Strings 2016, Tsinghua University, Beijing

Abelian and Discrete Symmetries in F-theory

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Outline (Summary): F-theory Compactification

I. Non-Abelian gauge symmetries
   brief overview of key ingredients

II. Abelian gauge symmetries
   rational sections and Mordell-Weil group
   Highlight insights into Heterotic duality

III. Discrete gauge symmetries
   multi-sections and Tate-Shafarevich group
   Highlight Heterotic duality and Mirror symmetry

IV. Global particle physics models
   three family Standard Model & with R-parity

Time permitting

Emphasize geometric perspective

Apologies: Upenn-centric

Heterotic/F-theory work based on:

M.C., A.Grassi, D.Klevers, M.Poretschkin and P.Song, ``Origin of Abelian Gauge Symmetries in Heterotic/F-theory Duality,” arXiv:1511.08208 [hep-th]

M.C., A.Grassi and M.Poretschkin, ``Discrete Symmetries in Heterotic/F-theory Duality and Mirror Symmetry,” arXiv:1607.03176 [hep-th]

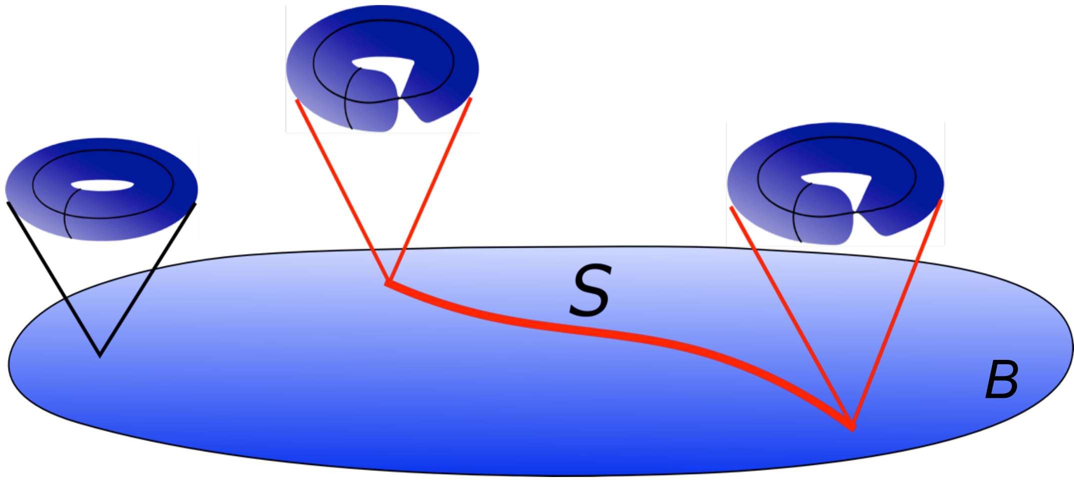
Type IIB perspective

F-THEORY BASIC INGREDIENTS

F-theory compactification

[Vafa'96], [Morrison,Vafa'96],...

Elliptically fibered Calabi-Yau manifold X



Modular parameter of two-torus (elliptic curve)

$$\tau \equiv C_0 + i g_s^{-1}$$

Weierstrass normal form for elliptic fibration of X

$$y^2 = x^3 + f x z^4 + g z^6$$

f, g - sections on (holomorphic functions of) B

[z:x:y] - homogeneous coordinates on $\mathbf{P}^2(1,2,3)$

F-theory compactification

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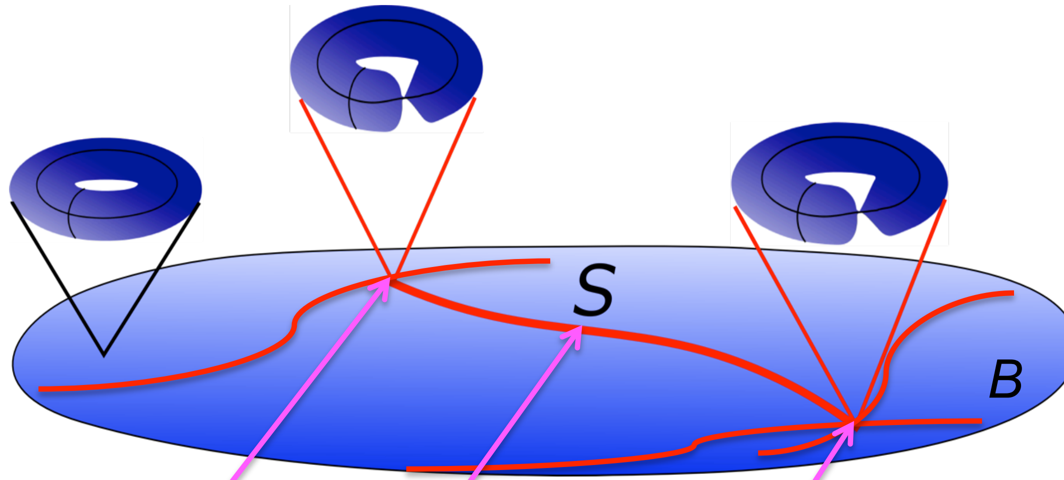

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Matter
(co-dim 2; chirality- G_4-flux)

Yukawa couplings
(co-dim 3)

singular elliptic-fibration, $g_s \to \infty$
location of (p,q) 7-branes

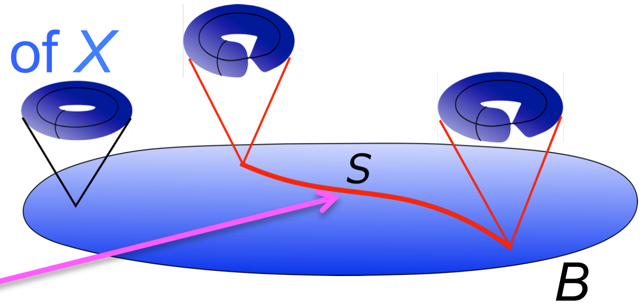
non-Abelian gauge symmetry
(co-dim 1)

Non-Abelian Gauge Symmetry

[Kodaira],[Tate], [Vafa], [Morrison,Vafa],...[Esole,Yau], [Hayashi,Lawrie,Schäfer-Nameki],[Morrison], ...

- Weierstrass normal form for elliptic fibration of *X*

$$y^2 = x^3 + fxz^4 + gz^6$$

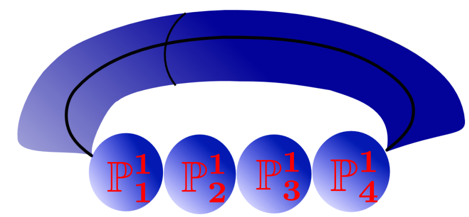


- Severity of singularity along divisor S in B

  specified by [$ord_S(f)$,$ord_S(g)$,$ord_S(\Delta)$]

- Resolution: structure of a tree of $\mathbb{P}^1$'s over *S*

  Resolved I_n-singularity ←→ SU(n) Dynkin diagram



Cartan gauge bosons: supported by (1,1) form $\omega_i \leftrightarrow \mathbb{P}^1_i$ on resolved X

(via M-theory Kaluza-Klein reduction of C_3 potential $C_3 \supset A^i\omega_i$)

Deformation: [Grassi, Halverson, Shaneson'14-'15]

II. U(1)-Symmetries in F-Theory

Abelian Gauge Symmetries

Different: (1,1) forms ω_m, supporting U(1) gauge bosons, isolated & associated with I_1-fibers, only

[Morrison,Vafa'96]

(1,1) - form ω_m ⟺ rational section of elliptic fibration

Abelian Gauge Symmetry & Mordell-Weil Group

rational sections of elliptic fibr. ⟺ rational points of elliptic curve

Abelian Gauge Symmetry & Mordell-Weil Group

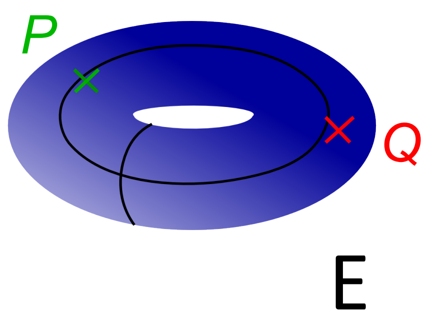
rational sections of elliptic fibr. ⬌ rational points of elliptic curve

Rational point Q on elliptic curve E with zero point P

- is solution (x_Q, y_Q, z_Q) in field K of Weierstrass form

$$y^2 = x^3 + fxz^4 + gz^6$$

- Rational points form group (addition) on E



Abelian Gauge Symmetry & Mordell-Weil Group

rational sections of elliptic fibr. ⇔ rational points of elliptic curve

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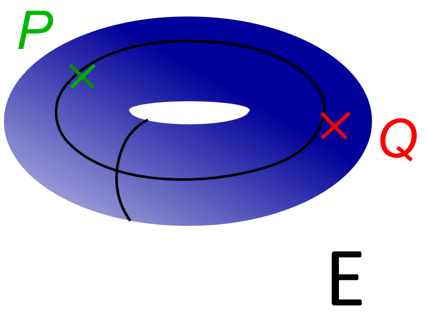
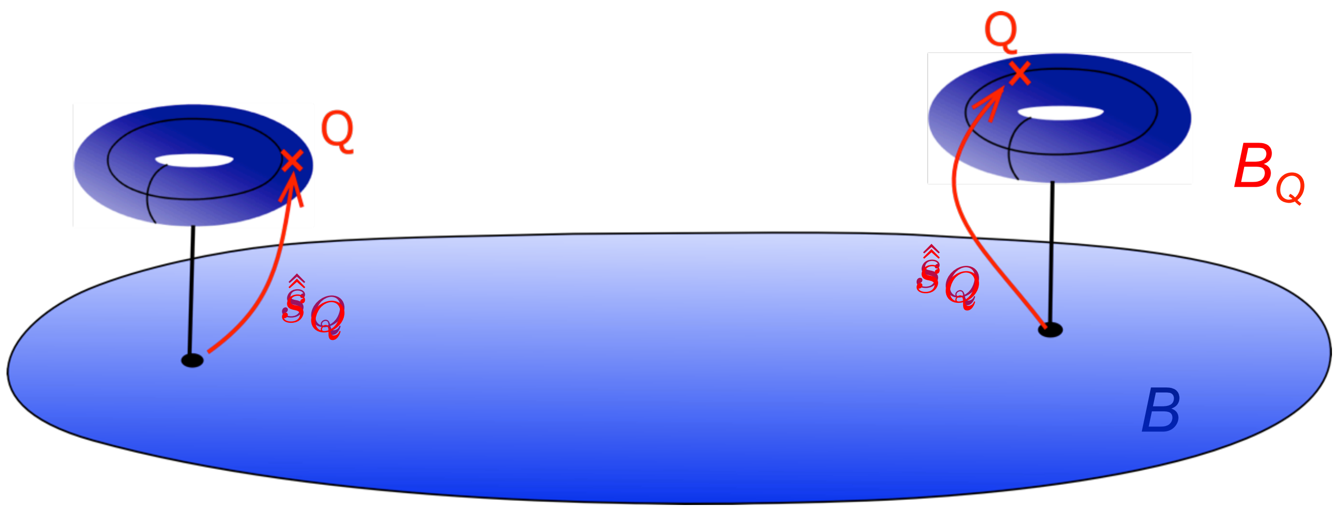


⇒ Mordell-Weil group of rational points

U(1)'s-Abelian Symmetry &Mordell-Weil Group

Point **Q** induces a rational section $\hat{s}_Q : B \to X$ of elliptic fibration



➡ $\hat{s}_Q$ gives rise to a second copy of *B* in *X*:

new divisor B_Q in *X*

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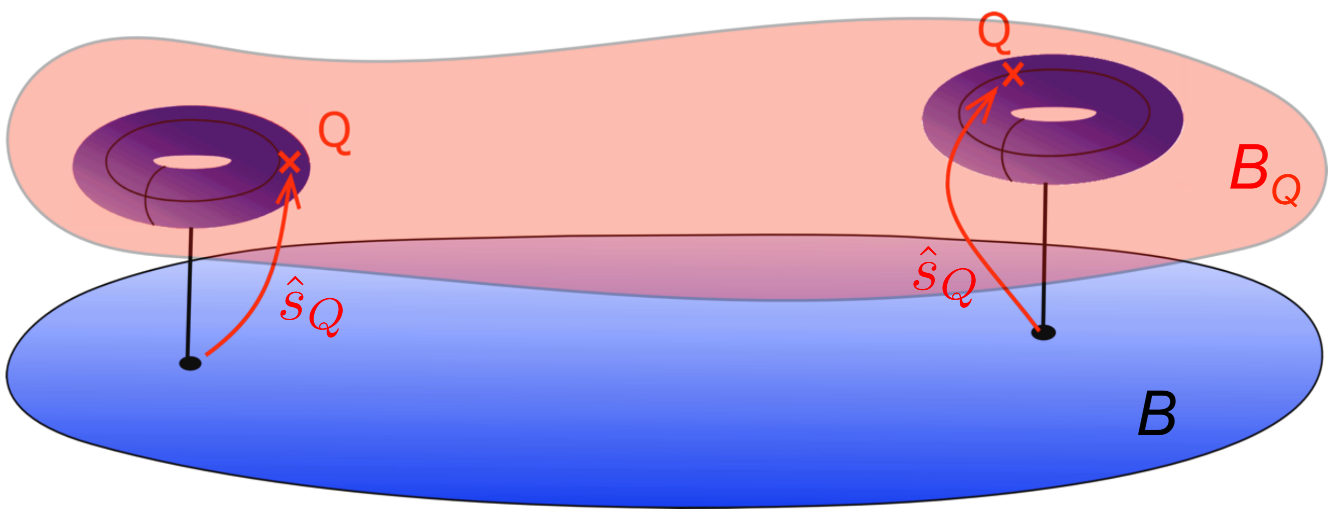

➡ $\hat{s}_Q$ gives rise to a second copy of *B* in *X*:

new divisor B_Q in *X*

➡ (1,1)-form ω_m constructed from divisor B_Q (Shioda map)

indeed (1,1) - form ω_m ⬌ rational section

Earlier work: [Grimm,Weigand 1006.0226]…[Grassi,Perduca 1201.0930] [M.C.,Grimm,Klevers 1210.6034]…

Explicit Examples: (n+1)-rational sections – $U(1)^n$

[Deligne]

[via line bundle constr. on elliptic curve E- *CY in (blow-up) of* $W\mathbb{P}^m$]

n=0: with P - generic CY in $\mathbb{P}^2(1,2,3)$ (Tate form)

n=1: with *P, Q* - generic CY in $\mathrm{Bl}_1\mathbb{P}^2(1,1,2)$ [Morrison,Park 1208.2695]…

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n=2: with *P, Q, R* - specific example: generic CY in dP_2

[Borchmann,Mayerhofer,Palti,Weigand 1303.54054,1307.2902]

[M.C.,Klevers,Piragua 1303.6970,1307.6425]

[M.C.,Grassi,Klevers,Piragua 1306.0236]

generalization to nongeneric cubic in $\mathbb{P}^2[u:v:w]$

[M.C.,Klevers,Piragua,Taylor 1507.05954]

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n=3: with *P, Q, R, S* - CICY in $\mathrm{Bl}_3\mathbb{P}^3$

[M.C.,Klevers,Piragua,Song 1310.0463]

n=4: determinantal variety in $\mathbb{P}^4$

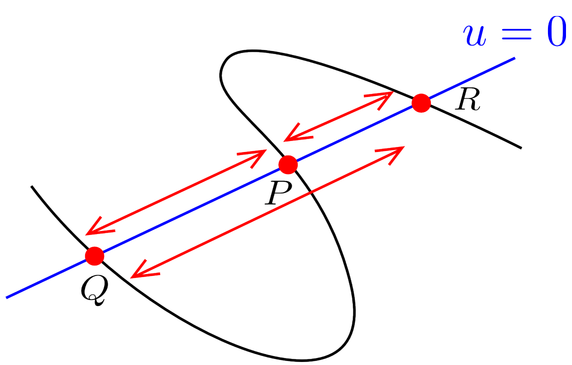
...

higher *n*, not clear…

U(1)xU(1): Further Developments

[M.C., Klevers, Piragua, Taylor 1507.05954]

General U(1)xU(1) construction:



non-generic cubic curve in $\mathbb{P}^2[u:v:w]$:

$$u f_2(u,v,w) + \prod_{i=1}^{3}(a_i v + b_i w) = 0$$

$f_2(u,v,w)$ degree two polynomial in $\mathbb{P}^2[u:v:w]$

Study of non-Abelian enhancement (unHiggsing) by merging rational points P, Q, R [first symmetric representation of SU(3)]

higher index representations [Klevers, Taylor 1604.01030]

[Morrison, Park 1606.0744]

non-local horizontal divisors (Abelian) turn into local vertical ones (non-Abelian) →

both in geometry (w/ global resolutions) & field theory (Higgsing matter)

U(1)'s in Heterotic/F-theory Duality

[M.C., Grassi, Klevers, Poretschkin, Song 1511.08208]

[Morrison,Vafa '96], [Friedman,Morgan,Witten '97]

Basic Duality (8D):

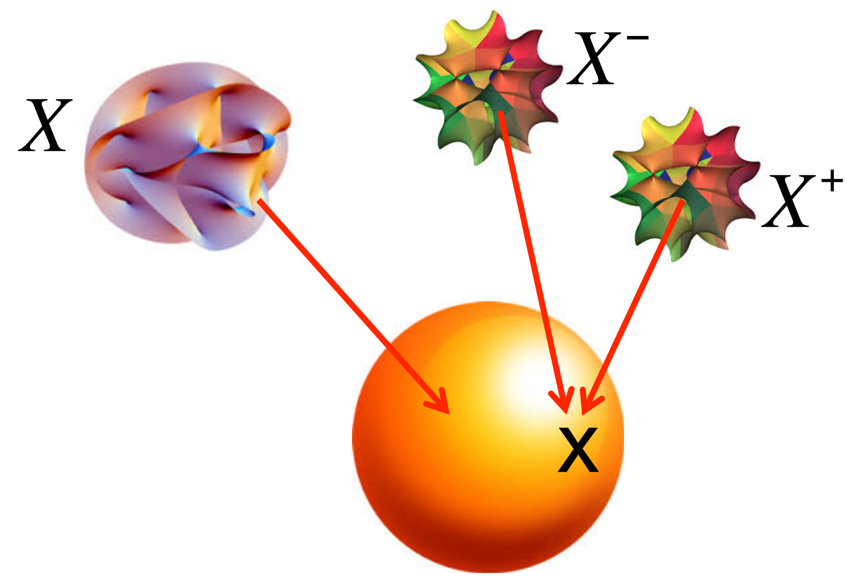
Heterotic E_8 x E_8 String on T^2

⬍ dual to

F-Theory on elliptically fibered
K3 surface X

Manifest in stable degeneration limit:

K3 surface X splits into
two half-K3 surfaces X^+ and X^-



K3-fibration over P^1
(moduli)

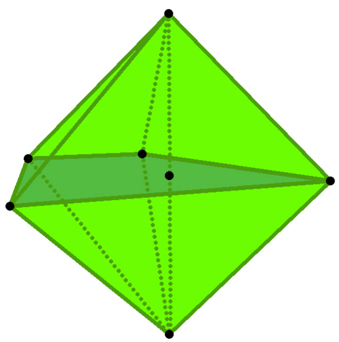
Dictionary:

- X^+ and X^- → background bundles V_1 and V_2
- Heterotic gauge group $G = G_1 \times G_2$ $G_i = [E_8, V_i]$
- The Heterotic geometry T^2 : at intersection of X^+ and X^-

U(1)'s in Heterotic/F-theory Duality

Employ toric geometry techniques in 8D/6D to study stable degeneration limit of F-theory models with one U(1)
[elliptic curve: CY hypersurface in $\mathrm{Bl}_1\mathbb{P}^2(1,1,2)$]

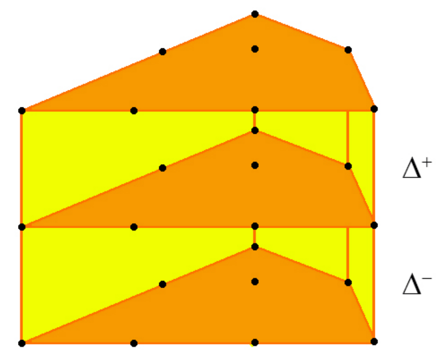
Toric polytope:



specifies the ambient space $P^1 x Bl_1 P^{(1,1,2)}$

Dual polytope:

Newton polytope



specifies the elements of O($-K_{P^1 \times Bl_1 P^{(1,1,2)}}$)

6D: fiber this construction over another P^1

Decomposing the F-Theory Geometry

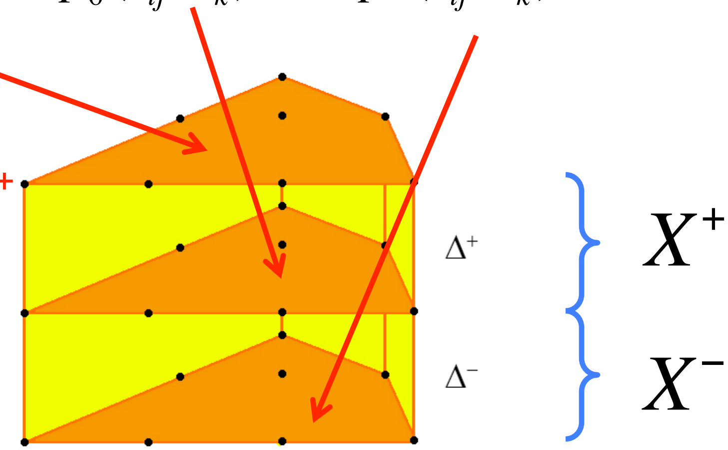
[Morrison, Vafa '96], [Berglund, Mayr '98]

$$\chi : p^{+}(s_{ij},x_k)U^2 + p_0(s_{ij},x_k)U^2 + p^{-}(s_{ij},x_k)V^2$$

p^+ specifies spectral cover C^+

p_0 specifies elliptic curve E

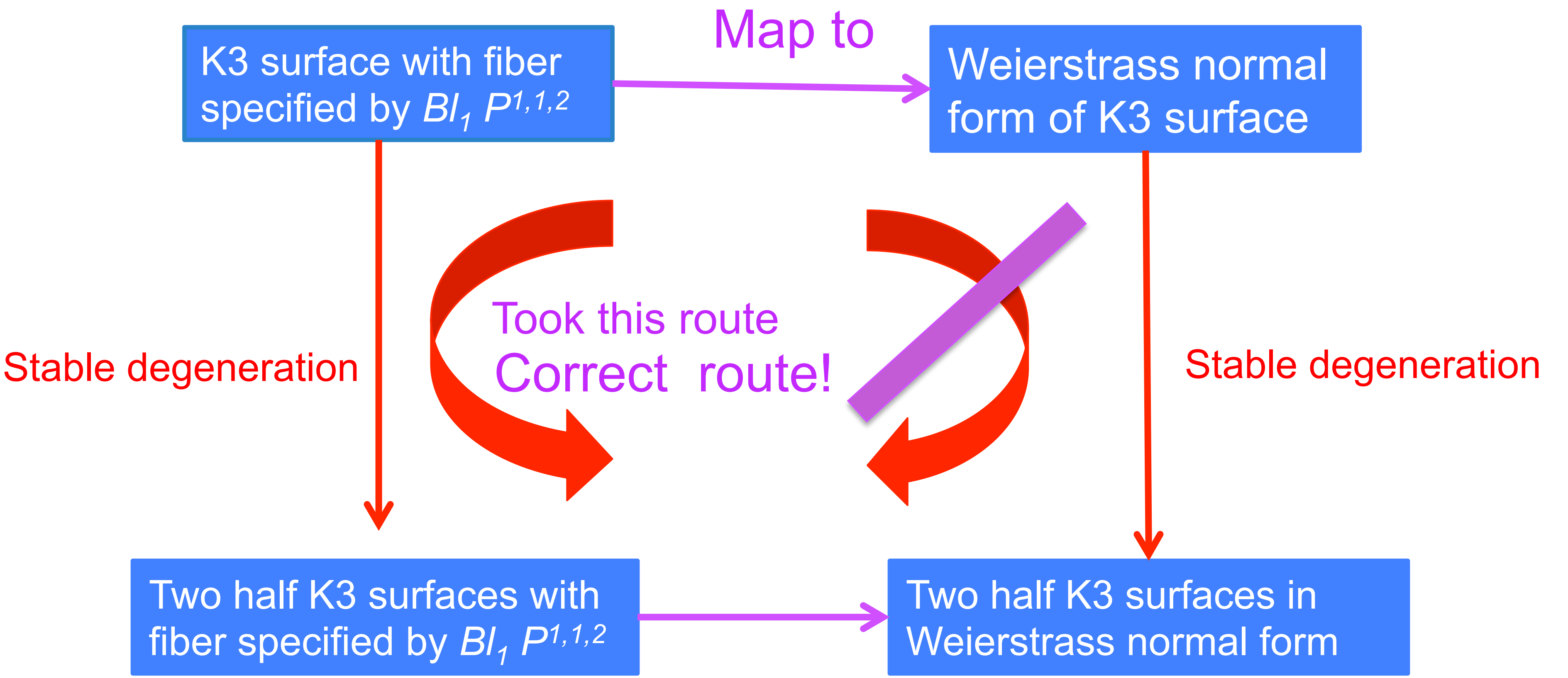
p^- specifies spectral cover C^-



- Spectral cover defines a SU(N) vector bundle on E
- Specialize to large gauge groups to keep spectral cover under control

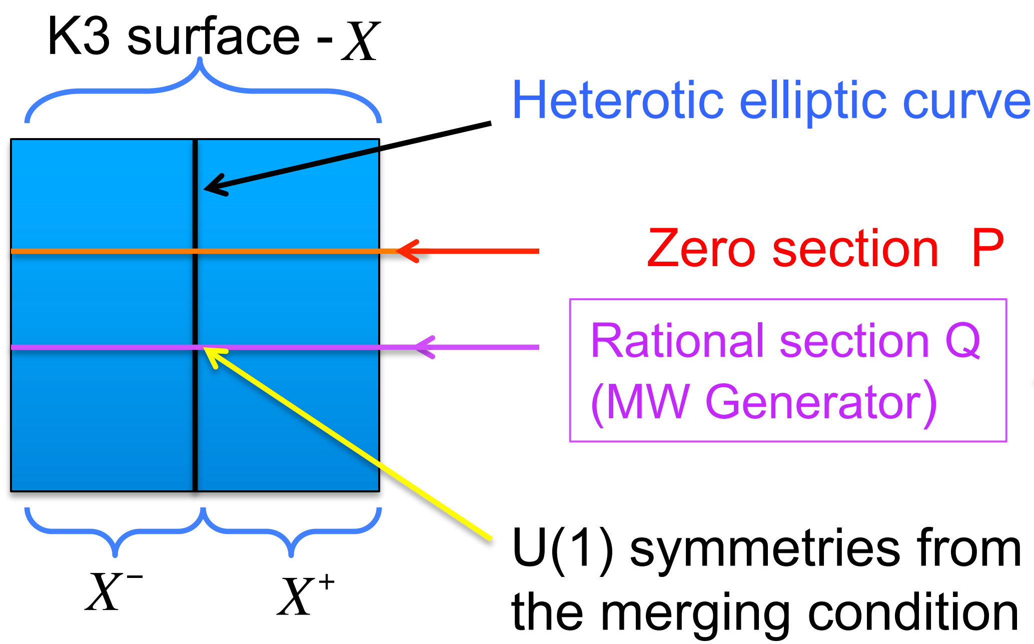
Developed for toric geometry with U(1)

Weierstrass form and stable degeneration with MW



do not commute!

Tracing U(1)s through duality



Case I

Split vector bundle-symmetric:

S(U(N-1) x U(1)) ^{2}

Example N=2: (E_7 x U(1))^2 gauge symmetry

6D: U(1)^2-massive (U(1)-background bundle)

[Witten]

→ only symmetric comb. U(1)-massless

Vector bundle with torsion:

S(U(N-1) x Z_2)

E_8xE_6xU(1)

6D: U(1)-massless

Case III

SU(N)xSU(M) bundle

6D: U(1)-massless

III. Discrete Symmetries in F-Theory

Discrete Symmetry

Calabi-Yau geometries with genus-one fibrations

These geometries do not admit a section, but a multi-section

Earlier work'00-ies: [Witten],[deBoer,Dijkgraaf,Hori,Keurentjes,Morgan,Morrison],[Sethi].
Recent extensive efforts'14-'15: [Braun, Morrison], [Morrison, Taylor], [Klevers, Mayorga-Pena, Oehlmann, Piragua, Reuter], [Anderson,Garcia-Etxebarria, Grimm, Keitel], [Braun, Grimm, Keitel], [Mayrhofer, Palti, Till, Weigand], [M.C.,Donagi,Klevers,Piragua,Poretschkin], [Grimm, Pugh, Regalado]

Key features:

Brief!

- Relation to models with U(1) symmetries via conifold transition
- Geometries with n-section ⟷ Tate-Shafarevich Group Z_n

Z_2 [Morrison, Taylor 1404.1527]
[Anderson, Garcia-Etxebarria, Grimm, Keitel 1406.1580]
[Mayrhofer, Palti, Till, Weigand 1408.6831]

Z_3 [M.C.,Donagi,Klevers,Piragua,Poretschkin 1502.06953]

Abelian & Discrete Gauge Symmetry in F-theory

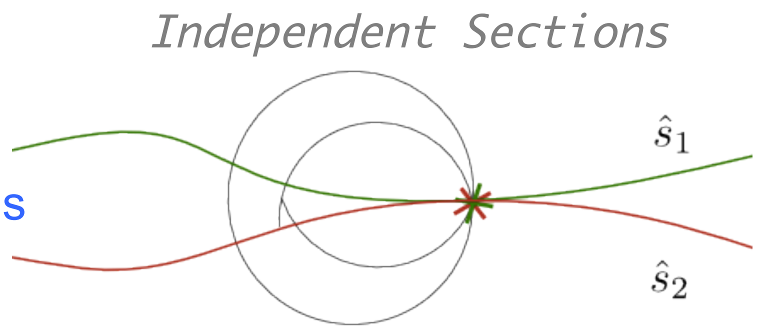
F-theory compactification with n sections (Abelian $U(1)^{(n-1)}$)

n=2 example

Singular codim-2 locus

I_2 -fiber

[Morrison,Taylor]
[Anderson,García-Etxebarria, Grimm,Keitel]
[Mayrhofer,Palti,Till,Weigand]



blow-down
(P^1 in the geometry with multiple sections collapses)

deformation

Abelian & Discrete Gauge Symmetry in F-theory

F-theory compactification with n sections (Abelian $U(1)^{(n-1)}$)

n=2 example

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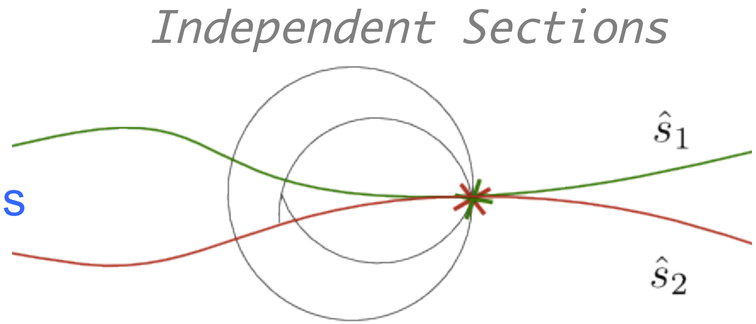
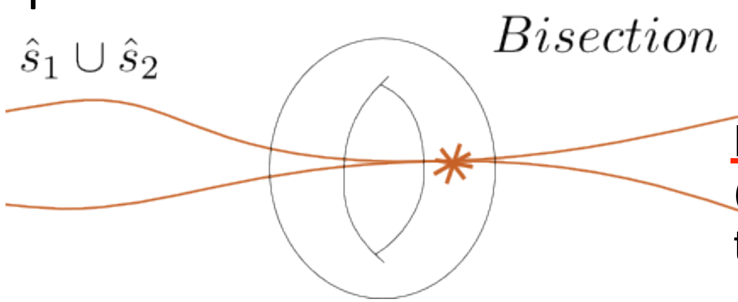
I_2 -fiber



blow-down
(P[1] in the geometry with multiple sections collapses)

appearance of massless field ϕ with charge 2

F-theory compactification with an n-section (discrete Z_n symmetry)



Deformation
(S^3 glues several sections to a multi-section)

massless field acquires VEV

$\langle\phi\rangle \neq 0$

$U(1) \rightarrow Z_2$

Conifold transition - Effective theory

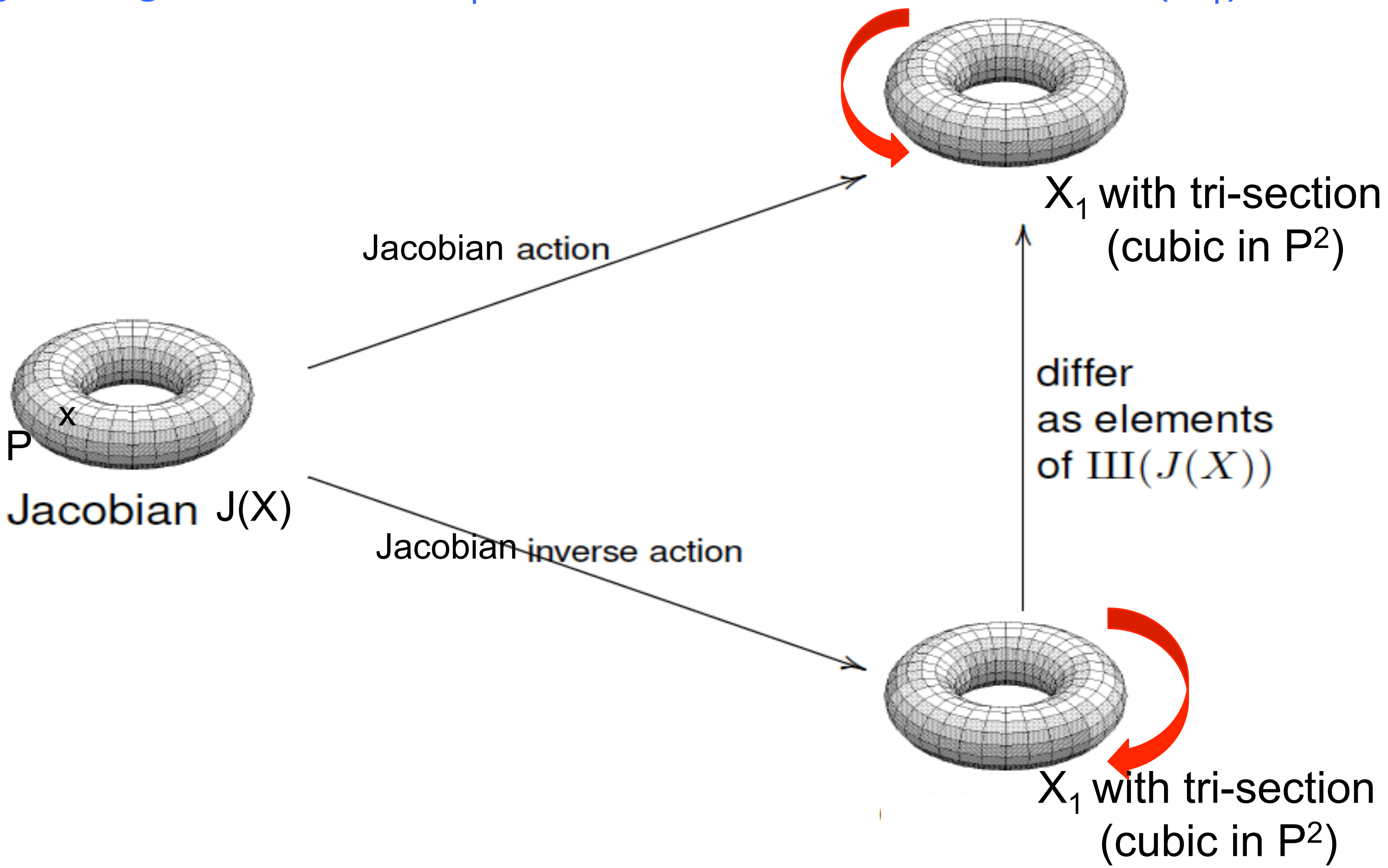
Explicit construction of U(1) geometries with matter charge n → Z_n

n=3: [Klevers, Peña, Piragua, Oehlmann, Reuter 1408.4808]
[M.C., Donagi ,Klevers ,Piragua, Poretschkin 1502.06953]...

Tate-Shafarevich group and Z_3

[M.C., Donagi, Klevers, Piragua, Poretschkin 1502.06953]

Only two geometries: X_1 w/ trisection and Jacobian $J(X_1)$



Thus, there are three different elements of TS group!

Discrete Symmetries & Heterotic/F-theory Duality

[M.C., A. Grassi, M. Poretschkin 1607.03176]

Goal: Trace the origin of discrete symmetry D

- Conjecture for $P^2(1,2,3)$ fibration [Berglund, Mayr' 98]

  X_2 elliptically fibered, toric K3 with singularities of type
  G_1 in X^+ and G_2 in X^-

  ⇕

  its mirror dual Y_2 with singularities of type
  H_1 in X^+ and H_2 in X^- with $H_i=[E_8 , G_i]$

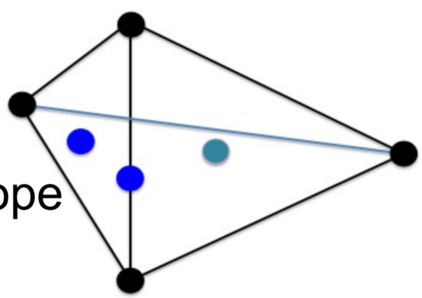
- Employ conjecture to construct background bundles with structure group G where $[E_8 , G] = D$ - beyond $P^2(1,2,3)$

- Explore ``symmetric'' stable degeneration with $H_1=H_2$
  → symmetric appearance of discrete symmetry D

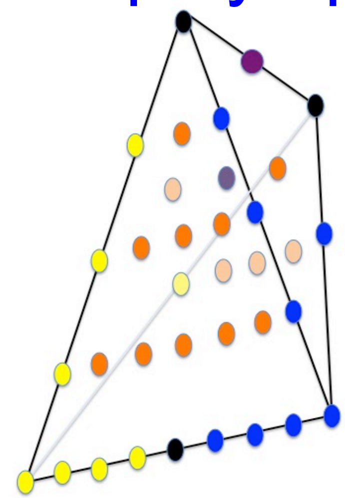
Example with Z_2 symmetry

Polytope:

Newton polytope



Dual polytope:



8D: $((E_7 \times SU(2))/\mathbb{Z}_2)^2$ - gauge symmetry

$\mathbb{Z}_2^2$ - gauge symmetry

c.f., [Aspinwall'05]

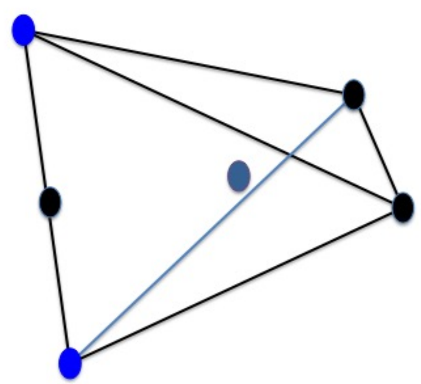
6D: $(E_7 \times E_7 \times SU(2))/\mathbb{Z}_2$ - gauge symmetry

$\mathbb{Z}_2$ - gauge symmetry

Field theory: Higgsing symmetric U(1) model:
only one (symm. comb.) U(1)-massless
→ only one Z_2 -``massless"

Example with Z_3 symmetry

Polytope:



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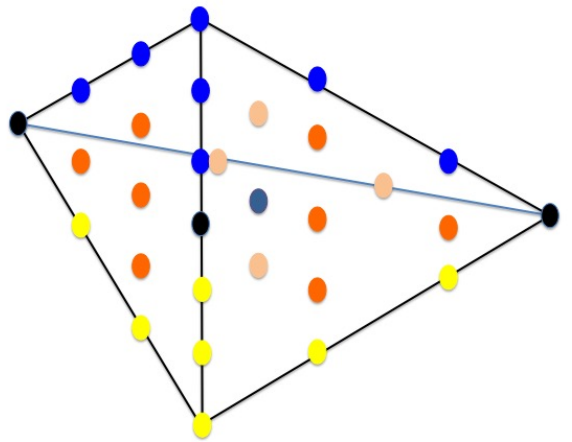


6D: $(E_6 \times E_6 \times SU(3))/\mathbb{Z}_3$ - gauge symmetry

$\mathbb{Z}_3$ - gauge symmetry

All these examples demonstrate:
toric CY's with MW torsion of order-n, via Heterotic duality related to dual toric CY's with n-section.

Related: [Klevers, Peña, Piragua, Oehlmann, Reuter 1408.4808]

IV. Particle Physics & F-theory

concrete examples

Initial focus: F-theory with SU(5) Grand Unification

[10 10 5 coupling,...] [Donagi,Wijnholt'08][Beasley,Heckman,Vafa'08]...

Model Constructions:

local [Donagi,Wijnholt'09-10]...[Marsano,Schäfer-Nameki,Saulina'09-11]...

Review: [Heckman]

global

[Blumehagen,Grimm,Jurke,Weigand'09][M.C., Garcia-Etxebarria ,Halverson'10]...
[Marsano,Schäfer-Nameki'11-12]...[Clemens,Marsano,Pantev,Raby,Tseng '12]...
[Braun,Grimm,Keitel 1306.05677]...[Lawrie,Schäfer-Nameki,Wong 1504.05593]...

Other Particle Physics Models:

Standard Model building blocks (via toric techniques)

[Lin,Weigand 1406.6071] SM x U(1) [1604.04292]

First Global 3-family Standard, Pati-Salam,Trinification Models

[M.C., Klevers, Peña, Oehlmann, Reuter 1503.02068]

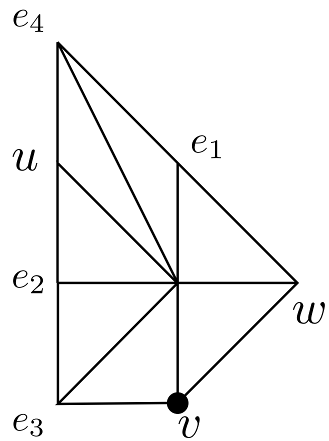
Standard Model with Z_2 (R-parity)

[M.C., Klevers, Oehlmann, Reuter 1608....] No-time!

Standard Model

[M.C., Klevers, Peña, Oehlmann, Reuter 1503.02068]

F_{11} polytope



Elliptic curve:

$$p_{F_{11}} = s_1 e_1^2 e_2^2 e_3 e_4^4 u^3 + s_2 e_1 e_2^2 e_3^2 e_4^2 u^2 v + s_3 e_2^2 e_3^2 u v^2 + s_5 e_1^2 e_2 e_4^3 u^2 w + s_6 e_1 e_2 e_3 e_4 u v w + s_9 e_1 v w^2$$

[hypersurface constraint in dP_4 ($\mathbb{P}^2$[u:v:w] with four blow-ups [e_1:e_2:e_3:e_4])

Gauge symmetry: $\mathbf{SU(3)} \times \mathbf{SU(2)} \times \mathbf{U(1)}$

Matter:

Representation	$(\mathbf{3}, \mathbf{2})_{1/6}$	$(\bar{\mathbf{3}}, \mathbf{1})_{-2/3}$	$(\bar{\mathbf{3}}, \mathbf{1})_{1/3}$	$(\mathbf{1}, \mathbf{2})_{-1/2}$	$(\mathbf{1}, \mathbf{1})_{-1}$

Construct G_4 for chiral index & D3-tadpole constraint

Standard Model

Base B = $\mathbb{P}^3$ Divisors in the base: $\mathcal{S}_7 = n_7 H_{\mathbb{P}^3}$

$\mathcal{S}_9 = n_9 H_{\mathbb{P}^3}$

Globally consistent solutions (#(families);n_{D3}) for allowed (n_7,n_9)

$n_7 \backslash n_9$	1	2	3	4	5	6	7
7							
6							
5							
4							
3							
2							
1							
0							
-1							
-2							

Standard Model

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Globally consistent solutions (#(families);n_{D3}) for allowed (n_7,n_9)

$n_7 \backslash n_9$	1	2	3	4	5	6	7
7	–	$(27;16)$	–	–			
6	–	$(12;81)$	$(21;42)$	–	–		
5	–	–	$(12;57)$	$(30;8)$	–	$(3;46)$	
4	$(42;4)$	–	$(30;32)$	–	–	–	–
3	–	$(21;72)$	–	–	–	$(15;30)$	
2	$(45;16)$	$(24;79)$	$(21;66)$	$(24;44)$	$(3;64)$		
1	–	–	–	–			
0	–	–	$(12;112)$				
-1	$(36;91)$	$(33;74)$					
-2	–						

Summary and Outlook

- Key ingredients of F-theory Compactification

  Geometric perspective - discrete data

  gauge symmetry, matter, Yukawa couplings

- Recent developments

  Abelian & Discrete Symmetries (related to respective MW & TS group)

  Highlight insights into heterotic duality

- Particle Physics Models

  SU(5) GUT's & three family Standard Model & with R-parity

  (tip of the iceberg)

Issues: continuous data such as coupling magnitudes,…
moduli stabilization,…

More work…

謝謝您