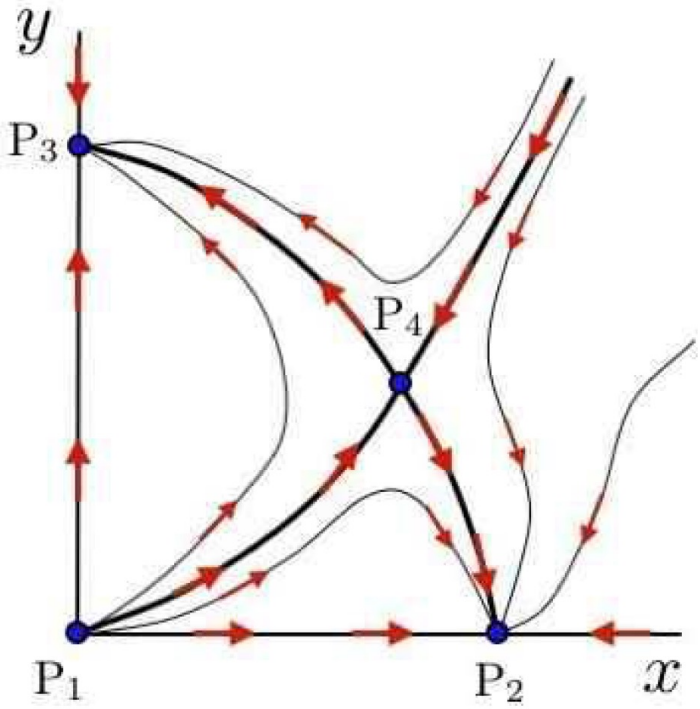
California Institute of Technology

Fun with RG flows

Sergei Gukov

based on: arXiv:1503.01474

work in progress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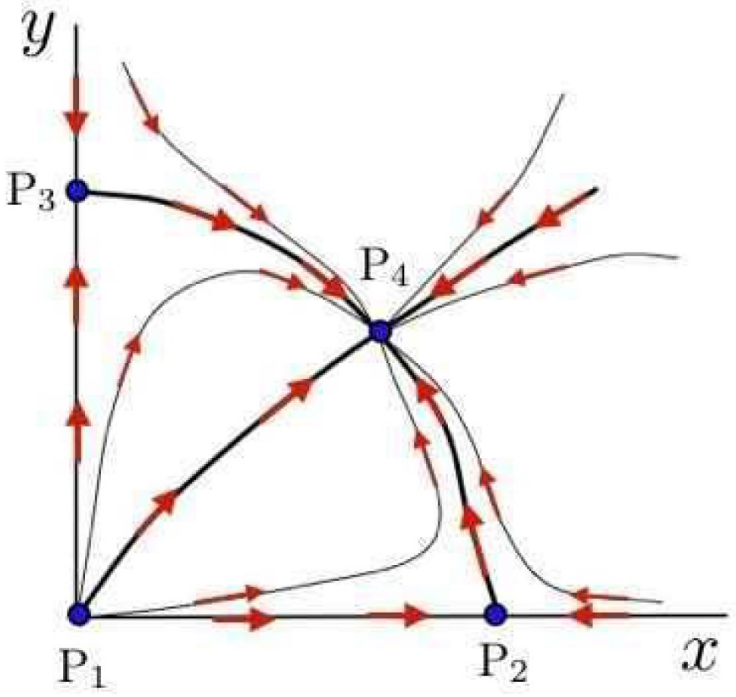
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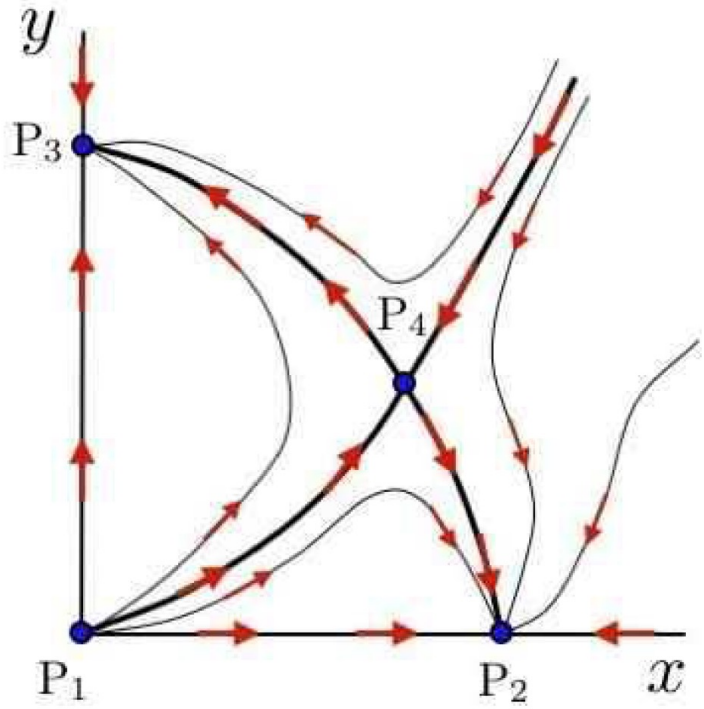


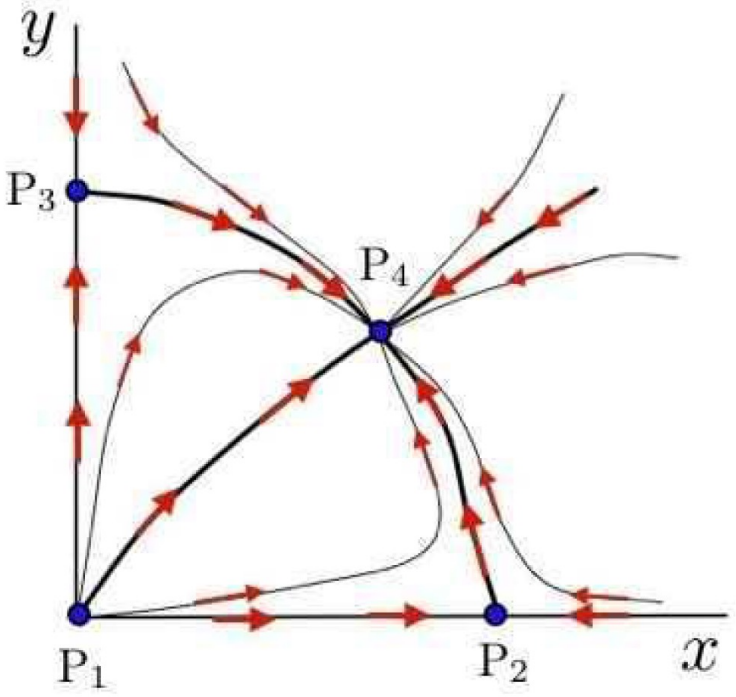


Figure 3.12: Two possible phase flows for the rabbits *vs.* sheep model of eqs. 3.61. Left panel: $k > r > k'^{-1}$. Right panel: $k < r < k'^{-1}$.

$$\dot{x} = x\,(r - x - ky)$$
$$\dot{y} = y\,(1 - y - k'x)$$

Exact Five-Loop Renormalization Group Functions of ϕ^4-Theory with $O(N)$-Symmetric and Cubic Interactions.

H. Kleinert and V. Schulte-Frohlinde

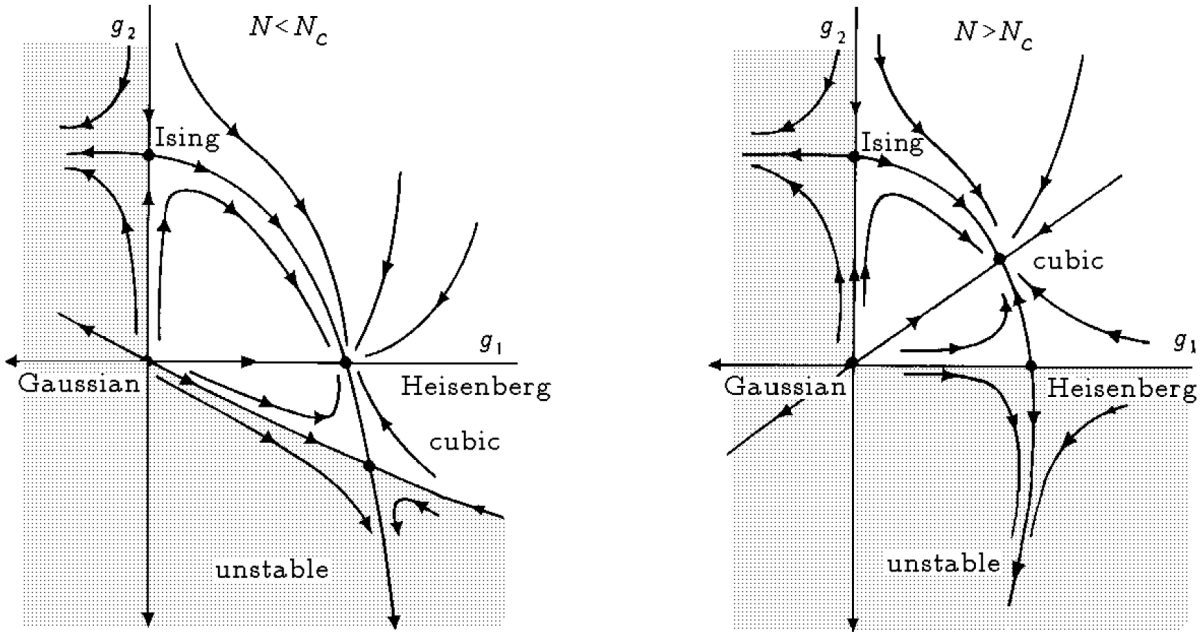
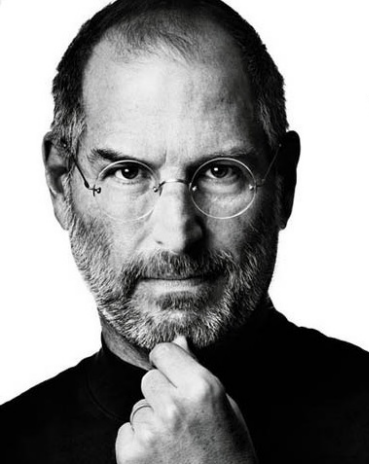


Figure 1: The Stability of the fixed points in the ϕ^4-theory with $O(N)$-symmetric and cubic coupling for $N < N_c$ and $N > N_c$. Our results are compatible with $N_c = 3$.

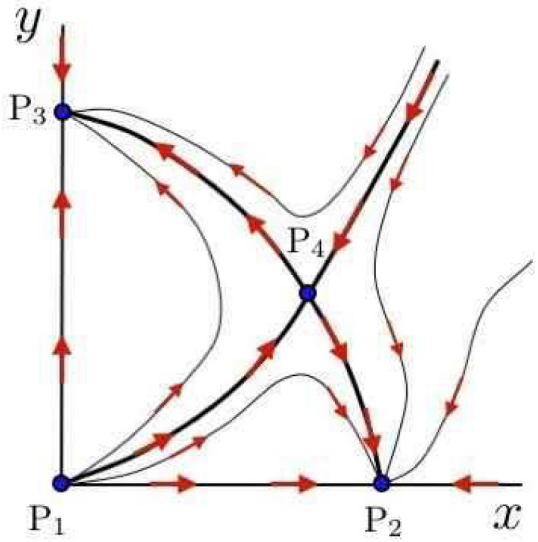


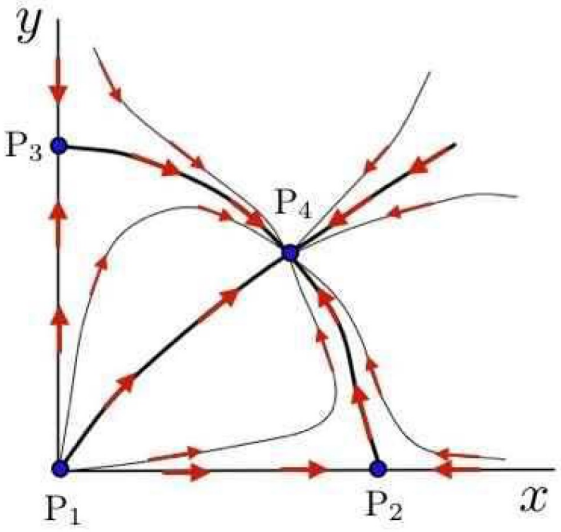
You can't connect the dots looking forward; you can only connect them looking backwards. So you have to trust that the dots will somehow connect in your future. You have to trust in something – your gut, destiny, life, karma, whatever.

Steve Jobs

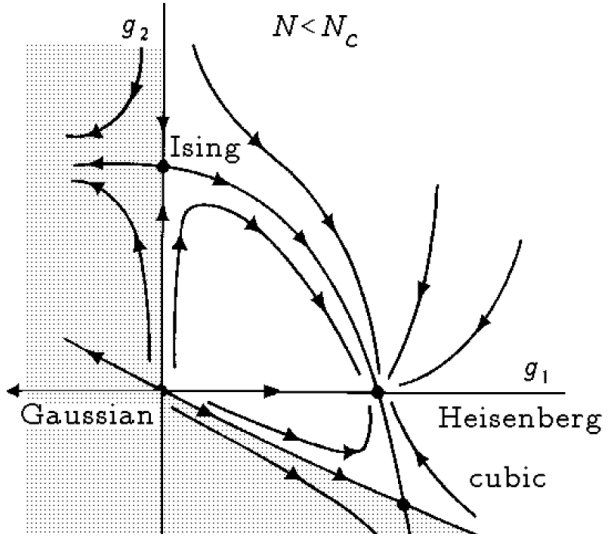
There are those who look at things the way they are, and ask why … I dream of things that never were, and ask why not?

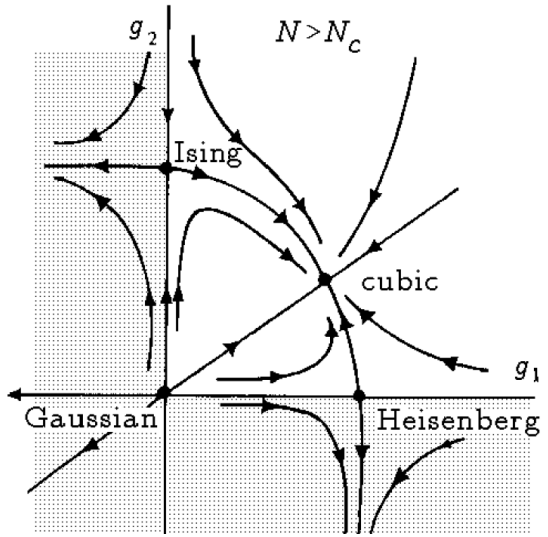
Robert Kennedy





RG Flow = Dynamical System



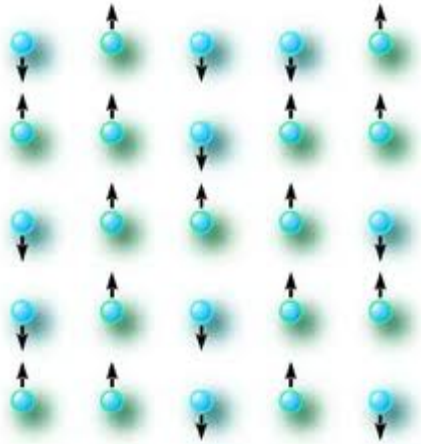


- New techniques:

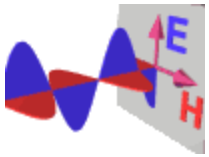
Bifurcation Theory

- New predictions:

3d O(N) model



3d QED



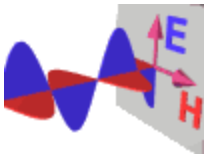
4d QCD

- New techniques:

Bifurcation Theory

- New predictions:

Conformal dimensions in 3d QED



$$\Delta_{\text{4-fermi}} - d \sim \sqrt{N_f - N_f^{\text{crit}}}$$

In fact, contrary to what some of the proposed scenarios might suggest!

$$\Delta_{\text{4-fermi}} - d \sim (N_f - N_f^{\text{crit}})$$

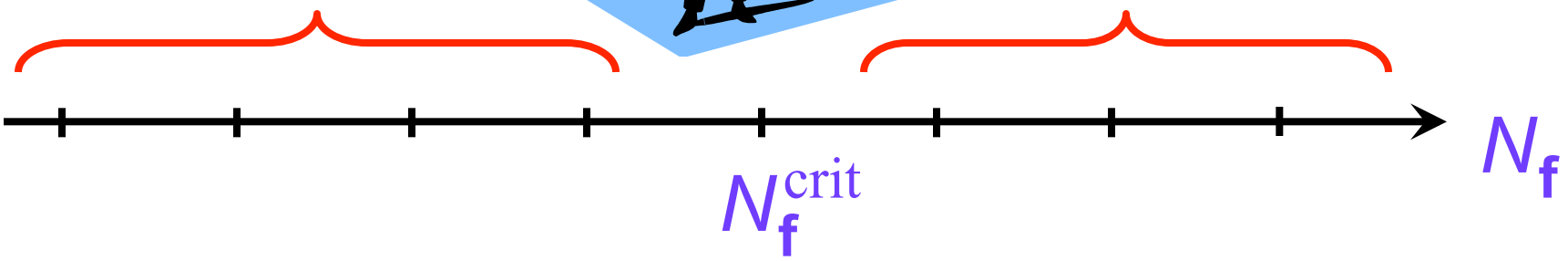
$$\mathcal{L} = -\frac{1}{4e^2} F_{\mu\nu} F^{\mu\nu} + \sum_{a=1}^{N_f} \overline{\Psi}_a (i\gamma^\mu D_\mu - m) \Psi^a$$

4-component Dirac spinors

spontaneous "chiral" symmetry breaking:

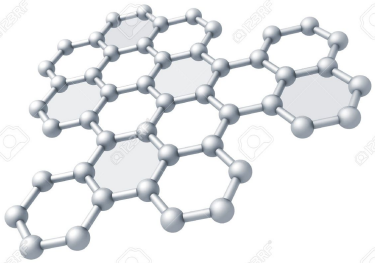
$$U(2N_f) \rightarrow U(N_f) \times U(N_f)$$

IR stable fixed point

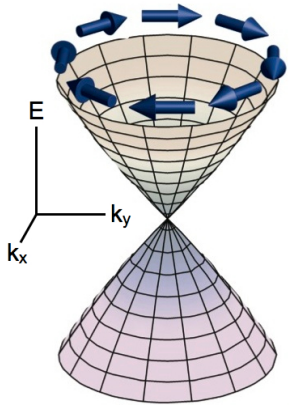


Condensed Matter Applications

- surface states in Topological Insulators



- unconventional quantum Hall effect in graphene

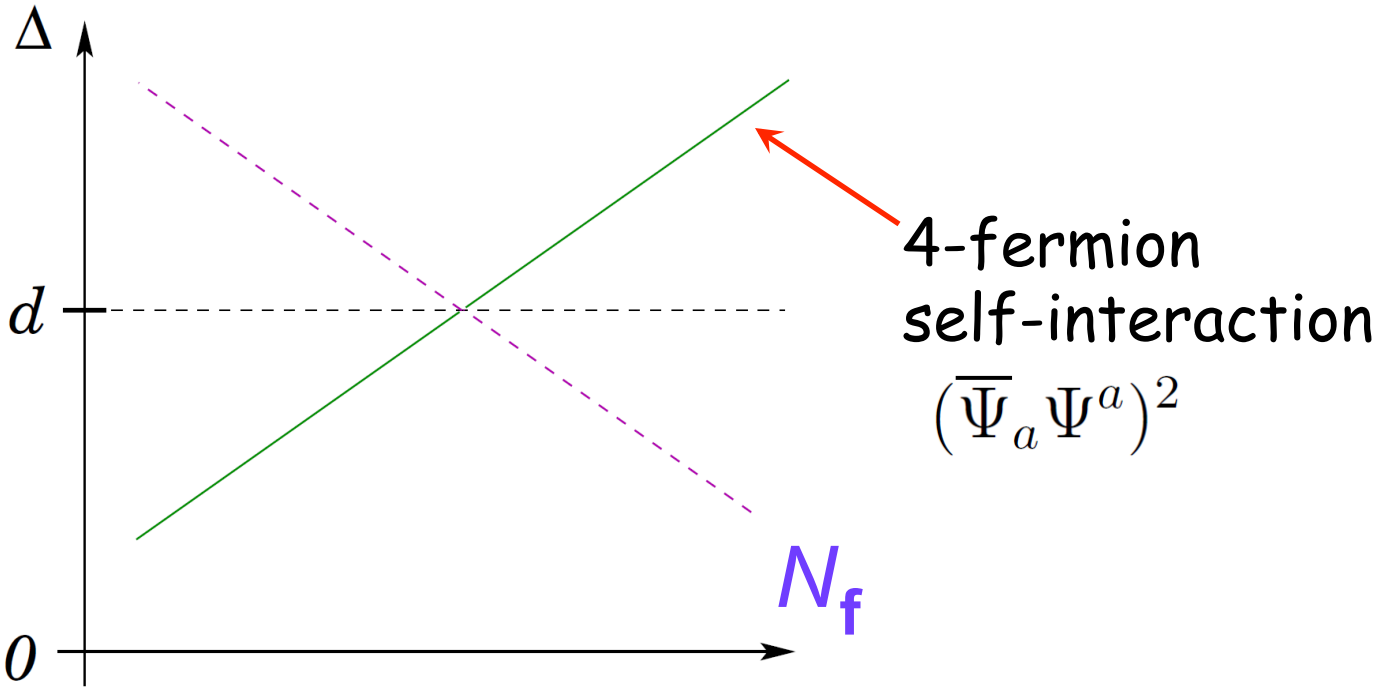


- underdoped and non-superconducting phase of high-T_c cuprate compounds:
  - $N_f^{\mathrm{crit}} < 2$ means the superconducting phase in these CuO_2 layers is separated from the antiferromagnetic phase by an unconventional non-Fermi-liquid phase
  - $N_f^{\mathrm{crit}} > 2$ means a direct zero-temperature phase transition from the d-wave superconducting phase to the antiferromagnetic phase

N_f^{crit}	Method	Year and Reference
$\leq \frac{3}{2}$	F-theorem	1999 [38, 61]
1.5	lattice simulations	2008 [62]
≤ 2	one-loop ϵ-expansion	2015 [63]
≤ 2	Hybrid Monte Carlo	2002 [55, 64, 65]
2.16	divergence of the chiral susceptibility	2002 [66]
2.89	ϵ-expansion	2016 [67]
$\frac{32}{\pi^2} \approx 3.24$	Schwinger-Dyson equations	1984-88 [37, 54]
≤ 4	F-theorem	2015 [68]
4	covariant solutions for propagators	2004 [69]
4.3	Schwinger-Dyson equations	1996-97 [70, 71]
6	perturbative RG in the large-N_f limit	2004 [56]
5.1...6.6	comparison to the Thirring model	2007-12 [72, 73]
$4 \approx N_f^{\chi\text{SB}} \leq N_f^{\text{conf}} \leq 10$	functional RG	2014 [52]

Table 5: Search for the critical value of N_f in non-compact QED$_3$.

- Large N_f: $N_f^{\text{crit}} = 6$ [K.Kaveh, I.Herbut, 2004]



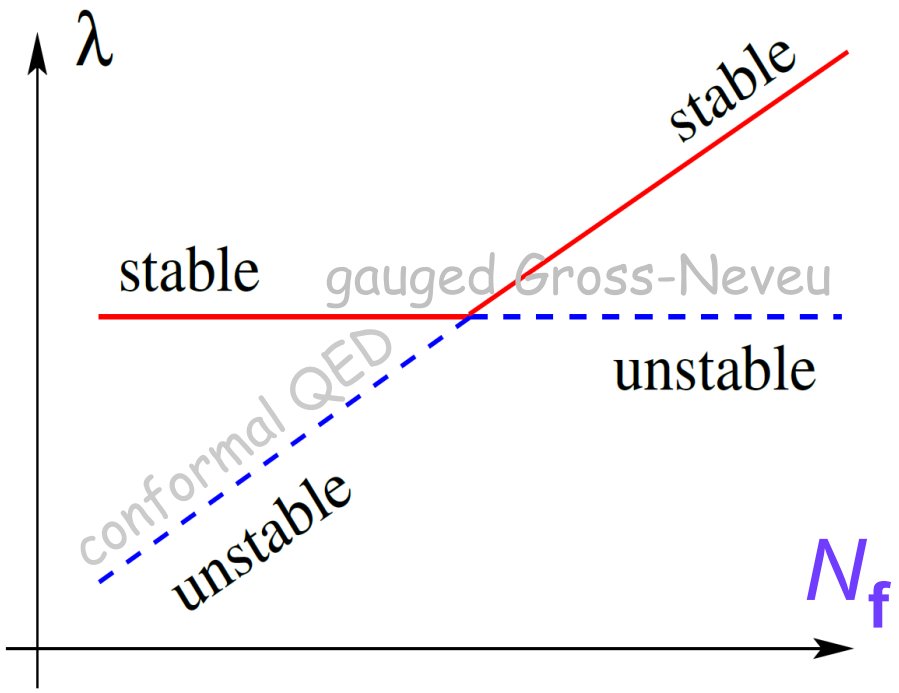
- One-loop 4-ε expansion: $N_f^{\text{crit}} \leq 2$

$$d - \Delta_{\text{4-fermi}} = -\frac{1}{2N_f}\left(4N_f + 1 \pm 2\sqrt{N_f^2 + N_f + 25}\right)$$

[L.Di Pietro, Z.Komargodski, I.Shamir, E.Stamou, 2015]

- Large N_f:

[K.Kaveh, I.Herbut, 2004]

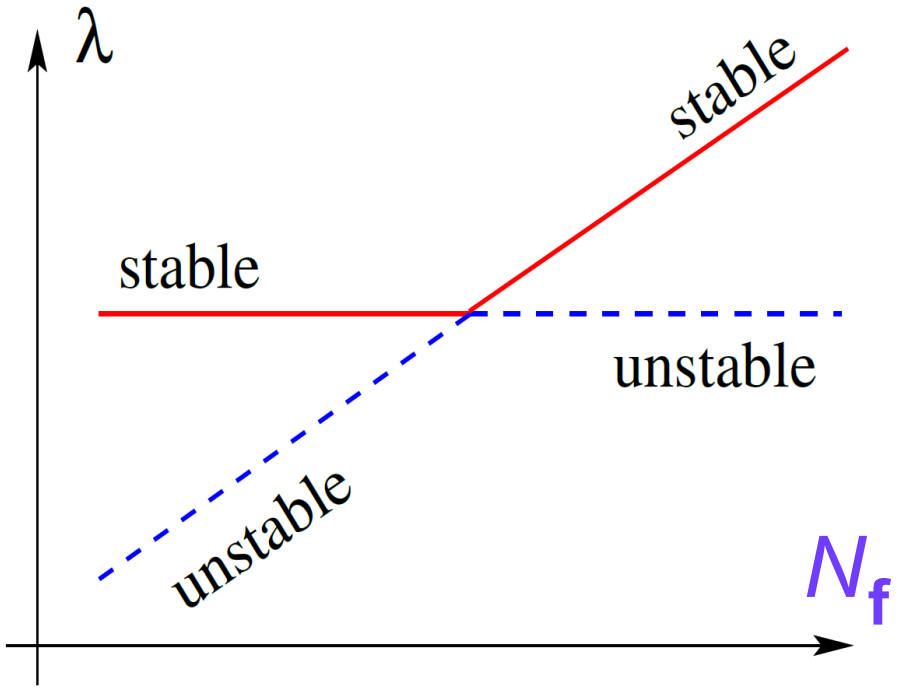


- One-loop 4-ε expansion: $N_f^{\text{crit}} \leq 2$

$$d - \Delta_{\text{4-fermi}} \approx 0.54 \left(N_f^{\text{crit}} - N_f \right)$$

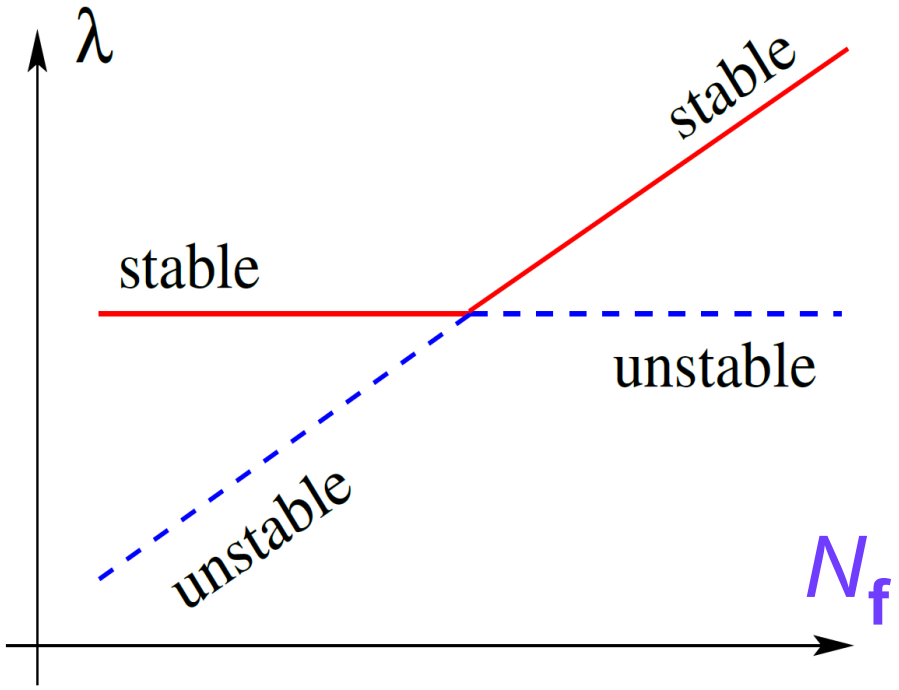
[L.Di Pietro, Z.Komargodski, I.Shamir, E.Stamou, 2015]

Transcritical Bifurcation



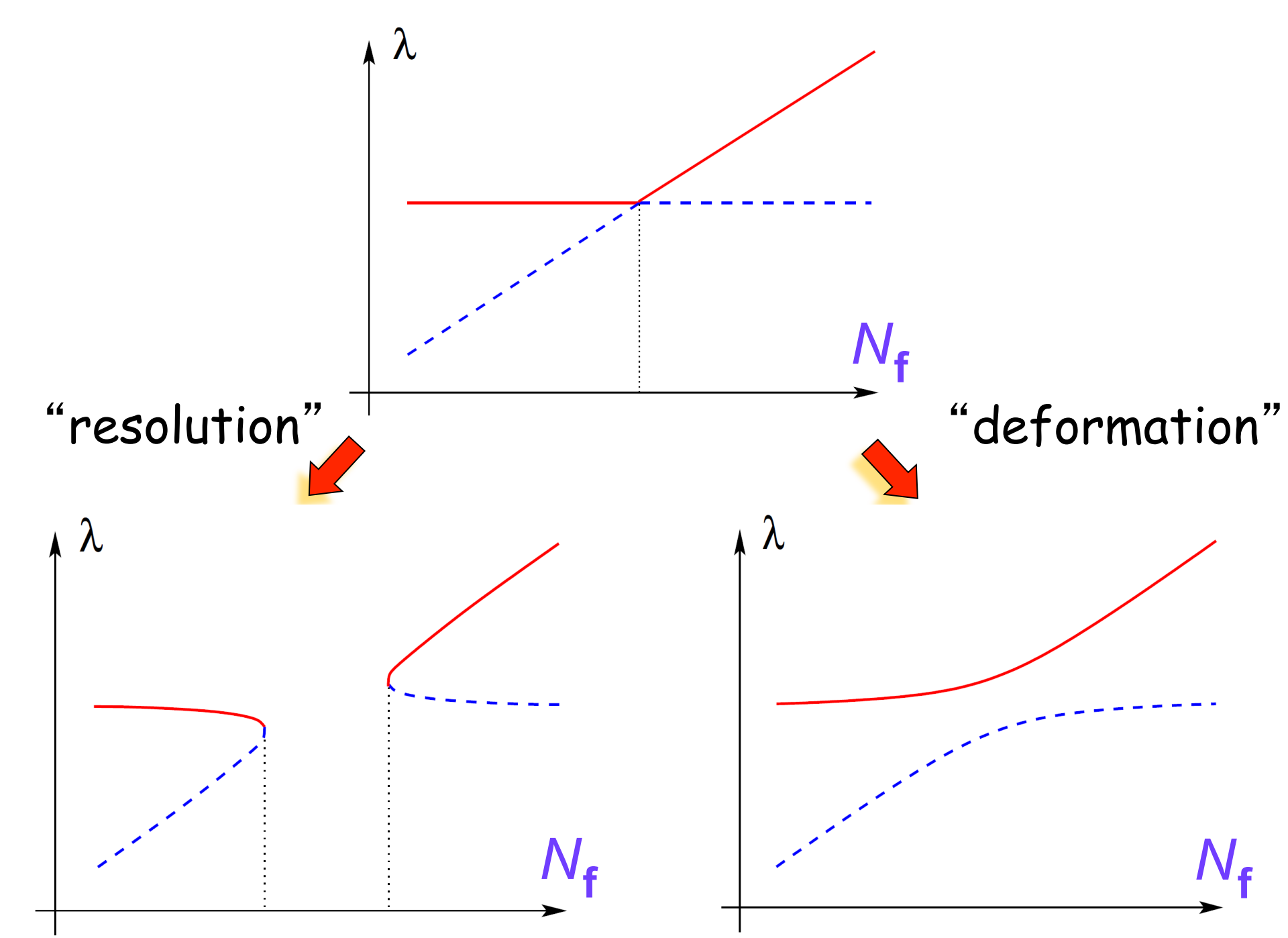
$$\begin{cases} \dot{\lambda}_1 &= (N_f - N_f^{\text{crit}})\lambda_1 - \lambda_1^2 \\ \dot{\lambda}_2 &= -\lambda_2 \end{cases}$$

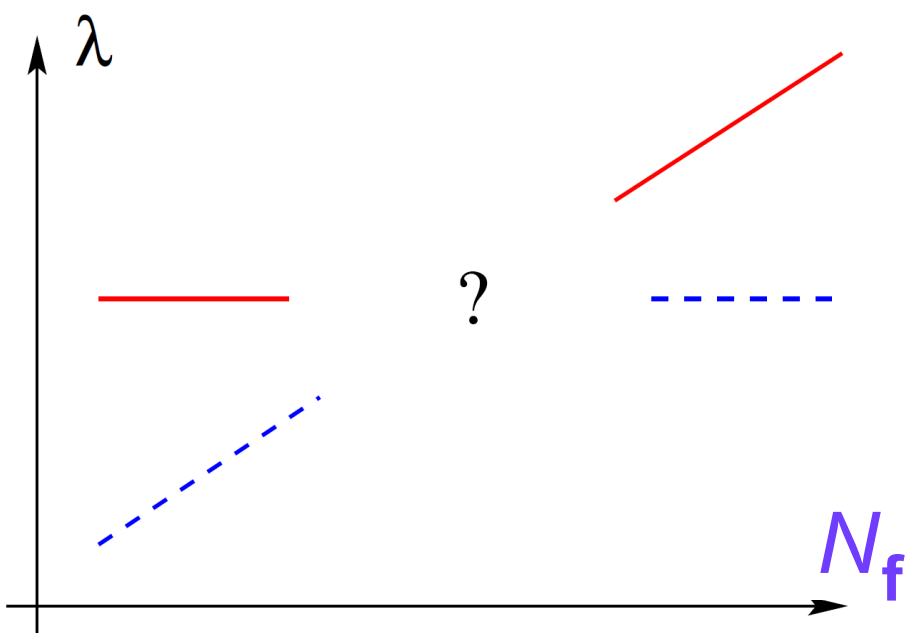
Transcritical Bifurcation



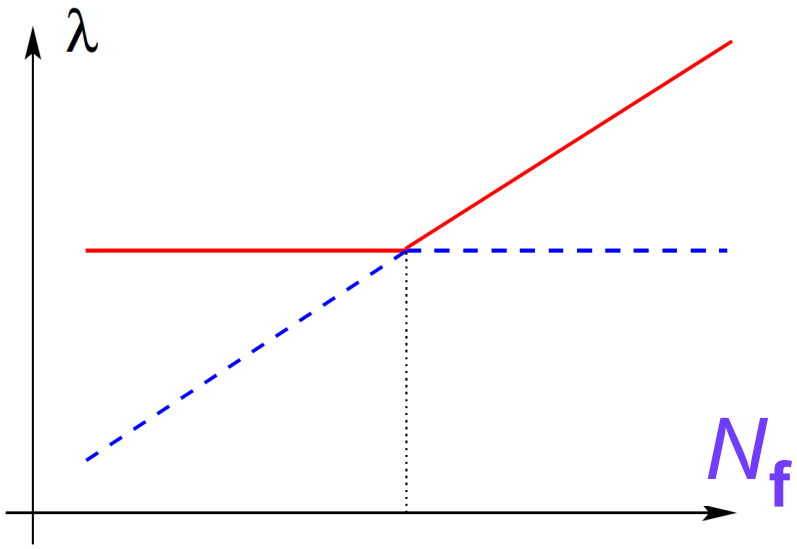
- Codimension-2
- Structurally unstable

$$\Delta - d \sim \sqrt{N_f - N_f^{\text{crit}}}$$

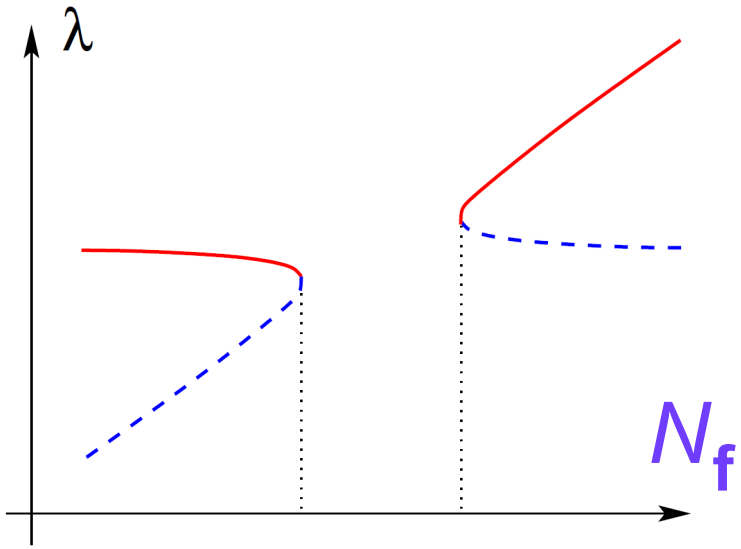

λ
N_f
"resolution"
"deformation"
λ
N_f
λ
N_f



* cf. “naturalness” a la K.G.Wilson, G.’ t Hooft, N.Seiberg, …



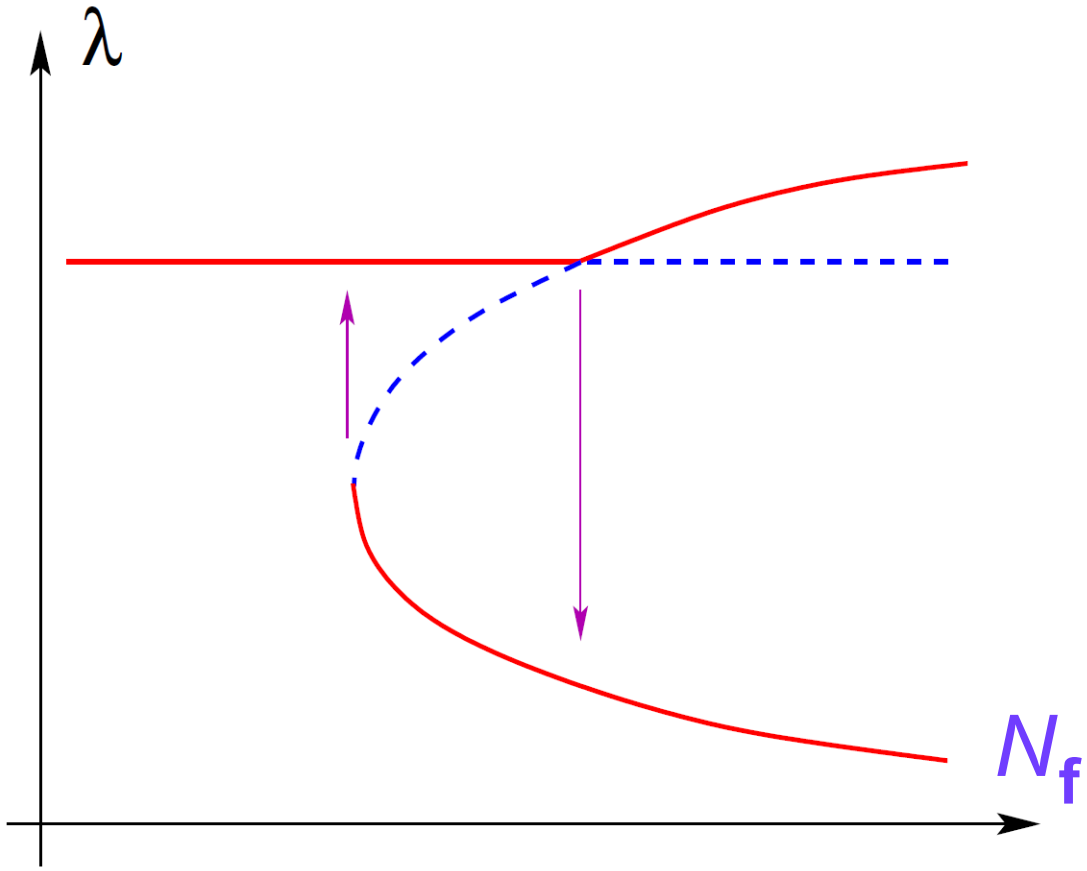
vs.

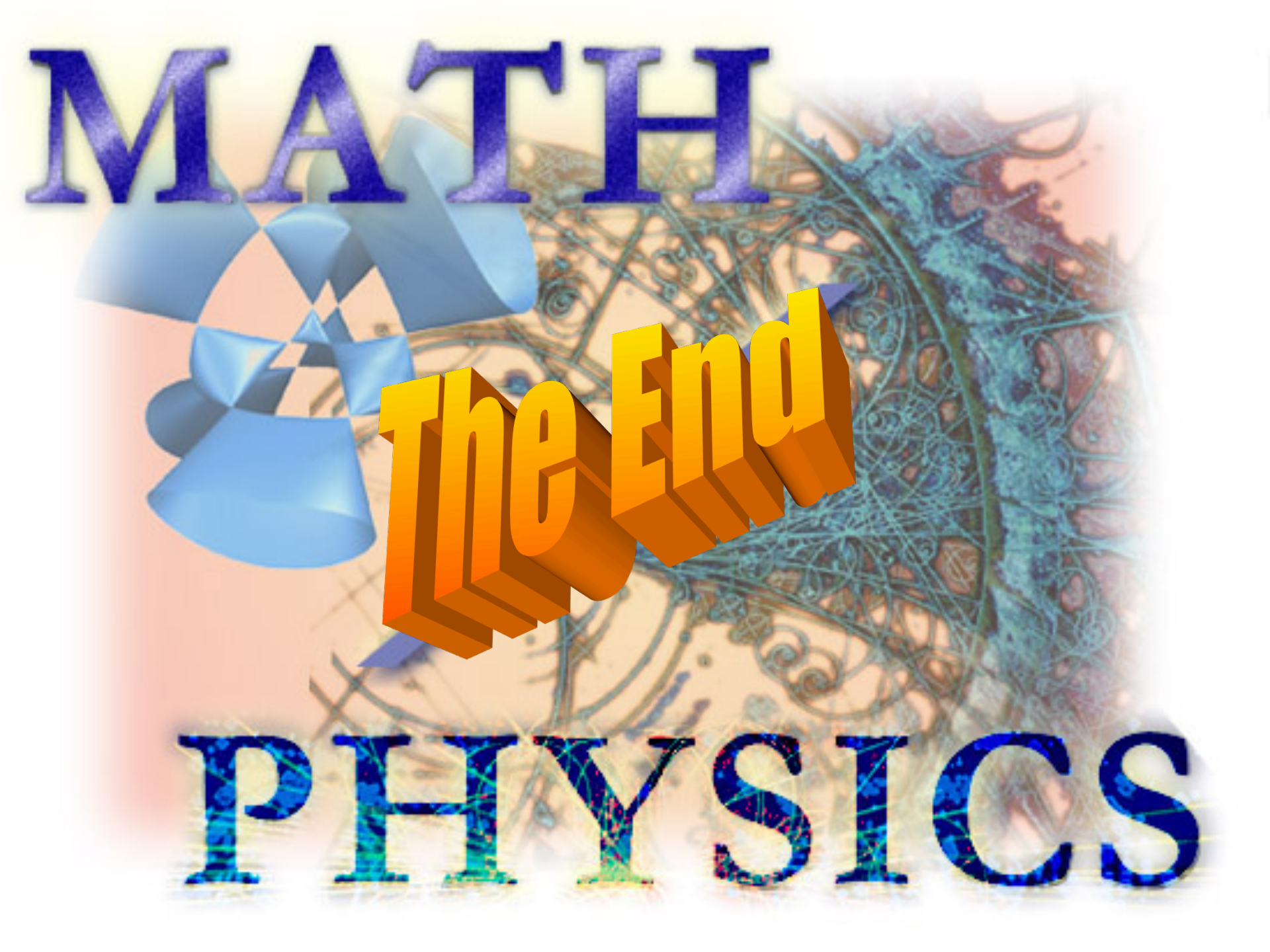


$$\Delta - d \sim (N_f - N_f^{\rm crit})$$

$$\Delta - d \sim \sqrt{N_f - N_f^{\rm crit}}$$

Adding higher-order terms

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