Negative Branes, Supergroups and The Signature of Spacetime

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Arxiv: with Ben Heidenreich, Patrick Jefferson & Cumrun Vafa

Controversial Ideas

- Supergroup gauge theories
- Negative-energy states
- Non-unitary theories
- Multiple times
- Signature change in quantum gravity
- Exotic string theories consistent with supersymmetry

Supergroup Gauge Theories

Gauge supergroup $U(N|M)$

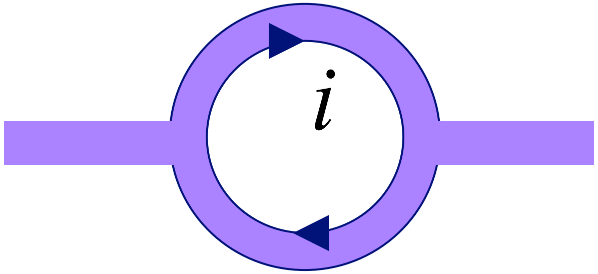
Symmetries of $v=(z,\theta)\in\mathbb{C}^{N|M},\quad |v|^2=\sum_{i=1}^{N}|z_i|^2+\sum_{a=1}^{M}\theta_a\bar{\theta}_a$

$$\Phi=\begin{pmatrix} A & B \\ C & D \end{pmatrix},\quad A,D\ \text{even},\ B,C\ \text{odd.}$$

B, C ghost fields

Large *N, M* expansion

$$\mathrm{Str}(1)=\sum_{i=1}^{N}1-\sum_{j=1}^{M}1=N-M$$



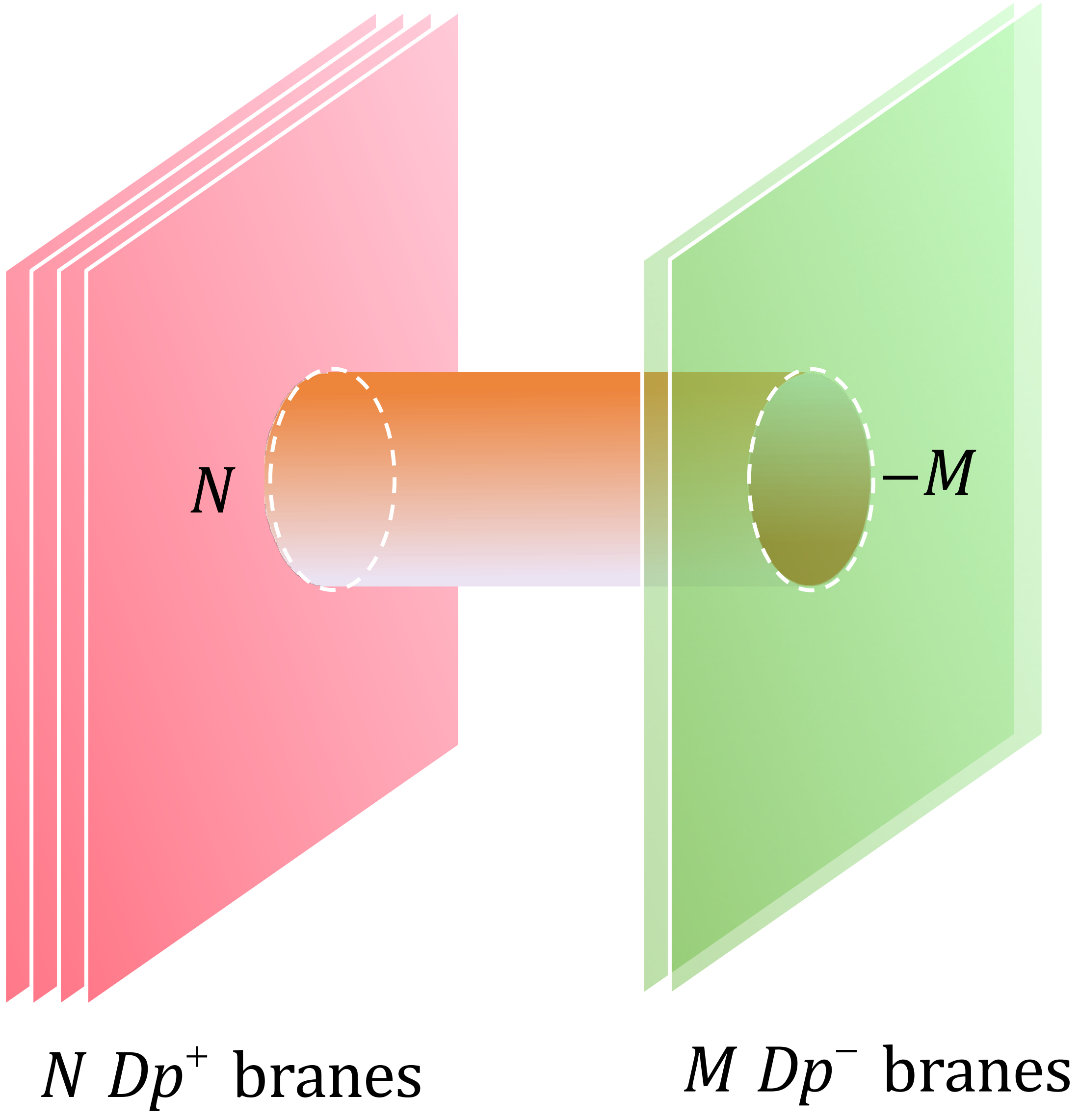
Perturbatively

$$U(N\,|\,M)\cong U(N-M)$$

Why consider supergroups?

- Determination of signature of space-time in string theory.
- Emergence of time, dS/CFT duality.
- Cosmological holography, away from S-matrix.
- Second-quantization of branes.
- Full spectrum of extensive objects in M/string theory.
- Different non-perturbative completions.
- Looking behind the horizon of black holes.

Negative-Brane Realization



Okuda, Takayanagi,
Ghost branes

Positive & Negative BPS Branes

- D^+ branes, charge $Q > 0$

  Tension $M = Q > 0$

- D^+ antibranes, charge $Q < 0$

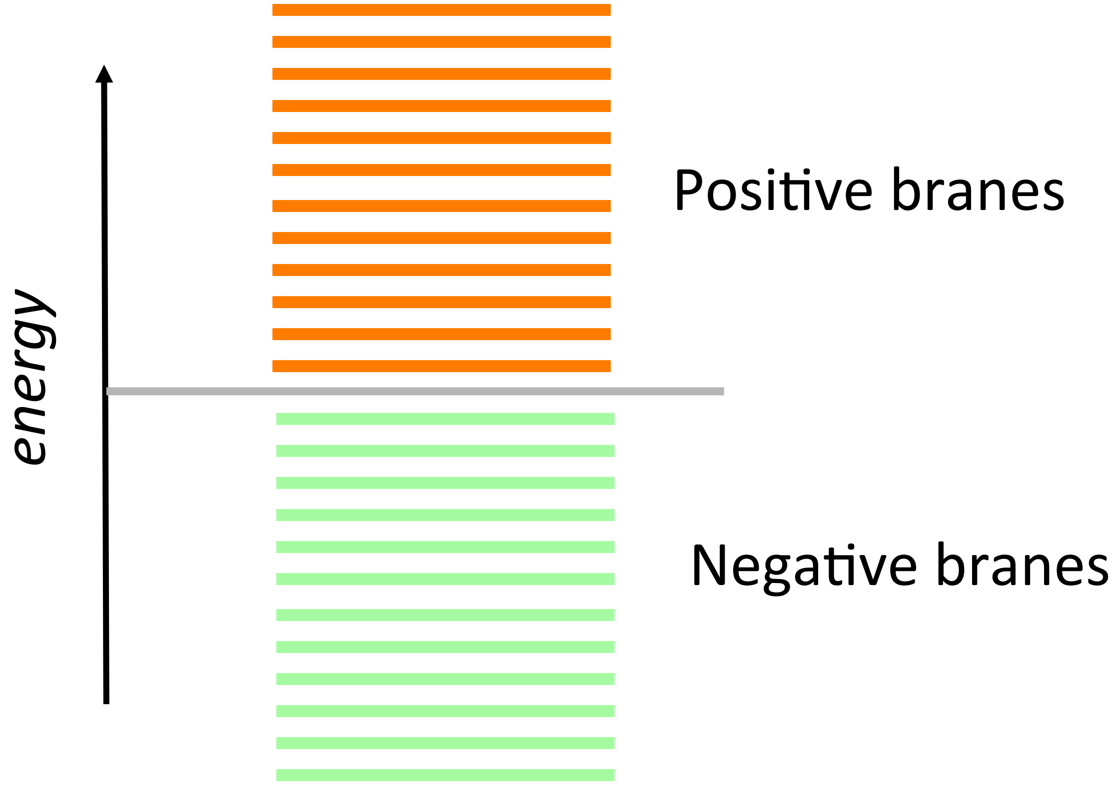
  Tension $M = - Q > 0$

- D^- branes, charge $Q < 0$

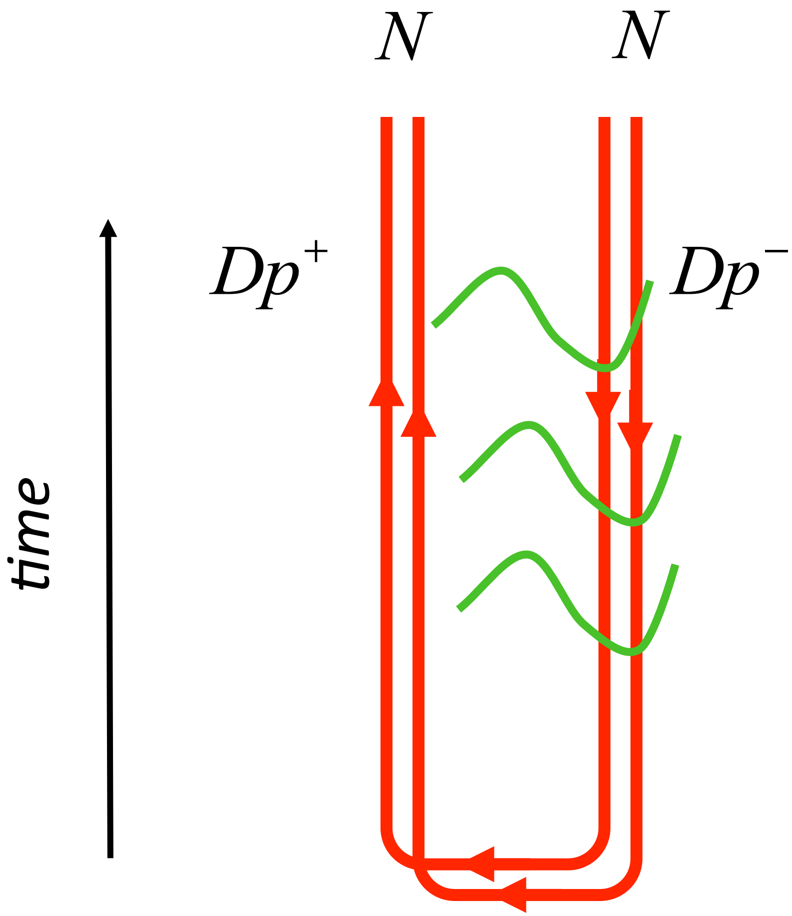
  Tension $M = Q < 0$

Preserve the same supersymmetry algebra

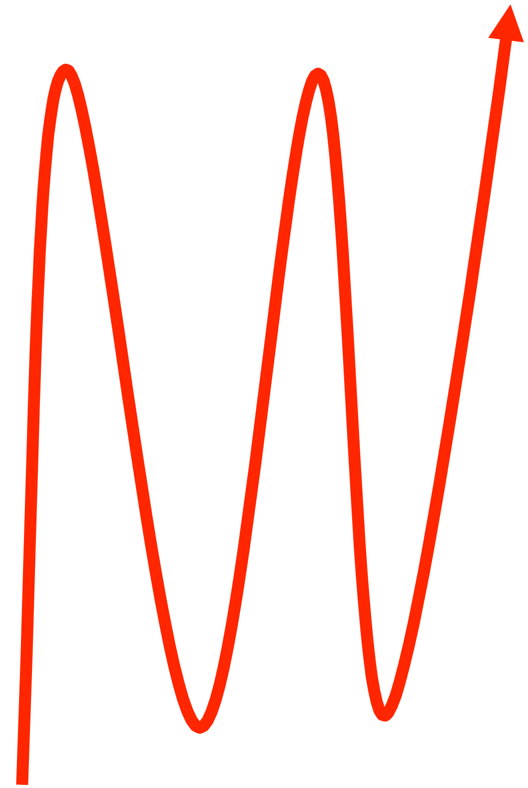
Dirac Sea



Second Quantization



$$U(N\,|\,N)$$



$$\sum_k U(1+k\,|\,k)$$

$$U(N) \xrightarrow{?} \sum_k U(N+k\,|\,k)$$

Supergroup Gauge Theories

$U(N+k|k)$ Yang-Mills theory

$$\mathrm{Str}(F^2 + D\Phi^2 + \ldots) = \mathrm{Tr}\, F_+^2 - \mathrm{Tr}\, F_-^2 + \mathrm{Tr}\, \mathrm{D}\Phi_+^2 - \mathrm{Tr}\, \mathrm{D}\Phi_-^2 + \ldots$$

- Does it exist non-perturbatively? Different completions of $U(N)$ for different values of *k*,
- Non-unitary: negative energy *v.* negative norms.

Non-Unitary QFTs

- Do exist in 2d, *e.g.* general (p,q) minimal models
- Lee-Yang edge singularity, ϕ^3, critical O(N),… analyzed numerically in dimension $d > 2$.
- In Euclidean space-time turn into oscillating integral

$$\int D\varphi \cdot e^{\frac{i}{\hbar}\int \partial\varphi_+^2 - \partial\varphi_-^2 + \cdots}$$

- Choose appropriate contour in complex field space: holomorphic blocks?

Exact RG

Arnone, Kubyshin, Morris, Tighe

Bosonic Yang-Mills with gauge supergroup, plus Higgs fields

$$SU(N\,|\,N) \to SU(N) \times SU(N)$$

$$A = \begin{pmatrix} A_+ & 0 \\ 0 & A_- \end{pmatrix}$$

- Second gauge field A_- Pauli-Villars field.
- Fermionic gauge field massive after symmetry breaking.
- Cut-off high enough A_- not excited.
- Keeping SU(N) gauge symmetry explicit.

Negative D-branes

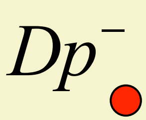
Black D-brane geometry in string frame

$$ds^2 = H^{-1/2} ds^2_{p+1} + H^{1/2} ds^2_{9-p}, \quad e^{-2\Phi} = g_s^{-2} H^{\frac{p-3}{2}}$$

Harmonic function

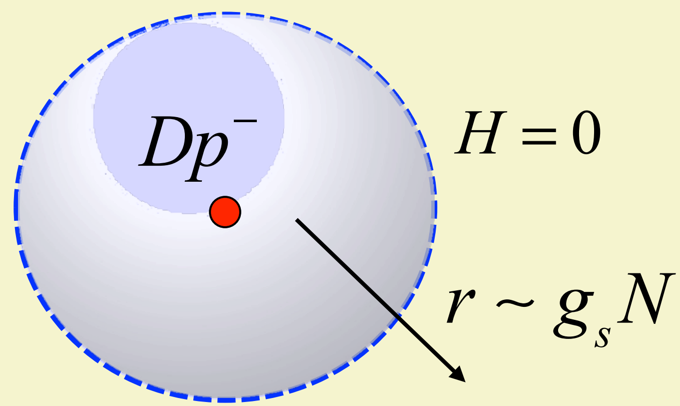
$$H(r) = 1 + c_p \sum_i \frac{g_s N_i}{|r - r_i|^{7-p}}$$

Perturbatively $g_s \sim 0$:



Flat space-time

Non-perturbatively $g_s \neq 0$:
naked singularity at $H = 0$



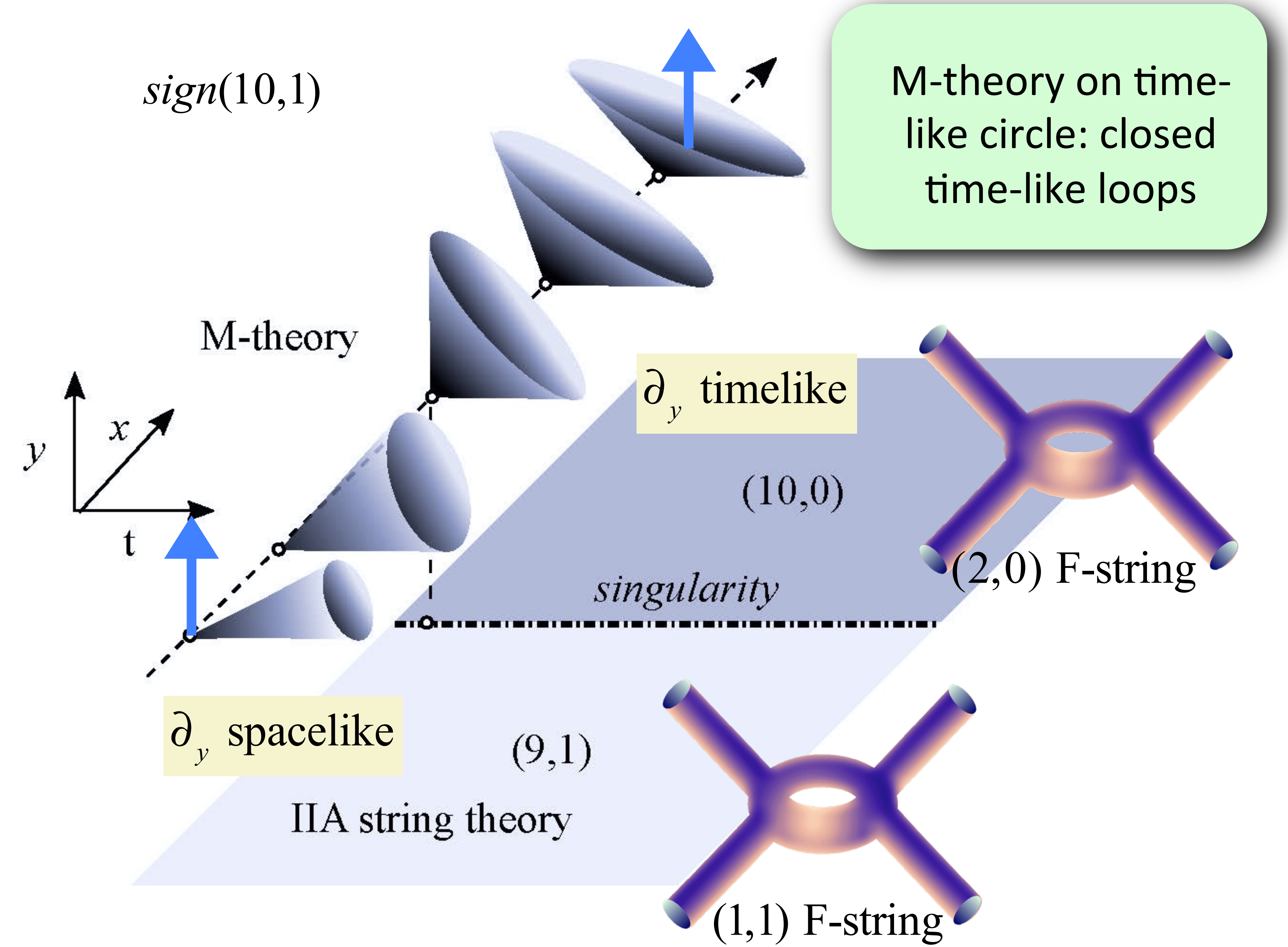
Negative D0-branes

M-theory metric in 11 dimensions $y = x_{11} \simeq y + g_s$

$$ds_{11}^2 = H(x)dy^2 + 2dtdy + ds_9^2$$

$$H(x) \sim (x - x_0)$$

pp-wave, smooth at $H = 0$



sign(10,1)
M-theory on time-like circle: closed time-like loops
M-theory
y
x
t
∂ y timelike
(10,0)
(2,0) F-string
singularity
∂ y spacelike
(9,1)
IIA string theory
(1,1) F-string

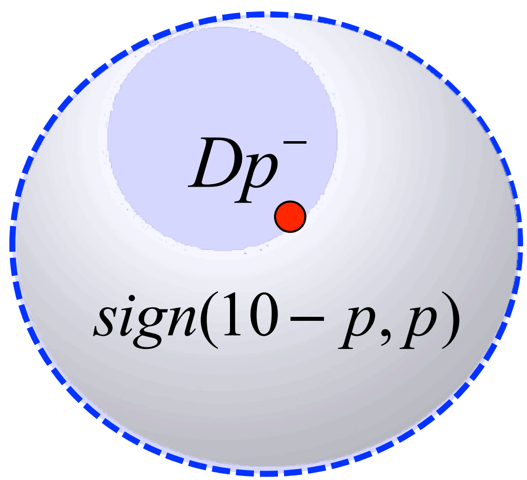
Analytic Continuation

After absorbing complex factors in $g_{\mu\nu}, \Phi,$ with $\tilde{H} = -H$

$$ds^2 = -\tilde{H}^{-1/2} ds^2_{p+1} + \tilde{H}^{1/2} ds^2_{9-p}, \quad e^{-2\Phi} = g_s^{-2} \tilde{H}^{\frac{p-3}{2}}$$

Bubble of space-time with different signature

$\mathbb{R}^{p+1} \times S^{9-p}$



$sign(9,1)$

Supersymmetric probes finite tension $L_{DBI} \sim H^{k-1}, \;\; k > 0$

Cauchy Problem

More than one time: ill-posed initial value problem

$$\partial_t^2 \phi = \Delta_{r,s} \phi$$

Spatial momentum space sign(*r*,*s*). Non-local constraint

$$\phi_0(x) = \int_{p^2>0} d^{r+s} p \cdot \tilde{\phi}_0(p) e^{ipx}$$

How to extend to interacting theory?

Modular theta-functions for indefinite signatures

$$\vartheta(\tau) \sim \sum_{p \in \Gamma^{r,s},\, p^2>0} \varepsilon(p) \cdot e^{i\pi p^2}$$

Hull Dualities

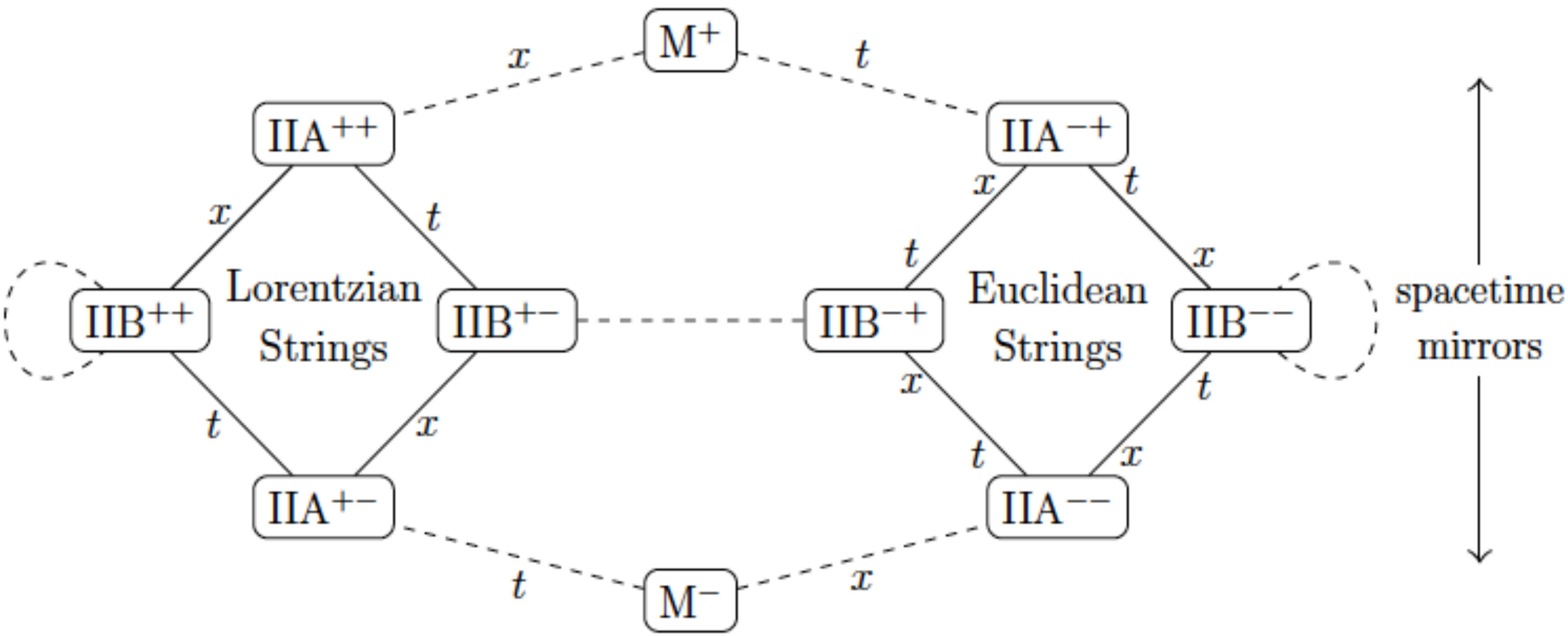
Hull, Khuri

T-dualities of Type II string theory on compactified timelike dimension: exotic string theories.

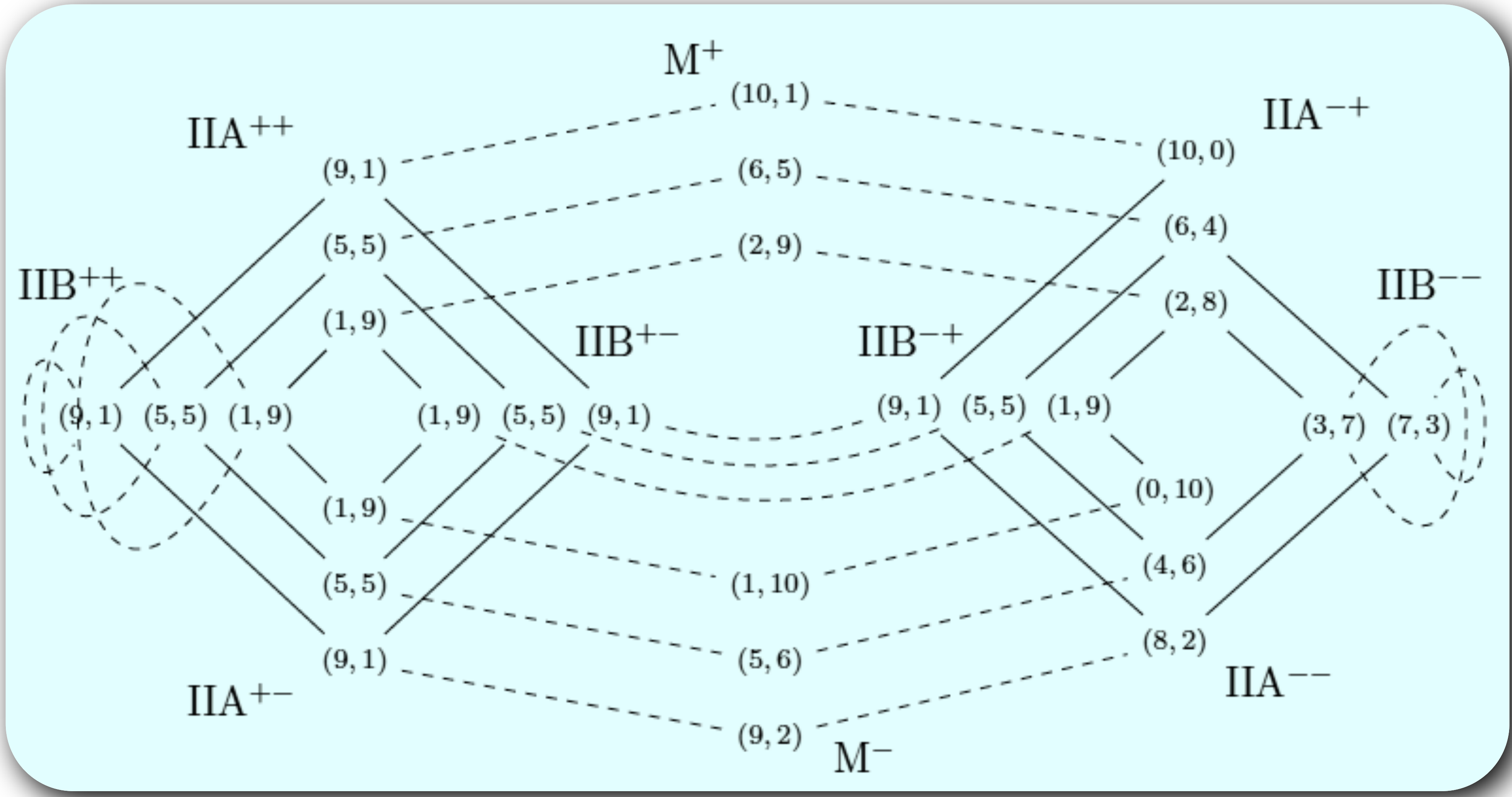
F-strings, D-branes, NS5-branes will also have different signatures,

Type IIA/B$^{\pm\pm}$ F1, D1/2 Lorentzian or Euclidean

M$^{\pm}$-theory: M2 Lorentzian or Euclidean



Web of Possible Signatures



Possible Spacetime & Worldvolume Signatures

	Spacetime	M2	M5
M^+	$(10,1),(6,5),(2,9)$	$(+,-)$	$(-,-)$
M^-	$(9,2),(5,6),(1,10)$	$(-,+)$	$(-,-)$

	Spacetime	D0	D2	D4	D6	D8	F1	NS5
IIA^{++}	$(9,1),(5,5),(1,9)$	$(+,-)$	$(+,-)$	$(+,-)$	$(+,-)$	$(+,-)$	$(-,-)$	$(-,-)$
IIA^{+-}	$(9,1),(5,5),(1,9)$	$(-,+)$	$(-,+)$	$(-,+)$	$(-,+)$	$(-,+)$	$(-,-)$	$(-,-)$
IIA^{-+}	$(10,0),(6,4),(2,8)$	$(-,+)$	$(+,-)$	$(-,+)$	$(+,-)$	$(-,+)$	$(+,+)$	$(-,-)$
IIA^{--}	$(8,2),(4,6),(0,10)$	$(+,-)$	$(-,+)$	$(+,-)$	$(-,+)$	$(+,-)$	$(+,+)$	$(-,-)$

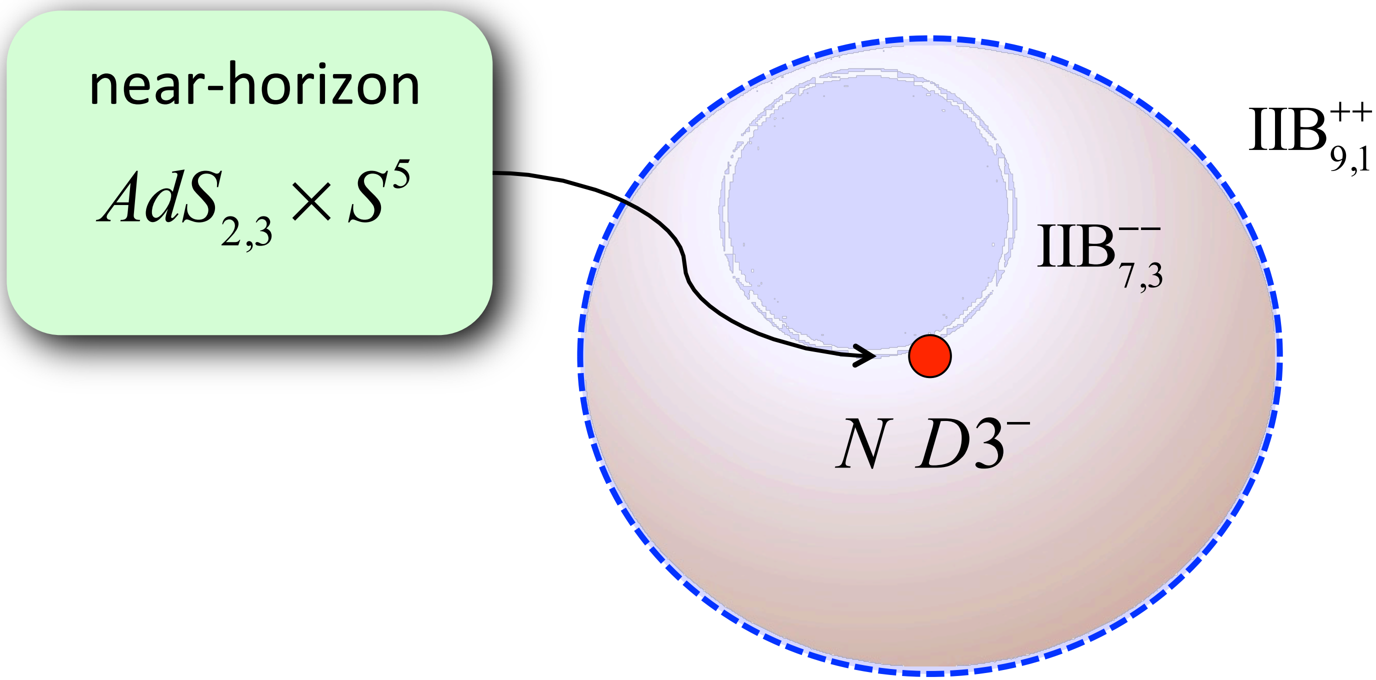
	Spacetime	D(−1)	D1	D3	D5	D7	D9	F1	NS5
IIB^{++}	$(9,1),(5,5),(1,9)$	—	$(-,-)$	$(-,-)$	$(-,-)$	$(-,-)$	✓	$(-,-)$	$(-,-)$
IIB^{+-}	$(9,1),(5,5),(1,9)$	✓	$(+,+)$	$(+,+)$	$(+,+)$	$(+,+)$	—	$(-,-)$	$(-,-)$
IIB^{-+}	$(9,1),(5,5),(1,9)$	✓	$(-,-)$	$(+,+)$	$(-,-)$	$(+,+)$	✓	$(+,+)$	$(+,+)$
IIB^{--}	$(7,3),(3,7)$	—	$(+,+)$	$(-,-)$	$(+,+)$	$(-,-)$	—	$(+,+)$	$(+,+)$

AdS/CFT

Near-horizon geometry for U(0|N) D3⁻ branes: $IIB_{7,3}$ string theory

$$AdS_{4,1} \times S^5 \rightarrow AdS_{2,3} \times S^5$$

Bubble of space-time with different signature

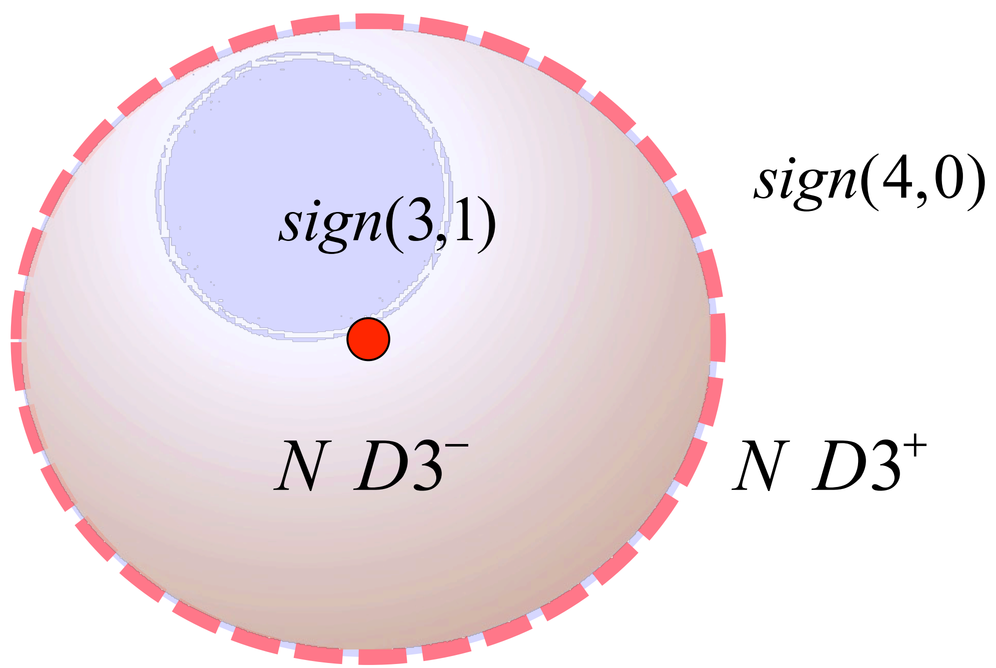


Cosmological Applications

3D gauge theory with one scalar

$$U(N|N) \to U(N) \times U(N)$$

Bubble of space-time with different signature



N=2 Supergroup Theories

N=2 super Yang-Mills with gauge supergroup

$$SU(N \mid M)$$

Beta function

$$\beta = -(N - M) < 0$$

Claim: SW curve depends on $N + M - 1$ parameters, equivalent to $SU(N) + 2M$ matter fields with pairwise equal masses.

Recall SW-curve

$$SU(N): \qquad z + \det(x - \Phi) + \frac{1}{z} = 0, \qquad \lambda = x\frac{dz}{z}$$

N=2 Supergroup Theories

Claim: SW-curve for supergroup

$$SU(N\,|\,M): \qquad z + \mathrm{sdet}(x-\Phi) + \frac{1}{z} = 0, \qquad \lambda = x\frac{dz}{z}$$

$$\Phi = (a_1, \ldots, a_N \,|\, b_1, \ldots b_M)$$

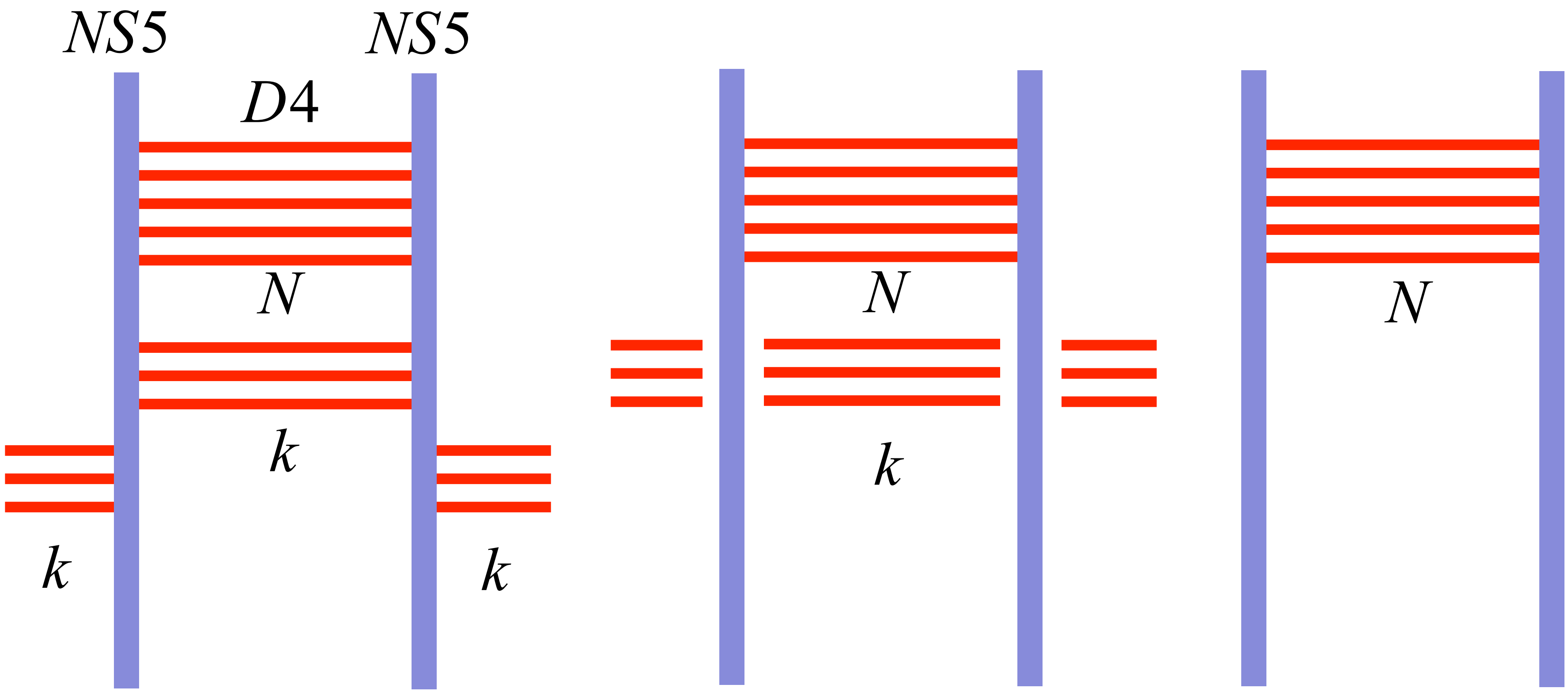
$$z + \frac{\prod_{i=1}^{N}(x-a_i)}{\prod_{j=1}^{M}(x-b_j)} + \frac{1}{z} = 0$$

SU(*N*) with 2*M* matter fields

$$z\prod_{j=1}^{M}(x-b_j) + \prod_{i=1}^{N}(x-a_i) + \frac{1}{z}\prod_{j=1}^{M}(x-b_j) = 0$$

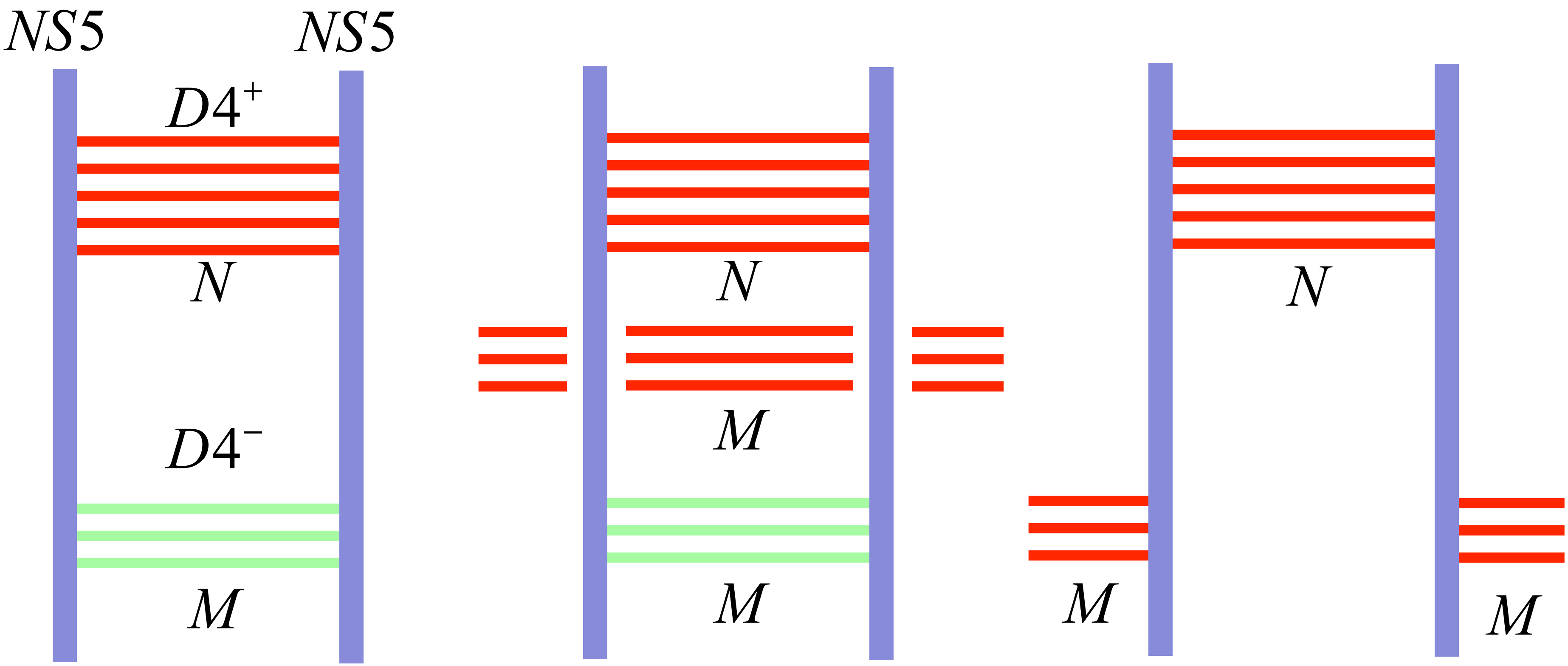
Exercise: Higgsing

$$SU(N+k)+2k \text{ flavors} \to SU(N)$$



Negative Branes

$$SU(N\,|\,M) \rightarrow SU(N) + 2M \text{ flavors}$$



Geometric Engineering

Multicenter Taub-NUT space for D3 branes

$$ds^2 = \frac{1}{V}(d\theta + A)^2 + V dy^2$$

$$V = 1 + \sum_{i=1}^{N} \frac{1}{\left|\vec{y} - \vec{a}_i\right|} - \sum_{j=1}^{M} \frac{1}{\left|\vec{y} - \vec{b}_j\right|}$$

Complex structure A_{N-1} singularity

$$uv = x^N \to uv = \prod (x - a_i)$$

Generalizes to

$$uv = x^{N-M} \to uv = \frac{\prod_{i=1}^{N}(x - a_i)}{\prod_{j=1}^{M}(x - b_j)}$$

Nekrasov Instanton Calculation

Start with

$$SU(N|N) \to SU(N) \times SU(N) + 2 \text{ bifundamentals}$$

Couplings

$$\tau_1 = -\tau_2 = \tau$$

Limit of exact SW curve

$$\lim_{q \to 0} \left(\frac{q_1}{q_2} \right)^{1/4} \frac{\vartheta_2\left(z^2; q^2\right)}{\vartheta_3\left(z^2; q^2\right)} = \frac{P_2(x)}{P_1(x)}, \quad q = q_1 q_2$$

$$s\det\left(x - \Phi\right) = \frac{P_2(x)}{P_1(x)} = 2q_1^{-1/2}(z + z^{-1})$$

Super Matrix Models

Supergroup

$$Z_{N+k|k} = \frac{1}{\mathrm{Vol}\, U(N\,|\,M)} \int d\Phi \cdot e^{Str\, W(\Phi)/g_s}$$

Volume Lie supergroup vanishes: need regularization

$$\int d\theta \cdot 1 = 0$$

Diagonalize: eigenvalues & anti-eigenvalues

$$\Phi \sim \begin{pmatrix} x_i & 0 \\ 0 & y_j \end{pmatrix} \qquad S = \sum_{i=1}^{N} W(x_i) - \sum_{j=1}^{M} W(x_i)$$

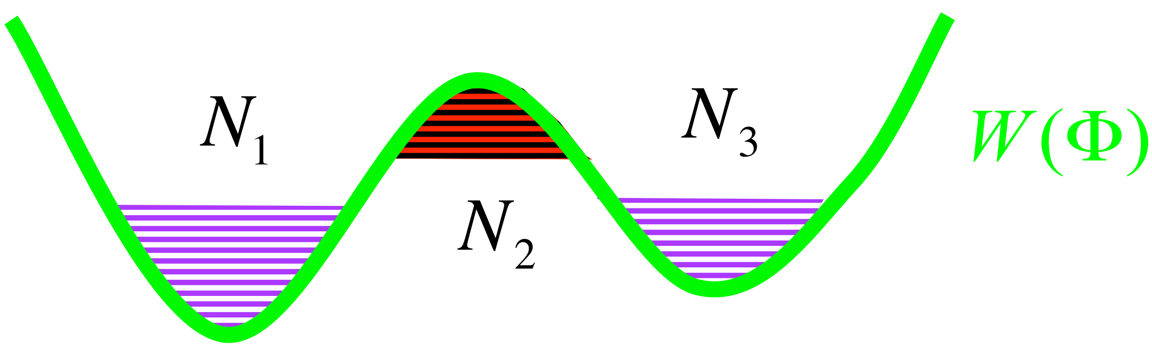
Bosonic Matrix Model

$$Z_{matrix} = \frac{1}{\mathrm{Vol}\, U(N)} \int_{N\times N} d\Phi \cdot e^{\mathrm{Tr}\, W(\Phi)/g_s}$$

't Hooft limit

$$N_I \to \infty, \;\; g_s \to 0, \;\; N_I g_s = \mu_I = fixed$$

Filling fractions (perturbative expansion)

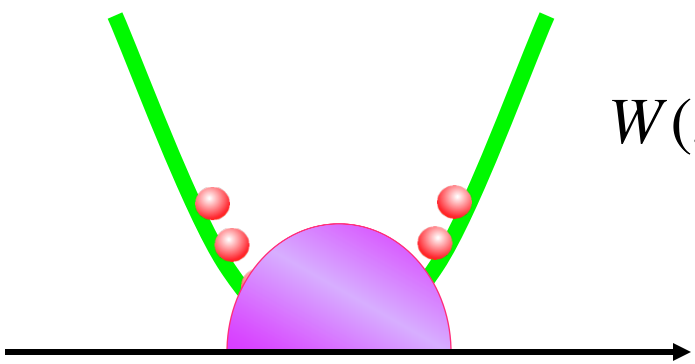


Eigenvalue Dynamics

$$Z_{matrix} = \int d^N x \cdot \prod \left(x_i - x_j\right)^2 \cdot e^{-\sum_I W(x_i)/g_s}$$

Effective action (repulsive Coulomb force)

$$S_{eff} = \sum_i W(x_i) - 2g_s \sum_{i<j} \log\left(x_i - x_j\right)$$



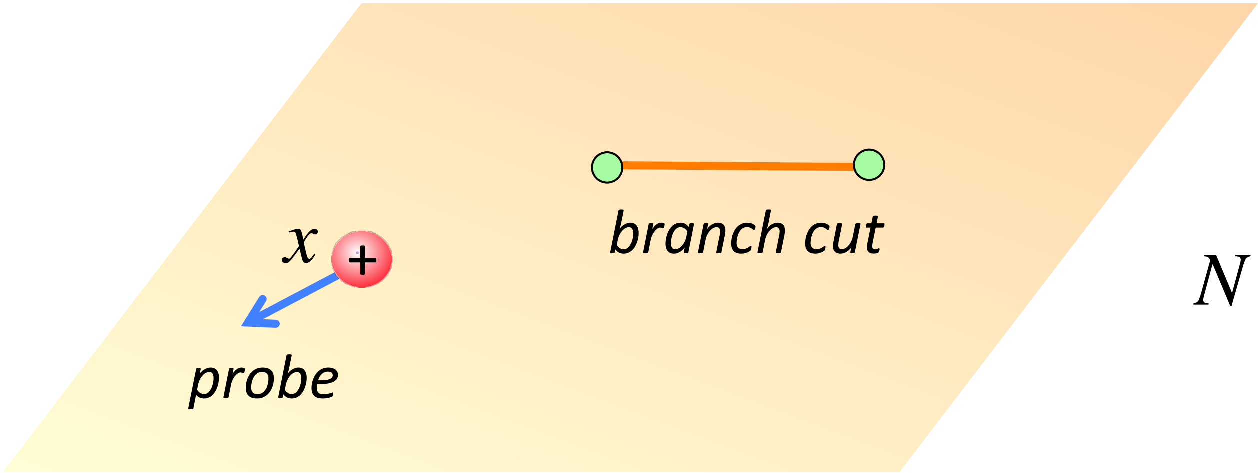
$$W(x) = x^2$$

Resolvent

$$y dx = dS_{eff} = dW(x) + 2g_s \mathrm{Tr} \frac{dx}{\Phi - x}$$

Resolvent

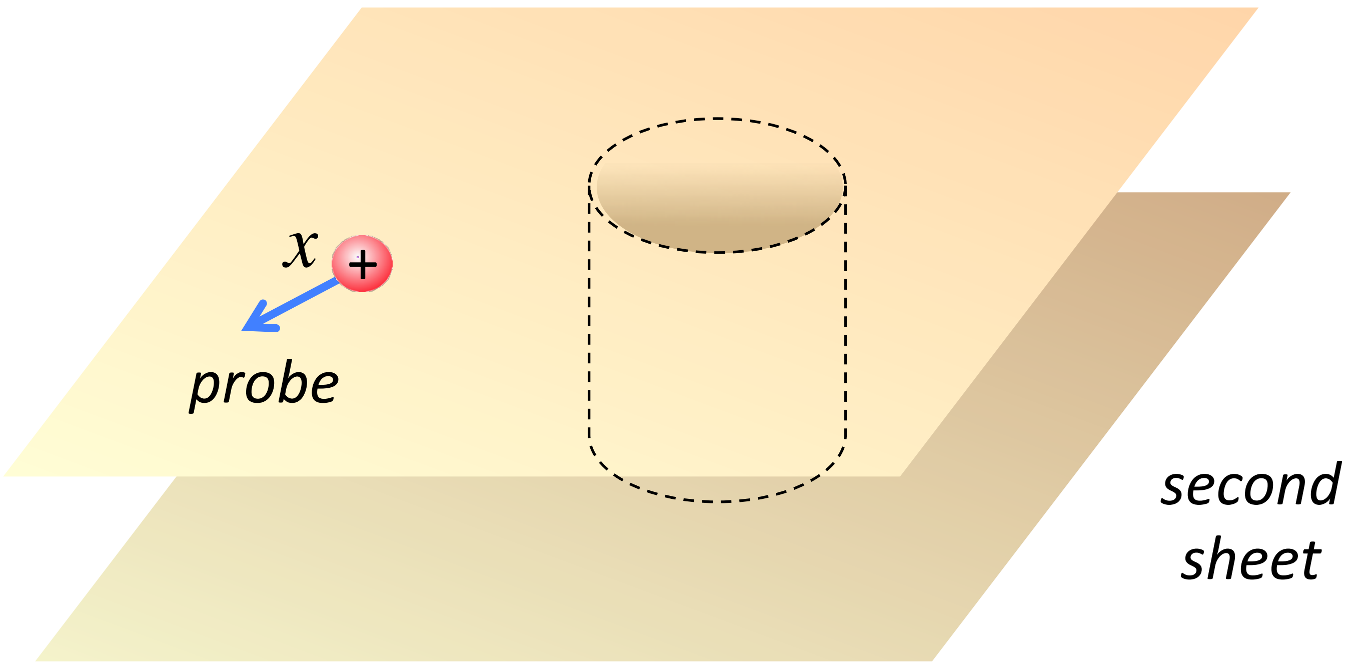
$$y dx = dS_{eff} = dW(x) + 2g_s \mathrm{Tr} \frac{dx}{\Phi - x}$$



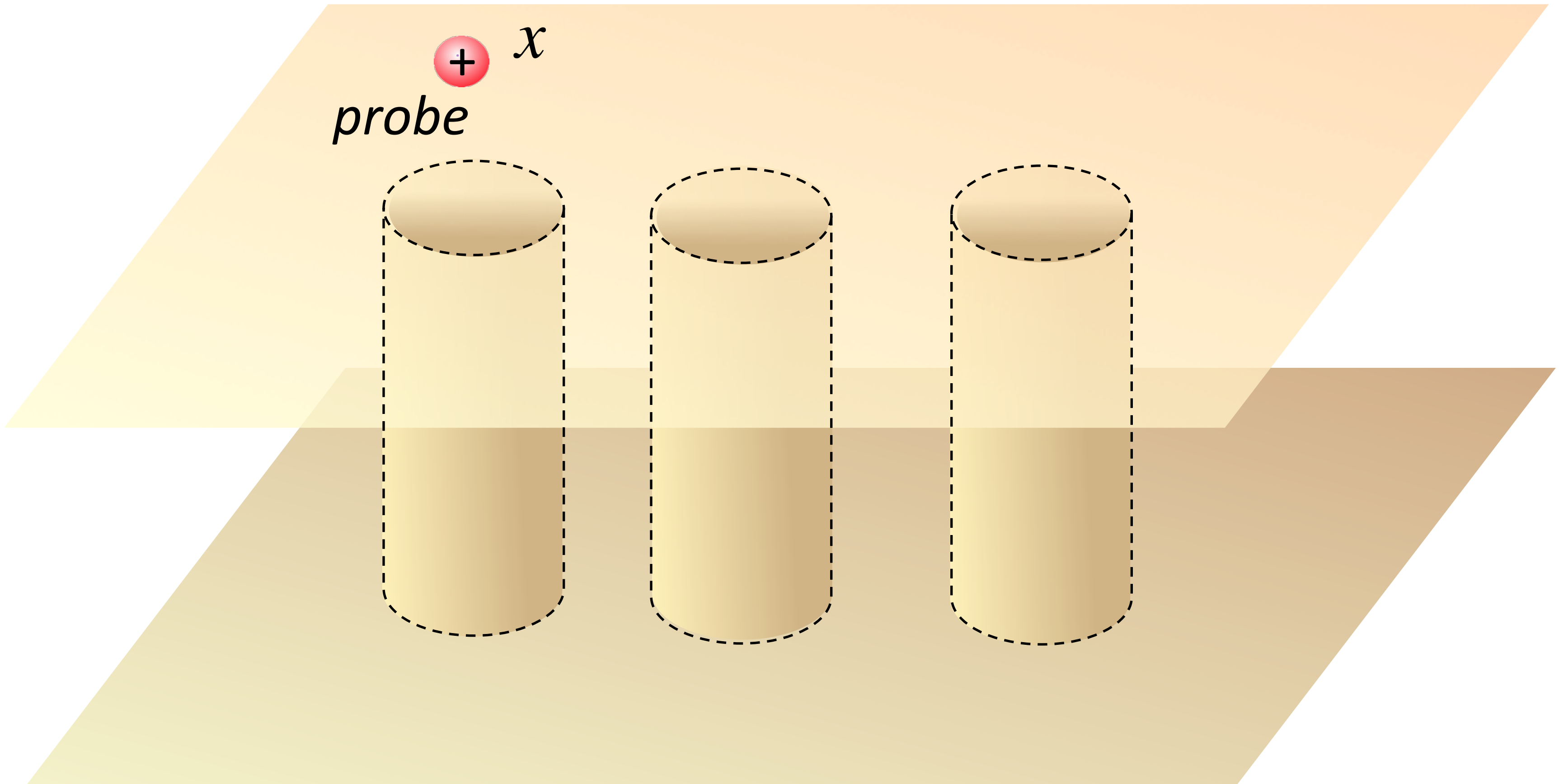
$N \to \infty$

Effective Geometry

$$y^2 = W_n'(x)^2 + f_{n-1}(x) = P_{2n}(x)$$



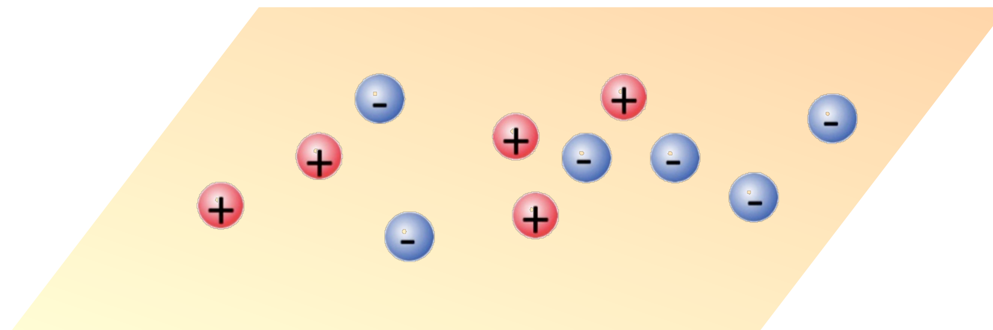
Effective Geometry



Are both sheets visible?

Super Jacobian

$$d\Phi \;\to\; d^N x \, d^M y \frac{\prod_{i<j}\left(x_i - x_j\right)^2 \prod_{a<b}\left(y_a - y_b\right)^2}{\prod_{i,a}\left(x_i - y_b\right)^2}$$



Eigenvalues of charge +1 and -1 (Coulomb gas model)
Resolvent

$$y dx \sim \mathrm{Str} \frac{dx}{\Phi - x} = \sum_i \frac{dx}{x_i - x} - \sum_i \frac{dx}{y_a - x}$$

Same periods as $U(N\text{-}M)$ model

Non-Perturbative Corrections

[Vafa] $U(N|M)$ has more Casimirs than $U(N)$, e.g.

$$\Delta = \prod_{i,a} \left(x_i - y_a \right)$$

General deformation

$$W = \sum_n t_n \mathrm{Str}\, \Phi^n$$

For example, for $U(2|1)$

$$\Delta = -\frac{1}{2}\left(\mathrm{Str}\, \Phi^2 - \left(\mathrm{Str}\, \Phi \right)^2 \right) = 0 \ \text{ for } \ U(1)$$

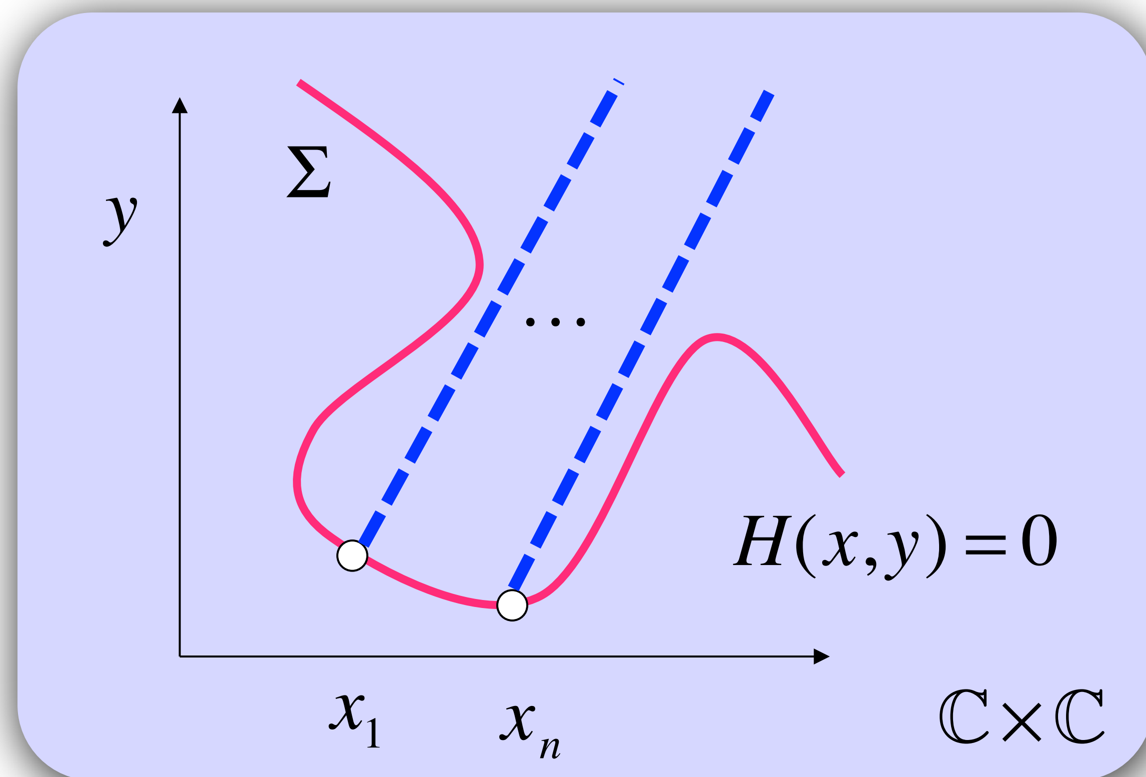
Non-Perturbative Corrections

So correlators $\left\langle \Delta \right\rangle_{N+k|k}$ are sensitive to *k.*

General pattern: calculation of partition function and correlation functions for $U(N+k\,|\,k)$ depend non-perturbatively (order e^{-N}) on *k.*

Working with supermatrix models seems to probe the full effective geometry.

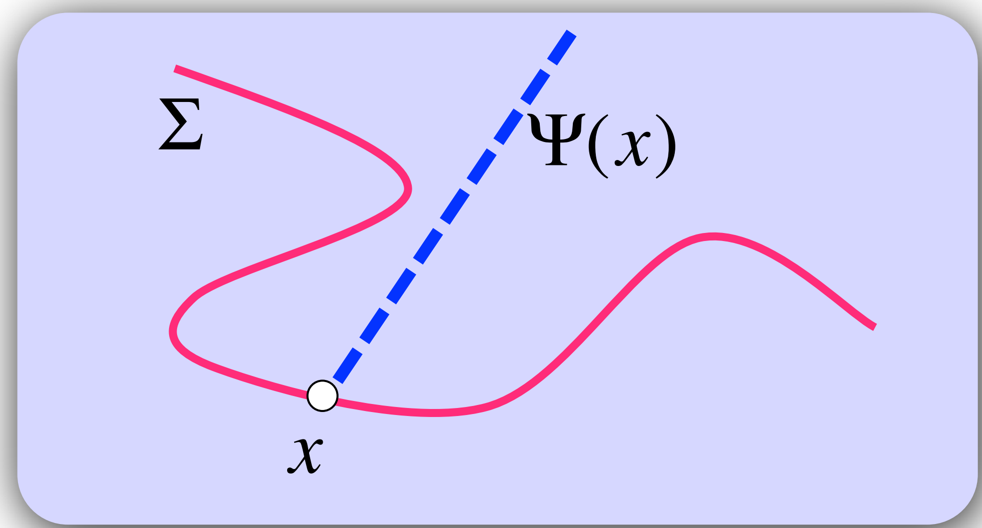
Topological D-Branes



Open string partition function

$$\Psi(x_1,\cdots,x_n)=\langle\psi(x_1)\cdots\psi(x_n)\rangle=\left\langle\prod_i \det(\Phi-x_i)\right\rangle$$

Quantum Wave Function



Single brane wave function $\Psi(x)$

$$\hat{H}\Psi = 0, \quad \hat{H} = \hat{H}(\hat{x}, \hat{y})$$

Quantization of phase space

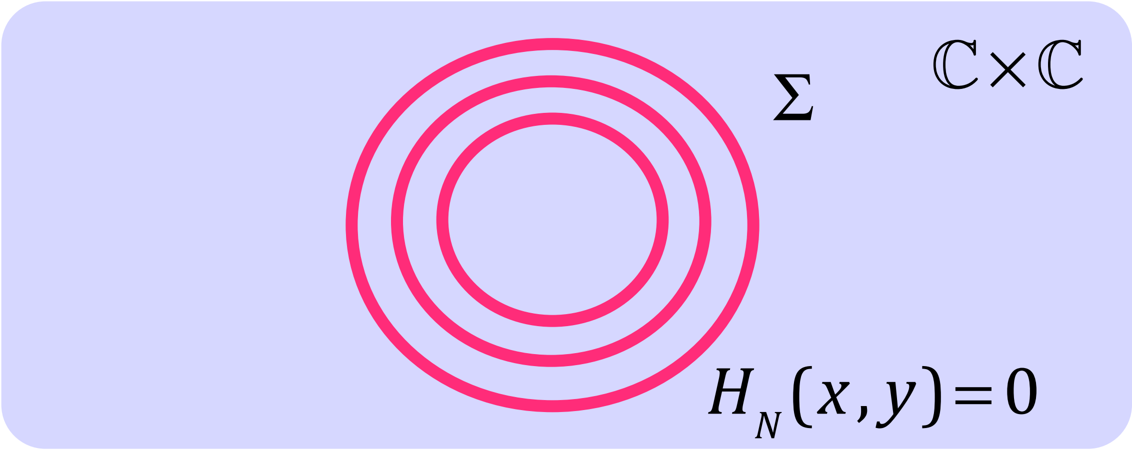
$$\hat{y} = -g_s \frac{\partial}{\partial x}, \quad [\hat{x}, \hat{y}] = g_s$$

Gaussian Matrix Model

$$\Psi_N(x) = \left\langle \det\left(\Phi - x\right)\right\rangle_N = H_{N-1}(x)\cdot e^{-x^2/2}$$

Eigenfunctions of harmonic oscillator

$$\left(-g_s^2\frac{\partial^2}{\partial x^2} + x^2 - g_s(2N-1)\right)\Psi_N(x) = 0$$



Negative Branes

$$\Psi^*_N(x) = \left\langle \frac{1}{\det(\Phi - x)} \right\rangle_N = \int \frac{\Psi_N(y)}{y - x} dy$$

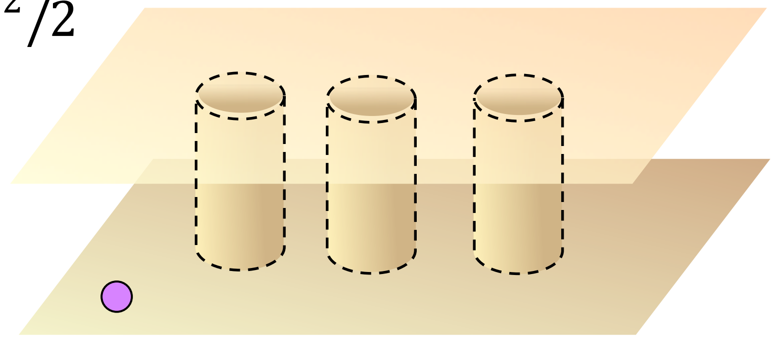
Non-normalizable eigenfunctions of harmonic oscillator

$$\widehat{H}_N \Psi^*_N(x) = 0$$

WKB behavior

$$\Psi^*(x) \sim e^{+x^2/2}$$

Probe other sheet



Conclusion

Unification of crazy ideas

Nee

Supergroup gauge theories

Non-unitary QFT

Negative tension branes

Spacetime signature change

Closed timelike loops