Effective Field Theory of Dissipative Fluids

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arXiv: 1511.03646
and to appear

Conserved quantities and hydrodynamics

Consider long wavelength disturbances of a system in thermal equilibrium:

non-conserved quantities: relax locally, $\tau_{\mathrm{relax}} \sim \tau_{\mathrm{mfp}}$

conserved quantities: cannot relax locally, only via transports

$$\lambda \to \infty, \quad \Rightarrow \quad \tau_{\mathrm{relax}} \to \infty$$

Conserved quantities → Gapless and long-lived modes (only ones in thermal equilibrium)

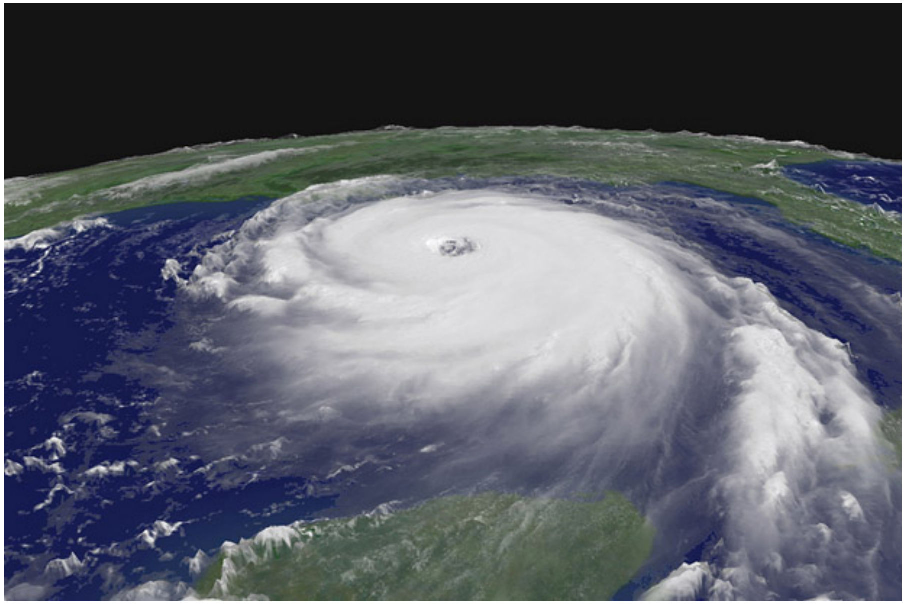
There should exist a universal low energy effective theory.

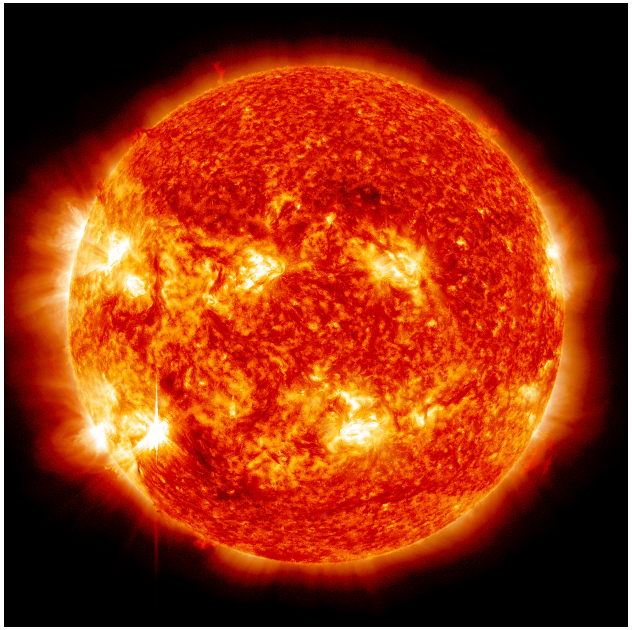
Phenomenological description: Hydrodynamics

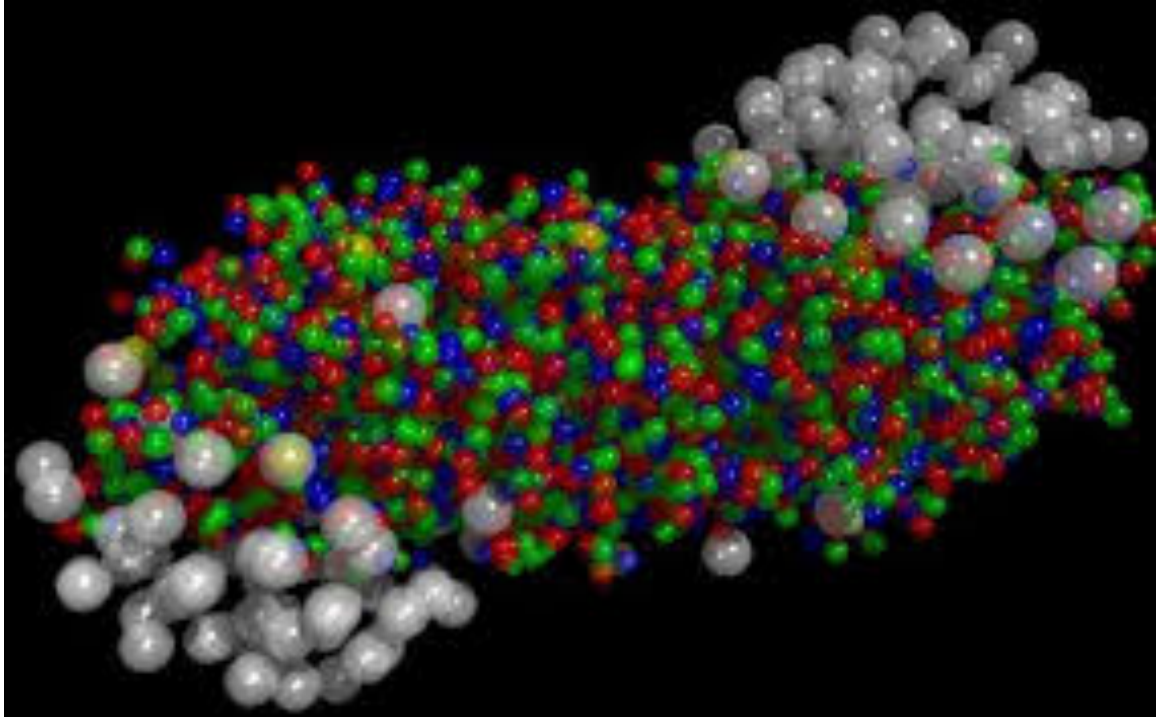
$$\partial_\mu \langle T^{\mu\nu} \rangle = 0, \qquad \partial_\mu \langle J^\mu \rangle = 0$$

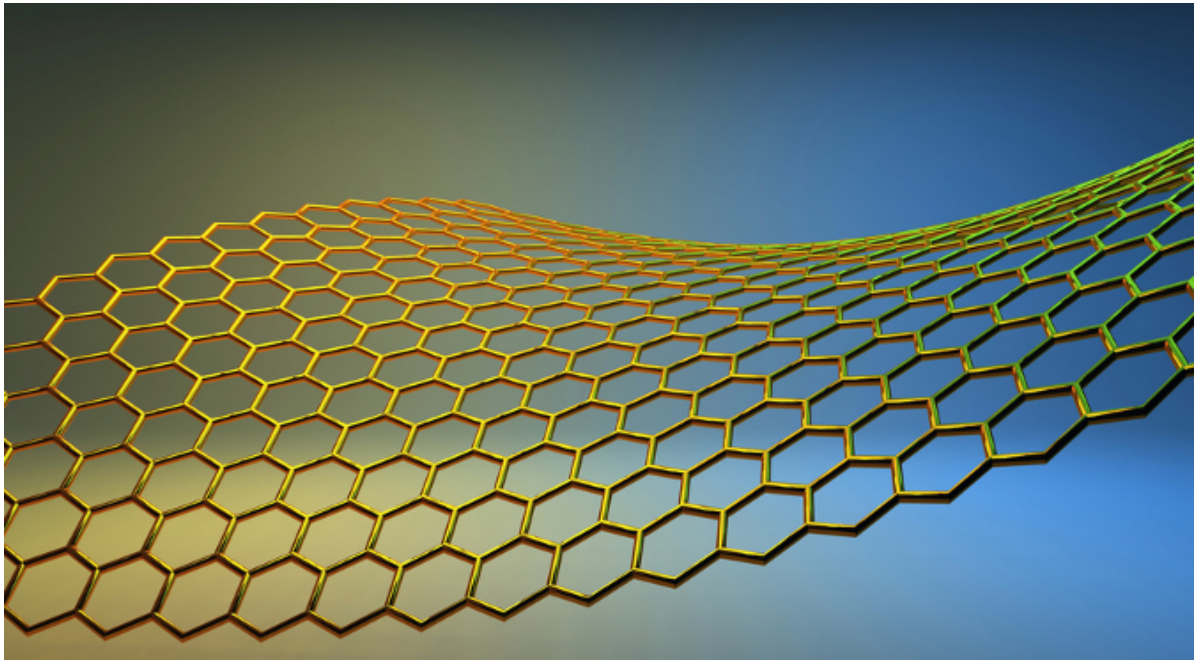
$$\beta(t, \vec{x}), \; u^\mu(t, \vec{x}), \; \mu(t, \vec{x})$$

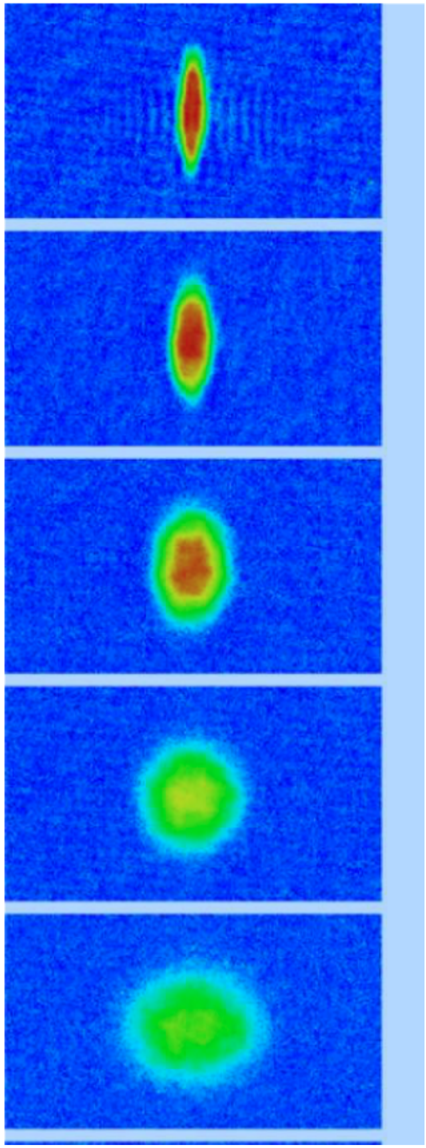
slowly varying functions of spacetime











O’Hara et al (2002)

Despite the long and glorious history of hydrodynamics

It does not capture fluctuations, which are always present, and are important in many physical contexts:

Transports (long time tail), dynamical aspects of phase transitions, non-equilibrium steady states, turbulence, finite size systems

……….

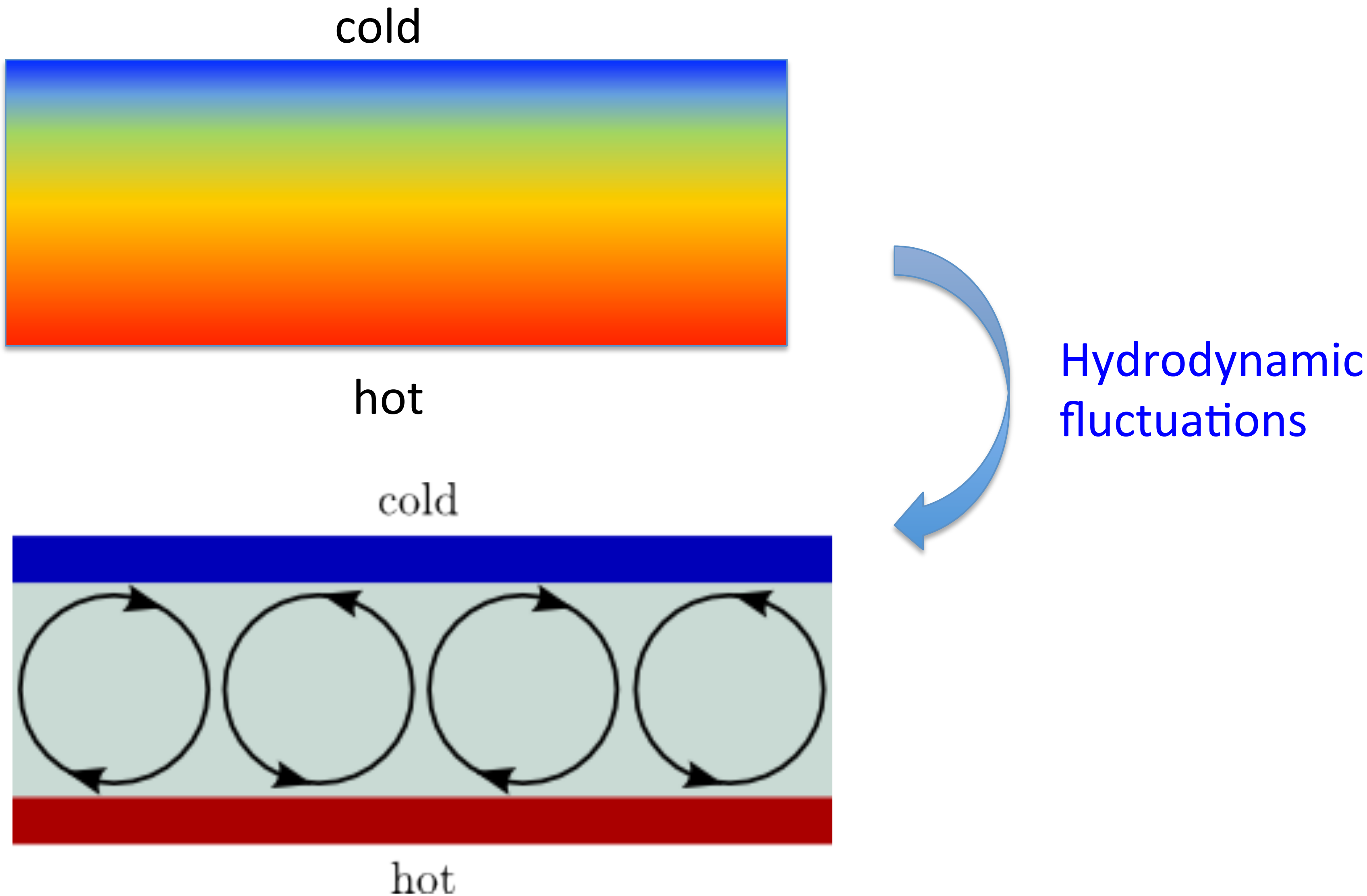
Phenomenological level: stochastic hydro (Landau, Lifshitz)

$$\partial_\mu \langle T^{\mu\nu} \rangle = \xi^\nu, \quad \partial_\mu \langle J^\mu \rangle = \zeta$$

Good for near-equilibrium disturbances (linearized)

But not adequate for far-from-equilibrium situations

Rayleigh-Benard problem



Constraints

Current formulation of hydrodynamics is awkward.

Constitutive relations : not enough to just write down the most general derivative expansion consistent with symmetries.

Phenomenological constraints: solutions should satisfy:

1. Entropy condition $\partial_\mu S^\mu \geq 0$

2. Onsager relations: linear response matrix must be symmetric

awkward: use solutions to constrain equations of motion

Clearly something more fundamental is missing

Microscopic origin? Are these complete?

Will address these issues by developing hydrodynamics as a low energy effective field theory of a general many-body system at finite temperature

Action principle for full fluctuating hydrodynamics applicable to far from equilibrium situations

Closed time path

Standard lore: Dissipative systems don't have an action formulation

This issue is naturally resolved by quantum mechanics.

Interested in describing real-time dynamics around a state, not transition amplitude

$\langle f | \cdots | i \rangle$

$t = -\infty$ → $t = +\infty$

Closed time path (CTP) or Schwinger-Keldysh contour

Requires doubling the number of d.o.f.

Closed time path

Standard lore: Dissipative systems don't have an action formulation

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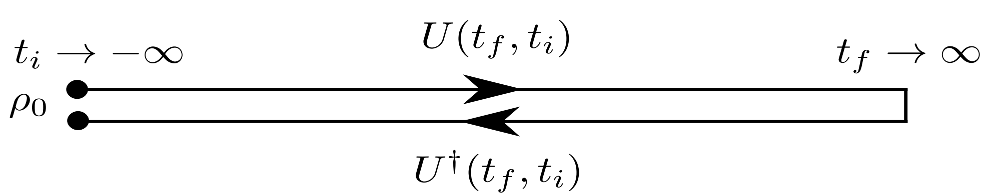
Interested in describing real-time dynamics around a state, not transition amplitude

$$\mathrm{Tr}\left(\rho_0 \cdots\right)$$

(b)



Closed time path (CTP) or Schwinger-Keldysh contour

Requires doubling the number of d.o.f.

Effective field theory for fluids

Microscopic description (in thermal density matrix)

$\Lambda \sim T$

Hydro EFT: $\int D\chi_1 D\chi_2 \, e^{iS_{\rm hydro}[\chi_1,\chi_2;\rho_0)]} \ldots$

hydrodynamic modes (2 sets)

RG

- thermal state reflected in coefficients of action
- Standard hydro: saddle point equations

Macroscopic phenomena

1. What are χ ? $\beta(t,\vec{x}), u^\mu(t,\vec{x}), \mu(t,\vec{x})$ do not work
2. What are the symmetries of $S_{\rm hydro}$?
3. Integration measure?

Searching for an action principle for dissipative hydrodynamics has been a long standing problem, dating back at least to the ideal fluid action of G. Herglotz in 1911.

Relativistic: Taub, Salmon, Carter, ……

Reviews: Jackiw, Nair, Pi and Polychronakos; Andersson and Comer

The last decade has seen a renewed interest:

Dubovsky, Gregoire, Nicolis and Rattazzi hep-th/0512260

Dubovsky, Hui, Nicolis and Son, arXiv:1107.0731

Including dissipation: Grozdanov and Polonyi; Harder, Kovtun, and Ritz; Kovtun, Moore and Romatschke; Endlich, Nicolis, Porto and J. Wang, ……

Holographic: Nickel, Son; de Boer, Heller, Pinzani-Fokeeva; Crossley, Glorioso., HL, Y. Wang.

An alternative approach: Haehl, Loganayagam and Rangamani, arXiv: 1510.02494, 1511.07809

Here for conceptual simplicity, I will describe the story for neutral normal fluids.

Description for full charged fluids is given in:

Crossley, Glorioso, and HL, arXiv: 1511.03646

Can also be generalized to superfluids (S. Rajagopal and HL, to appear)

Dynamical variables

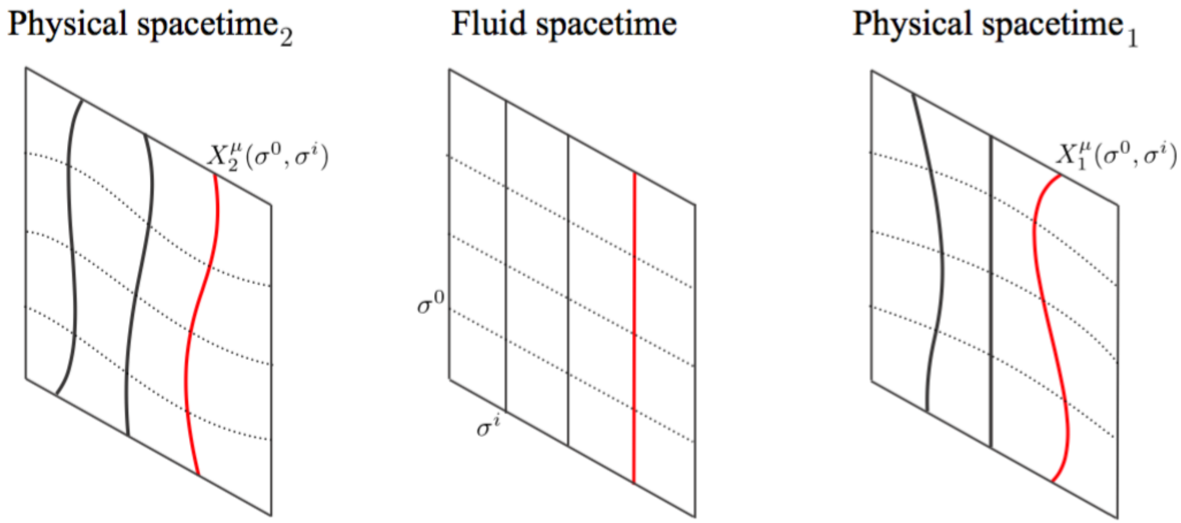
Recall Lagrange description of hydrodynamics:

σ^i : label fluid elements $x^i(\sigma^i, t)$: describe fluid motion

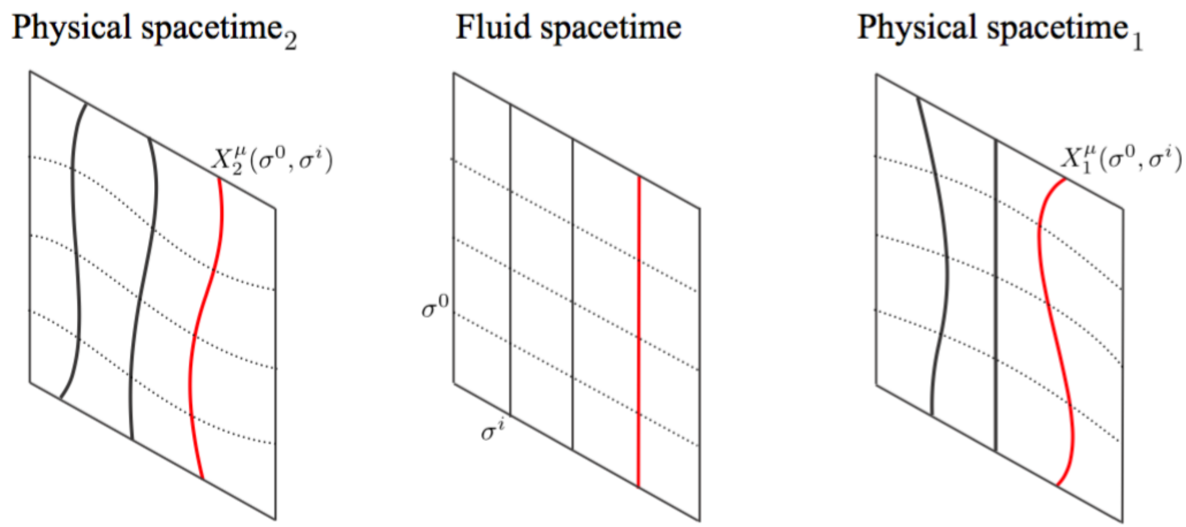
An "integrating-in" procedure to identify gapless and long-lived d.o.f. associated with a conserved stress tensor:

$$X_1^\mu(\sigma^i, \sigma^0), \quad X_2^\mu(\sigma^i, \sigma^0)$$

σ^0 : internal time



Also natural from Holography:
Nickel, Son,
arXiv:1009.3094



Standard hydro variables (which are now derived quantities)

$$u^\mu = \frac{1}{b}\frac{\partial X^\mu}{\partial \sigma^0}, \quad X^\mu = \frac{1}{2}(X_1^\mu + X_2^\mu) \quad e^{-\tau} = \frac{T}{T_0},$$

Noise: $X_a^\mu = X_1^\mu - X_2^\mu$

A significant challenge: ensure the eoms from the action of X can be solely expressed in terms of these velocity. (e.g. solids v.s. fluids)

Symmetries

σ^i label individual fluid elements, σ^0 internal time

Require the action to be invariant under:

$$\sigma^i \to \sigma'^i(\sigma^i), \quad \sigma^0 \to \sigma^0$$

$$\sigma^0 \to \sigma'^0 = f(\sigma^0, \sigma^i), \quad \sigma^i \to \sigma^i$$

define what is a fluid!

It turns out these symmetries indeed do magic for you:

at the level of equations of motion, they ensure all dependence on dynamical variables can be expressed in u^μ and temperature.

⇒ Recover standard formulation of hydrodynamics

(modulo phenomenological constraints)

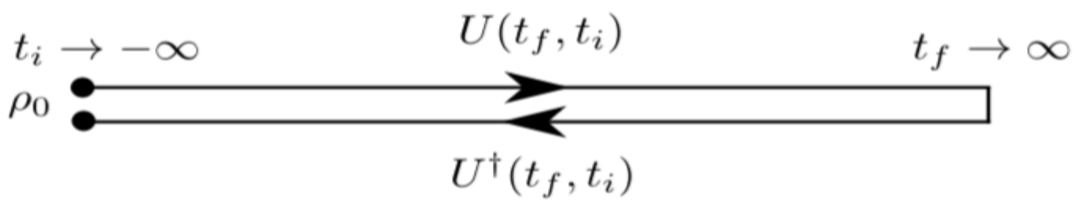
Full non-linear fluid fluctuating dynamics encoded in non-trivial differential geometry:

$$D_0,\ D_i,\ R_{ij}^{(1)},\ R_{ij}^{(2)},\ \mathfrak{t}_{ij}$$

This would be the full the story in a usual situation.

Consistency conditions and symmetries

For a system defined with CTP:

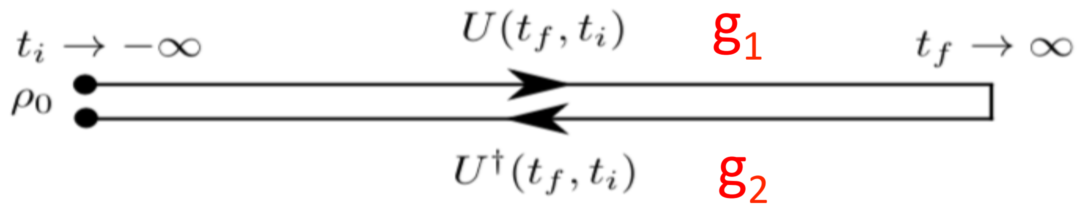


constraints on correlation functions,

Require further symmetries satisfied by the off-shell action so that these constraint conditions are satisfied after doing the path integral.

Reflectivity and KMS conditions

$W[g_1, g_2]$ =



- Reflectivity condition: $W^*[g_1, g_2] = W[g_2, g_1]$

Can be satisfied by imposing a Z_2 reflection symmetry on both dynamical and background fields

Complex action: non-negativity of imaginary part

- KMS conditions plus CPT imply a Z_2 symmetry on W:

$$W[g_1(x), g_2(x)] = W[g_1(-x), g_2(-t - i\beta_0, -\vec{x})]$$

characterize thermal state β_0 : inverse temperature at spatial infinities

Can be satisfied by imposing a Z_2 symmetry on the action: local KMS conditions

- All the constraints from entropy current condition and linear Onsager relations

- New constraints on equations of motion from nonlinear generalizations of Onsager relations.

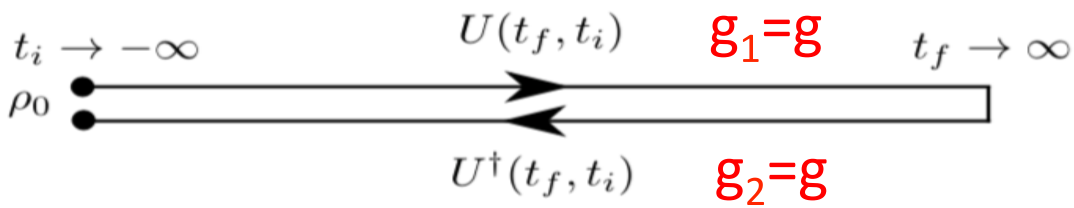
- Non-equilibrium fluctuation-dissipation relations

Crossley, Glorioso, HL
to appear

Local Z_2 KMS condition + equations of motion + non-negativity of imaginary part of the action

$\partial_\mu S^\mu \geq 0$ $\quad S^\mu$: conserved and entropy current at ideal level

Unitary condition



$$W[g,g] = 0 \qquad \mathrm{Tr}(U\rho_0 U^\dagger) = 1$$

This may be considered as a condition on path integral measure.

Resolution: Introduce a fermionic partner ("ghost") for each dynamical variable and require the action to have a "BRST" type symmetry δ :

$$\delta^2 = 0$$

See also Haehl, Loganayagam and Rangamani arXiv: 1510.02494

To describe hydrodynamical flucatuations properly anti-commuting objects are needed !

(BRST symmetry here is a global symmetry)

KMS condition and Supersymmetry

Strong indications that BRST + local KMS condition

⇒ emergent fermionic symmetry: $\bar{\delta}$

Classical limit: $\hbar \to 0$

$$\delta^2 = 0, \qquad \bar{\delta}^2 = 0, \qquad [\delta, \bar{\delta}] = \bar{\epsilon}\epsilon i \beta_0 \partial_t$$

See also Haehl, Loganayagam and Rangamani, arXiv: 1510.02494, 1511.07809

Standard supersymmetry in time direction.

Ping Gao and HL, in progress

Quantum level: indications of a "quantum-deformed" SUSY algebra

$$\delta^2 = 0, \quad \bar{\delta}^2 = 0, \quad [\delta, \bar{\delta}] = \bar{\epsilon}\epsilon 2 \tanh \frac{i\beta_0 \hbar \partial_t}{2}$$

Summary

1. Hydrodynamics with classical statistical fluctuations

is described by a supersymmetric quantum field theory

$$\hbar_{\text{eff}} \propto \frac{1}{s} \qquad s : \text{entropy density}$$

- Recovers the standard hydrodynamics, new constraints, non-equilibrium fluctuation-dissipation relations

- Full non-linear fluid fluctuating dynamics encoded in non-trivial differential geometry.

2. Hydrodynamics with quantum fluctuations also incorporated

is described by a "quantum-deformed" (supersymmetric) quantum field theory.

Bosonic action for full charged fluids

A double expansion in terms of noises and derivatives:

$$I = I^{(1)} + I^{(2)} + \cdots$$

$$I^{(1)} = \int d^d x \left[T^{\mu\nu}_{\text{hydro}} \partial_\mu X_{a\nu} + J^\mu_{\text{hydro}} \partial_\mu \varphi_a \right],$$

$$\begin{aligned} I^{(2)}_0 =& i \int d^d x \, [f_{25} \eta^{\mu\rho} \eta^{\nu\sigma} (2\partial_{<\mu} X_{a\nu>})(2\partial_{<\rho} X_{a\sigma>}) \\ &+ f_{26} \Delta^{\mu\rho} (2u^\nu \partial_{(\mu} X_{a\nu)})(2u^\sigma \partial_{(\rho} X_{a\sigma)}) + f_{28} \Delta^{\mu\nu} w_\mu w_\nu + f_{27} \Delta^{\mu\rho} (2u^\nu \partial_{(\mu} X_{a\nu)}) w_\rho \\ &+ f_{211} (u^\mu \partial X_{a\mu})^2 + f_{222} (\Delta^{\mu\nu} \partial_\mu X_{a\nu})^2 + f_{233} (e^\tau \partial \varphi_a)^2 \\ &- f_{212} \Delta^{\mu\nu} \partial_\mu X_{a\nu} u^\rho \partial X_{a\rho} + f_{223} (e^\tau \partial \varphi_a) \Delta^{\mu\nu} \partial_\mu X_{a\nu} - f_{213} u^\mu \partial X_{a\mu} (e^\tau \partial \varphi_a) \Big] . \end{aligned}$$

$$w_\mu = e^\tau \partial_\mu \varphi_a + 2\hat{\mu} u^\rho \partial_{(\mu} X_{a\rho)}$$

All coefficients are functions of $\tau, \hat{\mu} = \mu e^\tau$

Future directions

Formalism:

Non-relativistic, (to appear with Glorioso)

superfluids, (to appear with S. Rajagopal)

Deeper understanding of KMS, unitarity and supersymmetry (in progress with Ping Gao)

"quantum-deformed" Supersymmetry

…….

Applications:

Transports (longtime tails)

Non-equilibrium steady states, dynamical flows of QGP

Dynamical aspects of phase transitions

Novel fixed points of hydro EFT, such as KPZ scaling, turbulence ….

Thank you!