Black Hole Entropy from Gauge Theory

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Black Hole microstates from String Theory

String theory is a theory of quantum gravity

One of the great successes of String Theory:

Bekenstein-Hawking entropy of asymptotically-flat BPS black holes
from counting of microstates in field theory [Strominger, Vafa 96]

Black Hole microstates from String Theory

- Black hole = System of D-branes with a field theory description on their world-volume

- Black-hole microstates = States in the field theory

$$S = \log\left(d_{\text{micro}}\right)$$

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Leading Bekenstein-Hawking entropy typically from some 2D CFT and Cardy's formula:

$$S_{\text{BH}} = \frac{\text{Area}}{4G_{\text{N}}} \simeq 2\pi\sqrt{\frac{n\,c_{\text{2D}}}{6}}$$

Black Hole microstates from String Theory

The matching can be made much more precise!

E.g.: 4D $\mathcal{N}=8$ string theory:

- In field theory, exact quantum degeneracies from elliptic genus:

[Maldacena, Moore, Strominger 99; Shih, Strominger, Yin 05; Sen 08]

$$\sum_{n=-1,\,0 \pmod 4} d_{\mathsf{micro}}(n)\, q^{n/4} = \frac{\sum_{\ell\in\mathbb{Z},\,\mathbb{Z}+\frac{1}{2}} \; q^{\ell^2}}{\eta(q)^6}$$

Radamacher expansion: [Hardy, Ramanujan; Radamacher]

$$d_{\mathsf{micro}}(n) = \sum_{c=1}^{\infty} c^{-9/2} K_c(n)\, \widetilde{\mathcal{I}}_{7/2}\Big(\frac{\pi\sqrt{n}}{c}\Big)$$

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- In gravity, combine

Sen's entropy function + localization in SUGRA (+ some assumptions)

Reproduce *exactly* the same expansion.

[Sen; Dabholkar, Gomes, Murthy; Sen, Banerjee, Gupta, Mandal]

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↑ orbifolds of AdS_2 ↑ all perturbative orders with localization

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Black Hole microstates in AdS

Until recently, no similar result in AdS_{4+}

AdS/CFT gives a non-perturbative definition of quantum gravity in AdS

Non-perturbative computations in strongly-coupled CFT	$\Rightarrow$	quantum corrections to weakly-curved gravity

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Non-perturbative computations in strongly-coupled CFT	$\Rightarrow$	quantum corrections to weakly-curved gravity

$\rightarrow$ Development of *localization techniques* in SUSY QFTs

Ensemble of states in strongly-coupled CFT $=$ Large BH

Black holes in 4D gauged supergravity

Maximally SUSY example: $\frac{1}{16}$-BPS black holes in

M-theory on $\mathrm{AdS}_4 \times S^7$

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$\downarrow$

4D maximal $\mathcal{N} = 8$ $SO(8)$ gauged supergravity

$\downarrow$

4D $\mathcal{N} = 2$ $U(1)^4$ gauged supergravity (STU model)

- STU model: graviton, 4 vectors and 3 complex scalars (+ spinors)

Black holes in 4D gauged supergravity

Static spherically-symmetric magnetically charged (dyonic) BPS black holes:

[Cacciatori, Klemm 09; Gnecchi, Dall'Agata 10; Hristov, Vandoren 10; Halmagyi 13, 14; Katmadas 14]

- Metric: $$ds^2 = -f(r)\,dt^2 + \frac{1}{f(r)}dr^2 + g(r)\big(d\theta^2 + \sin^2\theta\, d\varphi^2\big)$$

  Asymptotically: global AdS_4

  Near horizon: $\mathrm{AdS}_2 \times S^2$ (BPS, 2 supercharges)

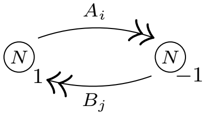
- Magnetic charges: $F_\Lambda = \mathfrak{n}_\Lambda\, d\mathrm{vol}_{S^2}$ $\quad \sum_\Lambda \mathfrak{n}_\Lambda = -2 \quad \Lambda = 1, \dots, 4$

- Possibly electric charges, non-trivial profile for scalars

Holography

[Aharony, Bergman, Jafferis, Maldacena 08]

M-theory on $\text{AdS}_4 \times S^7$ $\leftrightarrow$ 3D ABJM gauge theory with group $U(N)_1 \times U(N)_{-1}$



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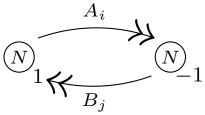


Asymptotic of BH determines a *relevant* deformation of the CFT:

- 3D theory on $S^2 \times \mathbb{R}$
- F^Λ $\Rightarrow$ topologically twisted on S^2 $(\mathfrak{n}_1, \mathfrak{n}_2, \mathfrak{n}_3$ family of twists)

$$\mathcal{L} = \mathcal{L}_{\text{ABJM}} + A_\mu^{\Lambda\,(\text{R})} J^{\mu,\Lambda\,(\text{R})} + \dots$$

- possibly with real masses

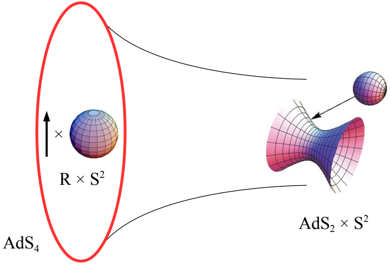
Holography

Relevant deformation triggers an *RG flow*:

3D ABJM

$\downarrow$

1D system



Near horizon AdS2 $\Rightarrow$ $\mathfrak{su}(1,1|1)$-invariant ensemble of ground states [Maldacena, Michelson Strominger 98]

$\updownarrow$

BH microstates

Counting the states

- Construct a "topologically twisted index" [FB, Zaffaroni 15]

  for 3D $\mathcal{N} = 2$ theories with $U(1)_R$, topologically twisted on S^2:

$$Z = \mathrm{Tr}\,(-1)^F\, e^{-\beta H}\, e^{i A_3^{\text{flav}} J^{\text{flav}}}$$

  H: Hamiltonian of the twisted theory on S^2

  A_3^{flav}: fugacities for flavor symmetry charges J^{flav}

- It counts ground states of the CFT twisted on S^2 : $\quad 0 = H - m^{\text{flav}} J^{\text{flav}}$

  (or "chiral" states of the massive theory if $m^{\text{flav}} \neq 0$)

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- Can be represented by a SUSY path-integral:

$$Z_{S^2 \times S^1}(y, \mathfrak{n}) = \int \mathcal{D}\varphi\, e^{-S[\varphi; y, \mathfrak{n}]} \qquad y = e^{i A_3^{\text{flav}} - \beta m^{\text{flav}}}$$

A localization formula

For gauge theories, the TT index [Nekrasov, Shatashvili 14]

can be computed *exactly* with localization: [FB, Zaffaroni 15]

$$Z_{S^2\times S^1}(y,\mathfrak{n}) = \sum_{\mathfrak{m}\in\Gamma_{\text{mag}}} \oint_{\mathcal{C}} \frac{1}{|\text{Weyl}|} \prod_{\text{Cartan}} \left(\frac{dx}{2\pi i x}\, x^{k\mathfrak{m}}\right) \underbrace{\prod_{\alpha\in G}(1-x^{\alpha}) \prod_{\rho_I\in\mathfrak{R}} \left(\frac{x^{\rho_I/2} y_I^{1/2}}{1-x^{\rho_I} y_I}\right)^{\rho_I(\mathfrak{m})+\mathfrak{n}_I-q_I+1}}_{Z_{\text{class}}\, Z_{\text{1-loop}}}$$

- Sum over lattice of magnetic charges
- Contour integral inside complexified maximal torus

  prescribed by Jeffrey-Kirwan residue [Jeffrey, Kirwan 95]

  already appeared in 2D elliptic genus [FB, Eager, Hori, Tachikawa 13]

  and 1D Witten index [Hori, Kim, Yi 14; Cordova, Shao 14; Hwang, Kim, Kim, Park 14]

  Picks specific residues and boundary terms according to ρ_I and k

Index at large N

We are interested in the large N limit of the TT index of ABJM

- Reduce to a sum of residues at zeros of BAEs: [Gukov, Pei 15]

$$1 = x_j^k \prod_{l=1}^{N} \frac{\left(1 - y_3 \frac{\tilde{x}_l}{x_j}\right)\left(1 - y_4 \frac{\tilde{x}_l}{x_j}\right)}{\left(1 - y_1^{-1} \frac{\tilde{x}_l}{x_j}\right)\left(1 - y_2^{-1} \frac{\tilde{x}_l}{x_j}\right)} \,, \qquad \text{similar for } x \leftrightarrow \tilde{x}$$

  Zeros are generalized critical points of the 2D effective twisted superpotential $\widetilde{\mathcal{W}}(x, \tilde{x})$ [Nekrasov, Shatashvili 14]

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- At large N, use continuous distribution (here $\sum_{\Lambda=1}^{4} u_\Lambda = 2\pi$) $iu_\Lambda = \log y_\Lambda$

$$\boxed{\log Z_{S^2 \times S^1} \simeq \frac{N^{3/2}}{3} \sqrt{2u_1 u_2 u_3 u_4} \sum_{\Lambda=1}^{4} \frac{\mathfrak{n}_\Lambda}{u_\Lambda}}$$

Further structure observed at large N [Hosseini, Zaffaroni 16; Hosseini, Mekareeya 16]

Black hole entropy from the index

Combine fermion number and flavor charges into

a "trial R-symmetry" of the SUSY QM:

$$Z_{S^2\times S^1} = \mathrm{Tr}_{H_{\mathsf{nh}}=0}\,(-1)^{R_{\mathsf{trial}}(A_3^{\mathsf{flav}})}\;e^{-\beta m^{\mathsf{flav}} J^{\mathsf{flav}}}$$

Near-horizon Hamiltonian (electric flux on AdS_2): $\qquad H_{\mathsf{nh}} = H - m^{\mathsf{flav}} J^{\mathsf{flav}}$

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- $\mathfrak{su}(1,1|1)$-invariance of ground states $\quad\Rightarrow\quad R_{\mathsf{sc}}(\text{ground states}) = 0$

Extremization principle:

$\left.\dfrac{\partial \log Z}{\partial u}\right\|_{u_{\mathsf{sc}}} = i\,\langle J^{\mathsf{flav}}\rangle$	black hole flavor charges
$\mathbb{R}\mathrm{e}\Big[\log Z - iu\langle J^{\mathsf{flav}}\rangle\Big]_{u_{\mathsf{sc}}} = S_{\mathsf{BH}}$	entropy

By evaluation, the ABJM TT-index reproduces the black hole entropies!

Index and attractor equations

Near horizon BH solutions are determined by attractor equations [Ferrara, Kallosh 96] [Dall'Agata, Gnecchi 10]

Gauged $\mathcal{N} = 2$ supergravity attractor equations (static BHs):

$$\partial_j \Big(-i \frac{\langle \mathcal{Q}, \mathcal{V} \rangle}{\langle \mathcal{G}, \mathcal{V} \rangle} \Big) = 0 \qquad\qquad -i \frac{\langle \mathcal{Q}, \mathcal{V} \rangle}{\langle \mathcal{G}, \mathcal{V} \rangle} = R^2_{S^2} \ \propto \ S_{\mathsf{BH}}$$

Special geometry:

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	$\mathcal{V} \propto (X^\Lambda, \frac{\partial \mathcal{F}}{\partial X^\Lambda})$	holomorphic sections
	$\mathcal{G} = (0, g)$	gaugings

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In fact

$$\boxed{ -i \frac{\langle \mathcal{Q}, \mathcal{V} \rangle}{\langle \mathcal{G}, \mathcal{V} \rangle} \quad \propto \quad \log Z_{S^2 \times S^1} - iu \langle J^{\text{flav}} \rangle }$$

Entropy function

Similarities with Sen's entropy function formalism. [Sen 07]

- Quantum entropy function:

$$d_{\mathsf{micro}}(q_a) = \left\langle e^{-iq_a \oint A^a} \right\rangle_{\mathsf{AdS}_2}^{\mathsf{finite}}$$

finite part of unnormalized path-integral on Euclidean AdS_2 with fixed charges.

- In grand canonical ensemble:

$$Z_{\mathsf{AdS}_2}^{\mathsf{finite}}(u_a) = \sum_{q_a} d_{\mathsf{micro}}(q_a)\; e^{i \sum q_b u_b}$$

Extremization is saddle-point approximation to the Fourier transform.

OSV conjecture

Similarities with the OSV conjecture. [Ooguri, Strominger, Vafa 04]

The TT index can be decomposed into a sum of *holomorphic blocks* [Beem, Dimofte, Pasquetti 12]

$$Z_{S^2\times S^1} = \sum_\alpha Z^\alpha_{D_2\times S^1}\cdot \widetilde{Z}_{D_2\times S^1}$$

The set of vacua $\{\alpha\}$ is 1-1 to the generalized vacua of the 2D effective twisted superpotential $\widetilde{\mathcal{W}}_{\text{eff}}$

$$e^{\frac{\partial \widetilde{\mathcal{W}}_{\text{eff}}}{\partial u_a}} = 1$$

Same set is 1-1 to solutions to the BAE [Nekrasov, Shatashvili 14; Gukov, Pei 15] [Closset, Kim 16]

At large N only one solution dominates

Conclusions

- Non-perturbative computations at strong coupling give information about quantum gravity in AdS

  Localization techniques provide interesting sets

- Extracted leading Bekenstein-Hawking entropy of BPS BHs in AdS_4 from the TT index

- Can we compute $\frac{1}{N}$ and e^{-N} corrections? [*cfr* Dabholkar, Drukker, Gomes 14]

  Can we compute the exact integer degeneracies?