Strings from Massive Higher Spins

Zohar Komargodski

Weizmann Institute of Science, Israel

with Simon Caron-Huot, Amit Sever, and Alexander Zhiboedov
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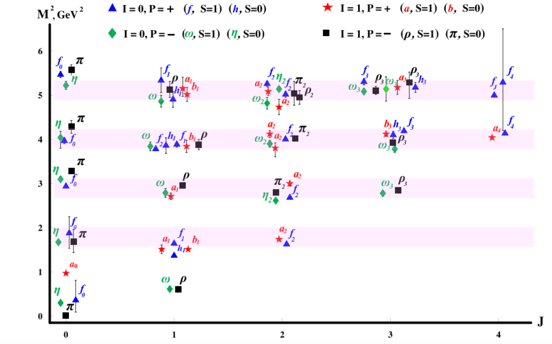
Consider a theory that has interacting particles with spins J and masses $M^2_{n,J}$. We assume that these particles interact weakly and they are stable if we turn off the interactions.



Examples

- Tree-Level String Theory: the spins are populated to ∞, large degeneracies and Hagedorn density.
- Yang-Mills Theory + Matter: if confinement occurs, expect the spins to be populated to ∞, don't expect exact degeneracies and expect Hagedron density of states. The resonances become exactly stable in the 't Hooft limit $N_c \to \infty$.
- Tree Level QFT: e.g. $L = (\partial\phi)^2 - M^2\phi^2 + \lambda\phi^3$ describes a stable spin 0 particle with weak self interactions if λ is small.

Historically, the investigation of the resonances of Yang-Mills theory led to String Theory and later String Theory was re-connected with Yang-Mills theory via Holography.

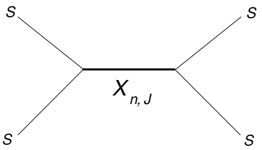
Here we will try to understand in what precise limit String Theory **in flat space** and Yang-Mills theory are connected.

The tool that we will use is the S-matrix. The scattering amplitude is a function of two complex variables s, t

$$A(s,t)$$

At the center of mass

$$s = E^2_{c.m.}\ , \qquad 1 + \frac{2t}{s - 4M_S^2} = \cos(\theta)\ .$$



The general properties of $A(s,t)$:

- Polynomial residues (and no other singularities):

$$\lim_{s\to M^2} A(s,t) = \frac{\sum_J f_J^2 P_J\left(1+\frac{2t}{M^2-4M_S^2}\right)}{s-M^2}$$

- Duality

$$A(s,t) = A(t,s)$$

These conditions by themselves are not very constraining. For example, in tree-level ϕ^3 theory we get

$$A(s,t) = \lambda \left[\frac{1}{s - M^2} + \frac{1}{t - M^2} \right]$$

There is a very natural way to eliminate such "uninteresting" solutions while retaining both Yang-Mills theories and tree-level String Theory.

We impose that there is some t_0 and spin J_0 particle such that

- Boundedness

$$\lim_{s\to\infty} A(s, t_0) < s^{J_0}$$

Equivalently,

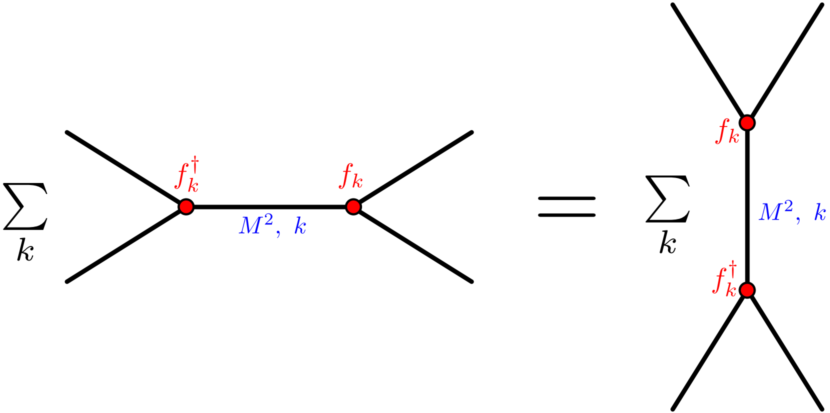
$$\lim_{s\to\infty} s^{-J_0} A(s, t_0) = 0 \ .$$

This 'Boundedness' condition eliminates all classical field theories. Moreover, if there is any particle with spin>2 in the spectrum then this condition must be satisfied if the theory makes sense in the ultraviolet [Camanho et al.].

Therefore this condition holds in Yang-Mills theory and String Theory. It allows to write the amplitude as *essentially* a sum over s-channel poles only

$$A(s,t) = \sum_{n,J} f_{n,J}^2 \frac{P_J\left(1+\frac{2t}{M_{n,J}^2-4M_S^2}\right)}{s-M_{n,J}^2}$$

The property $A(s,t)=A(t,s)$ is highly nontrivial.


$\sum_k$
$f_k^\dagger$
f_k
$M^2,\ k$
$=$
$\sum_k$
f_k
$M^2,\ k$
$f_k^\dagger$

- Such theories must have infinitely many particles.
- Such theories must have particles of unbounded spin.

Both conditions follow because otherwise there won't be appropriate poles in t. Therefore we refer to theories satisfying these conditions as **"Massive Higher Spin Theories."**

It is very interesting to consider in these theories the large s asymptotics with $t \ll s$

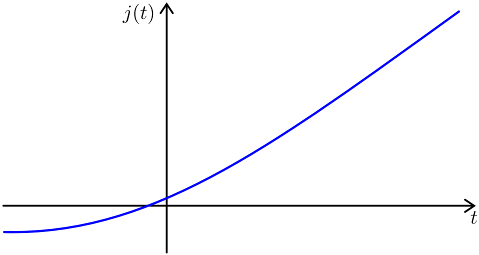
$$\lim_{s\to\infty} A(s,t) = F(t)s^{j(t)} \ .$$

importantly,

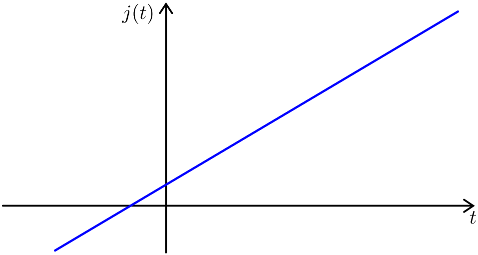
$$j(t_n) = n \ , \qquad n \geq 0$$

describe the fastest spinning particles (i.e. the leading Regge trajectory). If $t \leq 0$ and $s > 0$ then we are describing physical small angle scattering.

Confining Gauge Theory



Free String Theory



We see that at negative t Yang-Mills theory and String Theory are quite different. The Veneziano amplitude does not correctly describe the qualitative behaviour of small angle physical scattering. However, they seem rather similar at positive, large t.

Claim: Any Massive Higher Spin Theory behaves like

$$A(s,t) \sim e^{(s+t)\log(s+t)-s\log s-t\log t}$$

for large, positive, s, t (and arbitrary fixed s/t).

This form is, of course, also correct in Tree-Level String Theory [Veneziano...]. The consequences of this claim are

- Infinitely many *asymptotically linear and parallel* trajectories.
- In impact parameter space, if we take $b >> \Lambda_{QCD}^{-1}$ and if $s >> \Lambda_{QCD}^{2}$ the inelastic part of the amplitude is dominated by a saddle point off the contour of integration and we find

$$Im_s A(s,t) = e^{-\Lambda_{QCD}^2 b^2 / \log(s)} \ .$$

This signifies the existence of strings, because $\langle X_\perp^2 \rangle \sim \log(s)$ in free string theory.

Therefore, any weakly coupled theory which has $s > 2$ resonances must contain strings and agree with string theory in flat space in the high-energy imaginary-angle limit.

For $x > 1$, $P_J(x) > 0$. So it follows from

$$\lim_{s \to M^2} A(s,t) = \frac{\sum_k f_k^2 P_k \left(1 + \frac{2t}{M^2 - 4M_S^2}\right)}{s - M^2}$$

that for $t > 0$ all the residues are positive. Therefore, there is at least one zero between any two poles.

There may also be “excess” zeroes. But how many?!

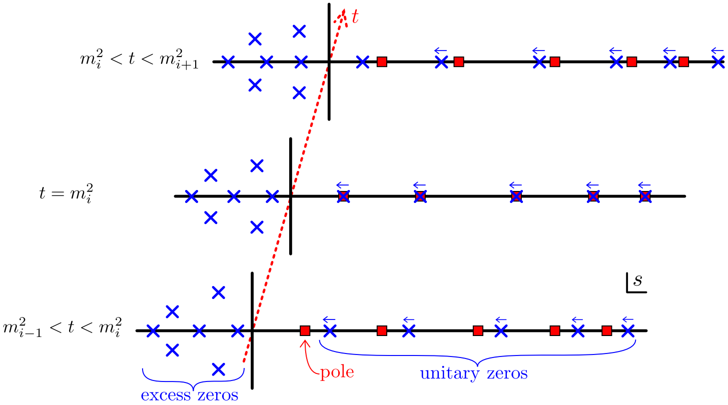
We can count them (reminds of Levinson's theorem from Quantum Mechanics) as follows:

$$\log A \sim j(t) \log(s)$$

implies that the discontinuity in s is given by $j(t)$. On the other hand, we can write the discontinuity as a sum over the number of zeroes minus poles

$$j(t) = \sum (zeroes - poles) \ .$$

So the number of excess zeroes is given by the number of fast-spinning bound states, i.e. $j(t)$!



t
$m_i^2 < t < m_{i+1}^2$
$t = m_i^2$
s
$m_{i-1}^2 < t < m_i^2$
excess zeros
pole
unitary zeros

The next key point is that for large *positive* s,t the amplitude is dominated by the excess zeroes. This is because the unitarity zeroes and poles give a contribution that is bounded by a constant. Therefore we denote the distribution of zeroes by $\rho(z, \bar{z}; t)$ and we write for the amplitude

$$\log A = \int d^2 z \ \rho(t; z, \bar{z}) \log(z - s) \ .$$

For very large t, s we can use dimensional analysis

$$\rho(t; z, \bar{z}) = \frac{j(t)}{t^2} \rho(z/t, \bar{z}/t)$$

$(\int d^2 z \rho(z, \bar{z}) = 1, \ \rho \geq 0)$

And thus we obtain (take $j(t) = t^k$)

$$\log A = t^k \int d^2 z \rho(z, \bar{z}) \log \left(1 - \frac{\beta}{z}\right)$$

with $\beta = s/t$.

This looks like the electric potential due to positive charges at $(z, \bar{z})$.

Duality and unitarity thus place nontrivial constraints on the allowed distributions $\rho(z, \bar{z})$. It would be convenient to define the "electric field" $F(\beta)$ as

$$F(\beta) = t^{1-k} \partial_s \log A = \int d^2 z \frac{\rho(z, \bar{z})}{\beta - z}$$

Unitarity 1

$$\partial_\theta^2 \log \left(\sum_{n=0}^{j(t)} C_n^2(t) \cosh(n\theta) \right) > 0$$

After some algebra one can see that this implies an inequality on the dipole moment

$$M_1 \equiv -\int d^2z \; z\rho(z,\bar{z}) \geq \frac{1}{2}$$

Veneziano: $\rho = \delta(Im(z))$ for $-1 \leq Re(z) \leq 0$ and thus $M_1 = 1/2$.

Crossing 1

At $s \gg t$ we have large β and we have the standard multipole expansion from electrostatics

$$F(\beta) = \frac{1}{\beta} - \frac{M_1}{\beta^2} + \cdots$$

but by Duality this implies the small β expansion (Duality takes $\beta \to \beta^{-1}$).

$$F(\beta) = -k \log(\beta)\beta^{k-1} + (k+1)M_1\beta^k + ...$$

which is consistent with the positivity of the second derivative only if

$$k > \frac{1}{2}$$

Furthermore, since we certainly have some positive charges away from the imaginary axis (as $M_1 \geq \frac{1}{2}$), the electric field cannot vanish at $\beta = 0$ and thus $F(\beta) = -k \log(\beta)\beta^{k-1} + ...$ is only consistent with unitarity if

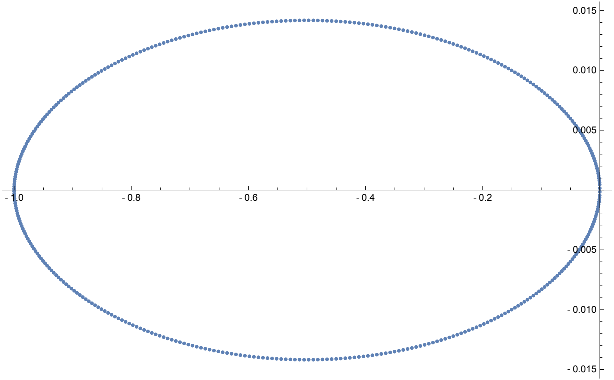
$$k \leq 1$$

Unitarity 2

We have a positive sum of partial waves

$$\sum_{n=0}^{j(t)} C_n^2(t) P_n\left(1+\frac{2s}{t}\right)$$

and the coefficients are not allowed to decrease too fast for otherwise the sum won't Reggeize. The zeroes in $\beta = s/t$ obviously all lie at $Re(z) \leq 0$. If the coefficients do not decay fast then the distribution has support only within the unit circle.


0.015
0.010
0.005
- 1.0
- 0.8
- 0.6
- 0.4
- 0.2
- 0.005
- 0.010
- 0.015

Crossing 2

Crossing $\beta \leftrightarrow \beta^{-1}$ is now very powerful as we are looking for a function that transforms nicely under $\beta \leftrightarrow \beta^{-1}$ and has branch points only at $0, -1$. This is because the electric field is analytic away from the charge distribution.

The solution to this electrostatics problem is unique:

$$F_k(\beta) = {}_2F_1(k, k, k+1, \frac{-1}{\beta}) \ .$$

The dipole moment can be read from the large β expansion. It is $k^2/(k+1)$. Hence we only remain with

$$k = 1$$

Thus we remain with $F_1(\beta) = \log\left(\frac{1+\beta}{\beta}\right)$ and uniform density between $[-1, 0]$. This fixes the amplitude uniquely to be, for large positive s, t,

$$\log A = (t+s)\log(t+s) - s\log(s) - t\log(t) \ .$$

Hence, every theory with spin>2 resonances, including Yang-Mills, must have strings and it is described by the Veneziano amplitude at large positive s, t.

- We see that the S-matrix Bootstrap program leads to new interesting results. The motivation for the question that we posed comes from recent lattice results [Teper...], calculations in gauge/gravity duality [Brower...], and effective string theories [Polchinski..., Hellerman...,]. It is therefore not surprising that it was not done 4 or 5 decades ago.
- Can we continue and determine the corrections to asymptotic linearity? Lifting the asymptotic degeneracy?
- Can we show that the Veneziano amplitude is the only amplitude with exactly linear trajectories?
- Implications for CFTs via the Mellin transform?
- The asymptotic, positive s, t regime (i.e. string theory) is separated from the physical small-angle high-energy scattering regime (the "AF" regime) by a phase transition in t. (Analytic continuations and asymptotic limits do not generally commute.) Can we characterise it?

And many more...

Thank you for your attention and see you in 2017 in Tel-Aviv!