$\mathcal{E}_N: \ (\alpha, \beta, \gamma, \delta) = (N(N^2+6N+3), -\frac{N}{2}(6N+5), \frac{7}{8}N, -\frac{N}{2})$$

Ohmori, Shimizu, Tachikawa

into

$$a = \frac{16}{7}(\alpha - \beta + \gamma) + \frac{6}{7}\delta$$

get:

$$a(\mathcal{E}_N) = \frac{64}{7}N^3 + \frac{144}{7}N^2 + \frac{99}{7}N$$

Also considered via string thy: Heckman & Herzog, to appear.

Vector multiplet issues

A free vector multiplet in d>4 is a unitary SFT: scale but not conformally invariant theory. Subsector of a non-unitary CFT.

El-Showk, Nakayama, Rychkov

Blithely applying our relation $a = \frac{16}{7}(\alpha - \beta + \gamma) + \frac{6}{7}\delta$

To the free (1,0) vector multiplet's 't Hooft anomalies gives

$$a(\text{vector}) = -\frac{251}{210}$$ negative...

Get unitary, interacting 6d (1,0) SCFTs from vectors + tensors & $\mathcal{L}_{\text{kin}} = \varphi F^2$ + specific matter to cancel gauge anomalies. **Seiberg**

Many examples from string/brane/F-theory constructions.

We verified for several classes of theories that $a_{\text{origin}} > 0$. **(Thy on tensor branch, on the other hand, is a SFT and indeed some $a_{\text{away}} < 0$.)**

Conjectured $a_{\text{origin}} > 0$ seems to be a non-trivial requirement.

Conclude

- Susy relation between a and R-symmetry and gravity ‘t Hooft anomalies, via tensor branch vev.
- Lots of new data: a-values for (1,0) SCFTs.
- Proved 6d a-thm for tensor defm’s. Higgs branch also in examples, proof under study.
- Positivity of a. Proof? Not obvious with vector multiplets.
- Thank you!