Recovering the spacetime metric from a holographic dual
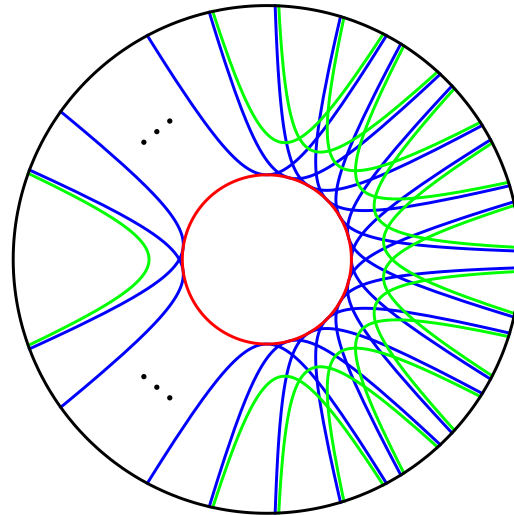
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(with N. Engelhardt,
1605.01070 and to appear)

Much previous work has focussed on quantum entanglement, and the Ryu-Takayanagi formula for entanglement entropy.
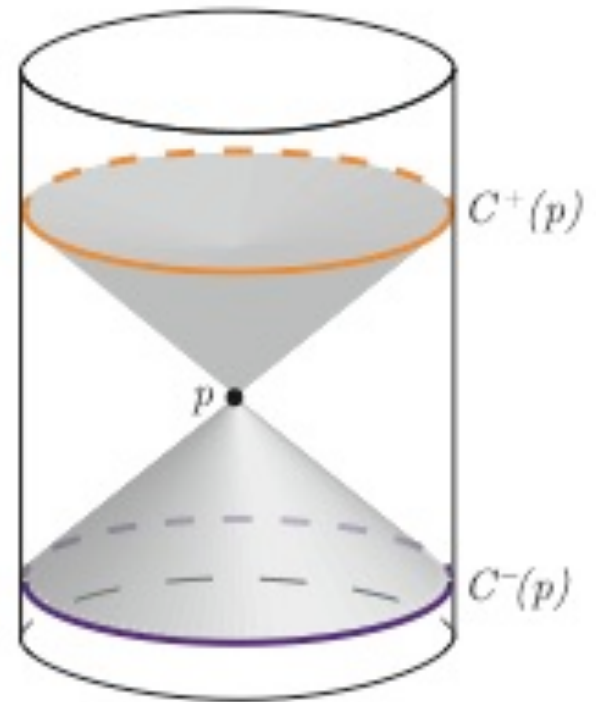

(Balasubramanian, Chowdhury, Czech, de Boer, Heller: 1310.4204; many follow-up papers…)

New approach

Consider a special set of spatial slices of the conformal boundary, called light-cone cuts.

(Similar to work by Newman et al for asymptotically flat spacetimes in the 70's and 80's.)

Outline

Light-cone cuts

Gravitational lensing can cause light rays from p to cross, producing caustics. After this, the light ray moves into the past of p.

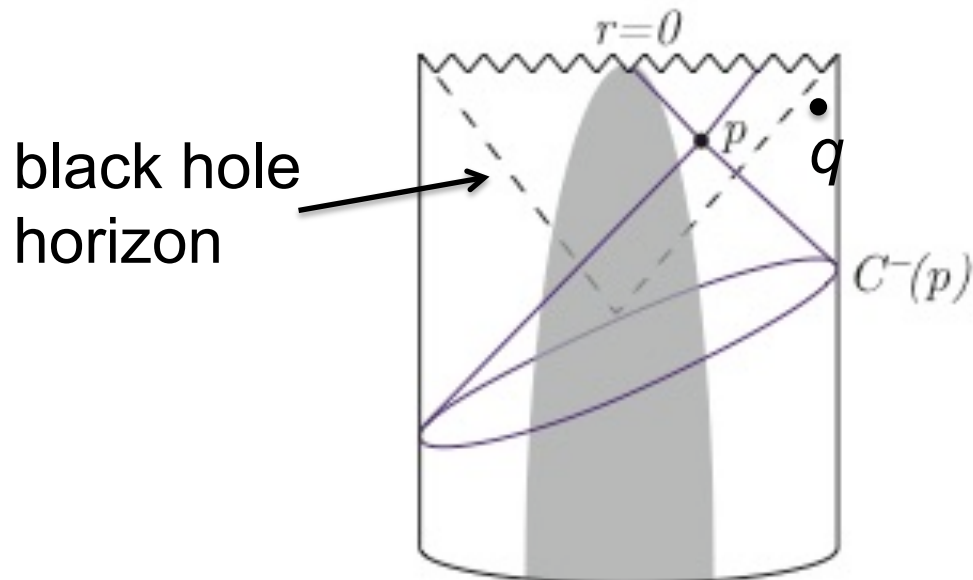Define I⁻(p) = (set of all q such that there is a future directed timelike curve from q to p). Set

$$C^-(p) \equiv \partial I^-(p) \cap \partial M$$

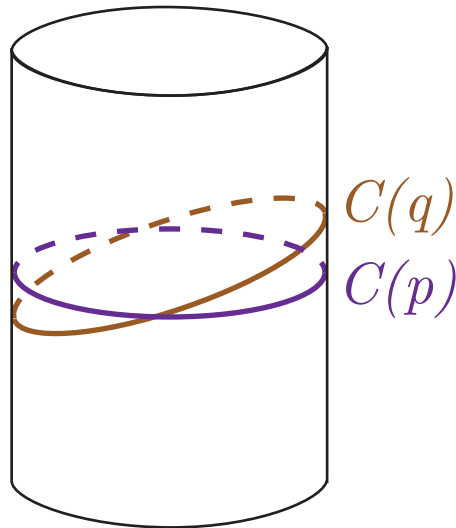C⁺(p) is defined similarly.

Properties

1) C(q) is a complete spatial slice of the boundary
2) There is a past cut for every point in the future of the boundary, even inside black holes.

3) Two distinct cuts C(p) and C(q) either
    a) do not intersect
    b) intersect at precisely one point
    c) cross

One cut cannot wiggle with respect to the other or agree on an open set.

Example: cuts of pure AdS$_{d+1}$

If the boundary is a static cylinder ds^2 = -dt^2 + dΩ^2 and bulk point is at t = 0, r = r$_0$, in θ = 0 direction, then

$$\tan t_\infty(\theta) = \frac{\sqrt{1 + r_0^2 \sin^2\theta}}{r_0 \cos\theta}$$

By time translations and rotations, get a d+1 dim. family of cuts labeled by t$_0$, r$_0$, and d-1 angles.

There is a direct connection between the cuts and causal relation between points in AdS.

Reconstructing the conformal metric from the cuts

The conformal metric is metric up to overall constant: $g_{ab} = \lambda^2\, g_{ab}$

Claim 1: The conformal metric is uniquely fixed by any open subset of the light cone.

Proof: Take any $d + 1$ null vectors v_i and view them as a basis. Let w_j be additional null vectors and expand them in terms of v_i. The fact that $w_j^2 = 0$ yields a set of linear algebraic equations for the inner products $v_i \cdot v_j$
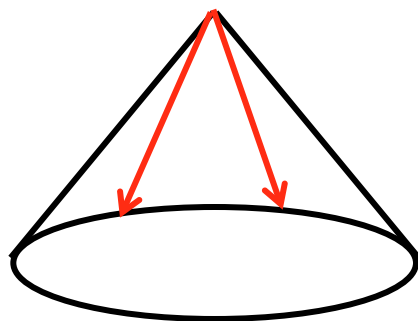
Given $\quad w_k = \sum\limits_i M_{ki} v_i.$

Then $\quad 0 = w_k \cdot w_k = \sum\limits_{i,j} M_{ki} M_{kj} (v_i \cdot v_j)$

where there is no sum on k.
By taking enough w_k, one fixes the inner products up to an overall constant.

Claim 2: If C(p) and C(q) intersect at precisely one point, and both cuts are C^1 at this point, then p and q are null-separated.

Proof: There is a unique inward directed null geodesic from each C^1 point on a cut. If cuts C(p) and C(q) are tangent at x, null geodesic from x must go through both p and q.
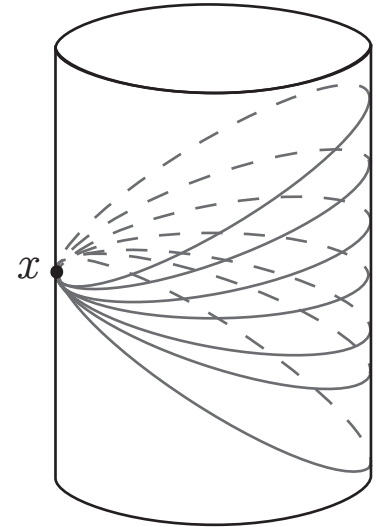
To reconstruct the conformal metric at p: Take C(p) and C^1 point x on it. The set of all cuts tangent at x form a line in the space of cuts. Define it to be null. Repeat at d+1 points to get null basis. Additional null vectors determine conformal metric.
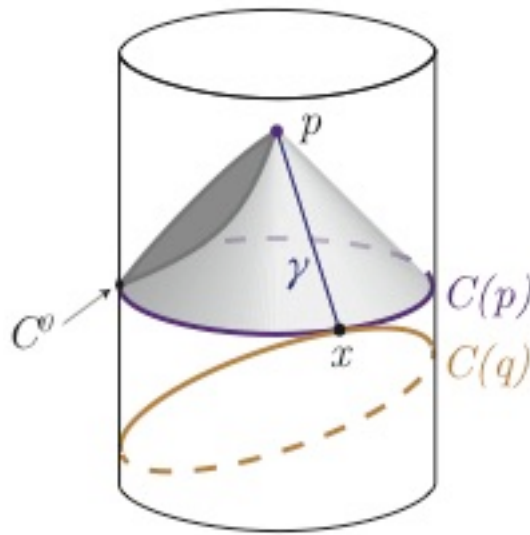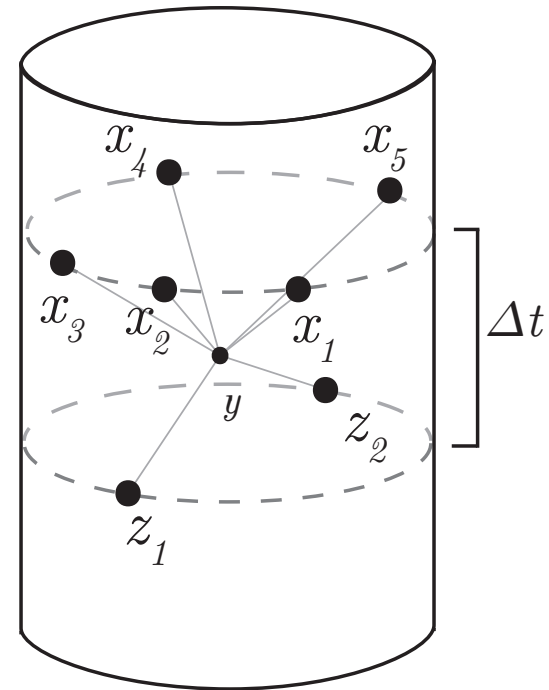

spacetime
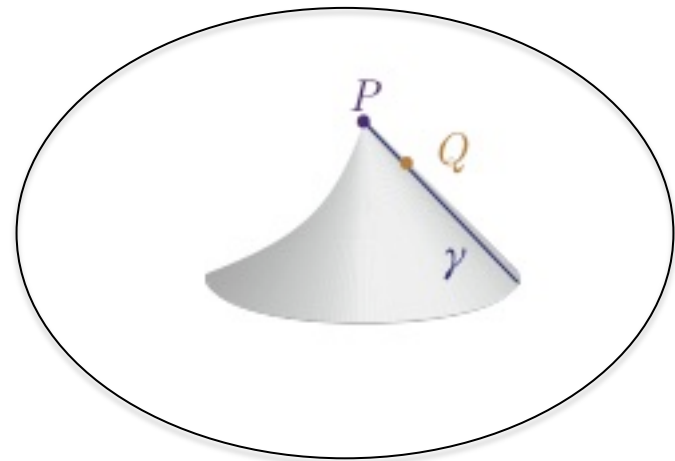
space of cuts

Finding the light-cone cuts

Refs: Maldacena, Simmons-Duffin, Zhiboedov, 1509.03612; Gary, Giddings, and Penedones, 0903.4437.

In the large N, large coupling limit: If O is a local CFT operator, the time-ordered Lorentzian correlator

$$\langle \mathcal{O}(z_1)\mathcal{O}(z_2)\mathcal{O}(x_1)\cdots\mathcal{O}(x_d)\rangle$$

diverges when there is a vertex y which is null related to each point and momentum is conserved at y.

This structure is very similar to our cuts: Every point in $C^\pm(p)$ is null related to p. To use it to find the cuts, we need two generalizations:

1) Rather than pure AdS, we work in an excited state which is asymptotically AdS.
2) Consider a d + 3 point correlator.

Fix the d+1 points in the future (which fixes y) and move z_i in spacelike directions keeping the correlator infinite. This traces out the past cut.

Comments

- This procedure only works for points in the causal wedge of the entire boundary.

- Since the cut corresponds to only part of the light cone, momentum conservation at the vertex may mean that only part of the cut can be reconstructed. This is enough to recover conformal metric.

- Singularity is expected to remain when perturbative corrections in 1/N are included, but not at finite N (or coupling).
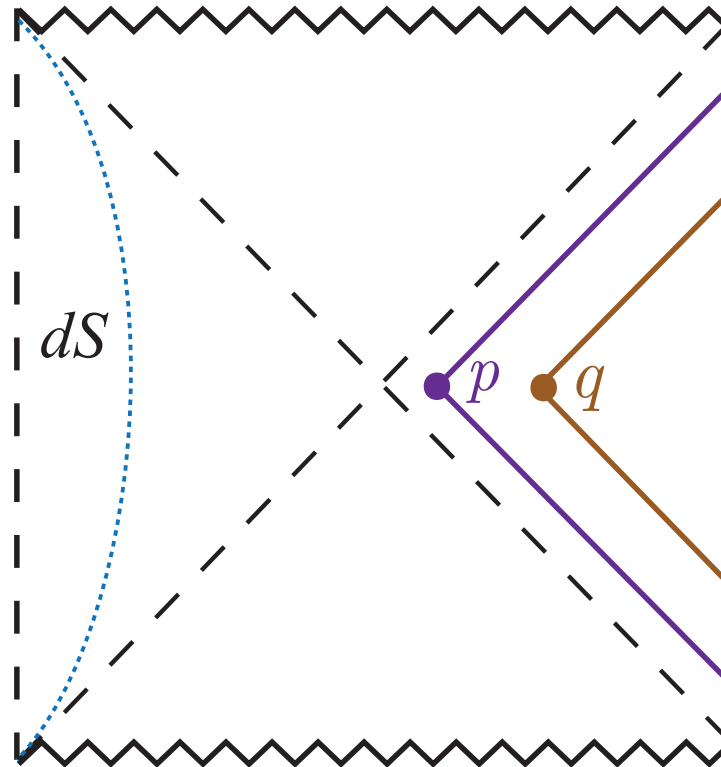
Global causal relations from cuts

If we are given the set of cuts, we can say if some points are spacelike or timelike separated:

p and q are spacelike separated if C(p) and C(q) cross, or if C±(q) both lie between C+(p) and C-(p).
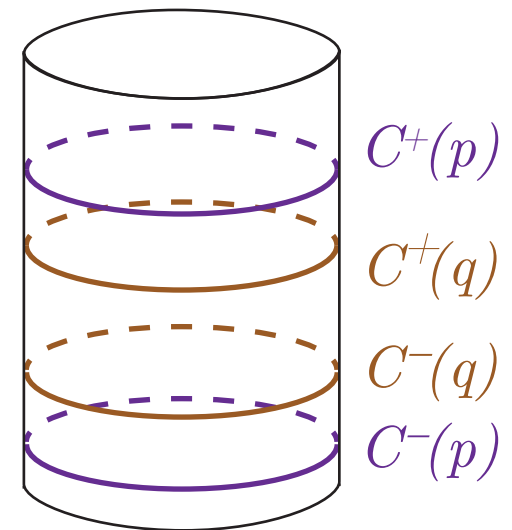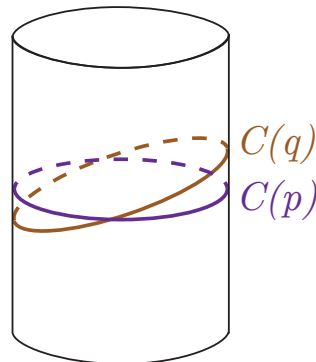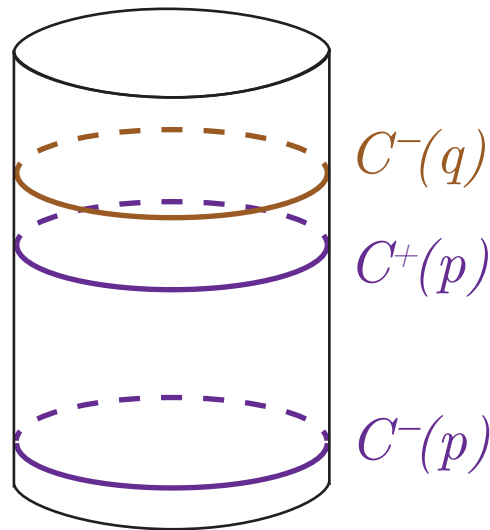
The second case can arise if one point is near the bifurcation surface of a black hole.
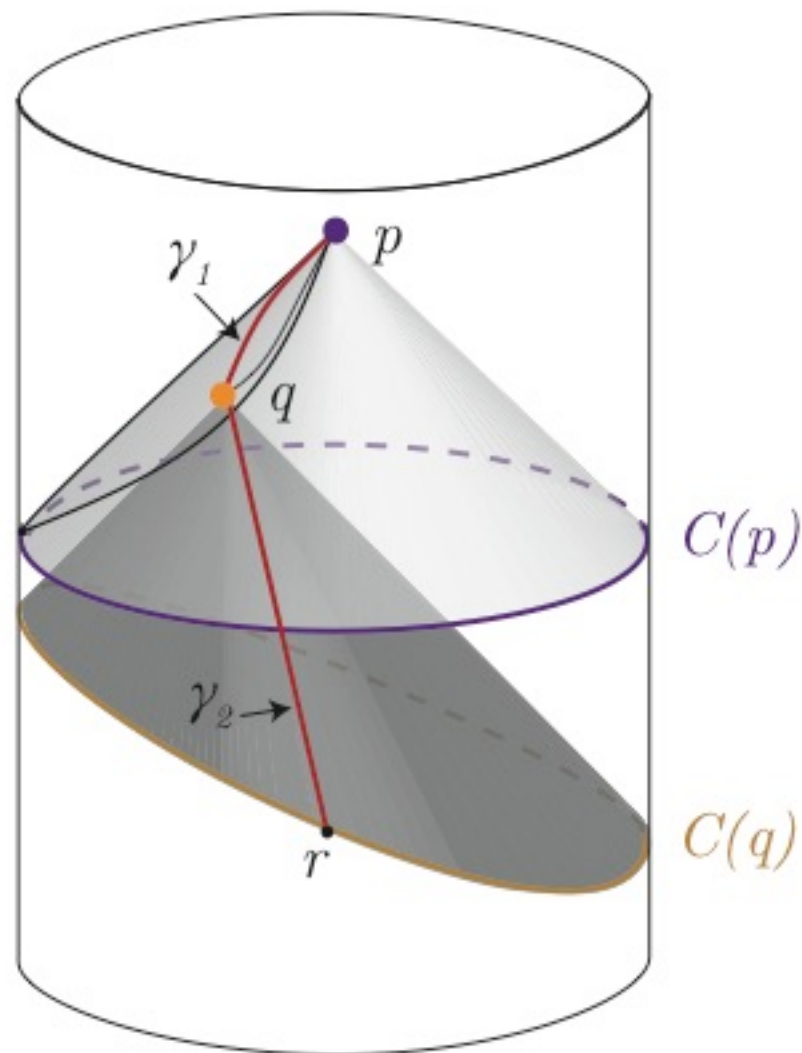

Bag of gold geometry with de Sitter region inside.

A second result:

q is to the future of p if $C^-(q)$ is to the future of $C^+(p)$.

If q is to the past of p, then $C^-(q)$ is to the past of $C^-(p)$. But the converse is false.

If q is on a line of caustics from p, then C⁻(q) is to the past of C⁻(p) even though p and q are null related.

One way to determine the conformal factor

Since we know the conformal metric, we know the Weyl curvature of the bulk.

If no bulk matter fields are turned on, the vacuum field equation fixes the Ricci curvature.

So the entire bulk geometry is fixed.

Open questions

1) Can one obtain other cuts from the CFT? (Some cuts can be obtained from symmetry: In spherical collapse to a black hole, $\langle T_{ab} \rangle$ is spherically symmetric. The past cuts of points at r = 0 are fixed by this symmetry.)

2) Can one obtain matter fields in the bulk using the cuts?

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- These cuts can be determined from singularities in gauge theory correlators
- The conformal metric in the bulk can be reconstructed just from the location of these cuts