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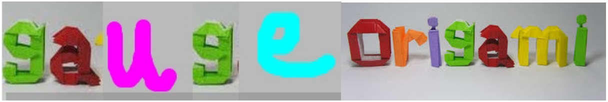
Strings'2016

NIKITA NEKRASOV

Beijing, August 2, 2016



Strings 2016 Beijing August 2

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Part of the project

NON-PERTURBATIVE

DYSON-SCHWINGER EQUATIONS

AND NOVEL SYMMETRIES OF QFT

DYSON-SCHWINGER EQUATIONS

INVARIANCE OF (PATH) INTEGRAL

$$\langle \mathcal{O}_1(x_1)\ldots\mathcal{O}_n(x_n)\rangle = \frac{1}{Z}\int_\Gamma D\Phi\, e^{-\frac{1}{\hbar}S[\Phi]}\,\mathcal{O}_1(x_1)\ldots\mathcal{O}_n(x_n)$$

UNDER "SMALL" DEFORMATIONS
OF THE INTEGRATION CONTOUR

$$\Phi \longrightarrow \Phi + \delta\Phi$$

DYSON-SCHWINGER EQUATIONS

QUANTUM EQUATIONS OF MOTION

$$\langle \mathcal{O}_1(x_1) \dots \mathcal{O}_n(x_n) \delta S[\Phi] \rangle = \hbar \sum_{i=1}^{n} \langle \mathcal{O}_1(x_1) \dots \mathcal{O}_{i-1}(x_{i-1}) \delta \mathcal{O}_i(x_i) \mathcal{O}_{i+1}(x_{i+1}) \dots \mathcal{O}_n(x_n) \rangle$$

DYSON-SCHWINGER EQUATIONS

WITH SOME LUCK

=

GOOD CHOICE OF (POSSIBLY NON-LOCAL) OBSERVABLES

$$\mathcal{O}_i(x)$$

IN SOME LIMIT (CLASSICAL, PLANAR, ...)

THE DS EQUATIONS CLOSE

For example, in

GAUGE THEORY

$$\Phi \longrightarrow A = A_\mu dx^\mu \in \mathrm{Lie}\, U(N)$$

$$\frac{1}{\hbar} S[\Phi] \longrightarrow S_{YM}[A] = -\frac{1}{4g^2} \int_{\mathbf{R}^4} \mathrm{tr} F_A \wedge \star F_A$$

$$\mathcal{O}_i(x_i) \longrightarrow W_R(\gamma) = \mathrm{tr}_R\, P\exp \oint_\gamma A$$

$$\mathcal{W}(\gamma) = \frac{1}{N} \langle W_\Box(\gamma) \rangle$$

GAUGE THEORY: PLANAR LIMIT

$$N \longrightarrow \infty, \qquad g^2 \to 0,$$

FINITE $\lambda = g^2 N$

$$\Delta_\gamma \mathcal{W}(\gamma) = \frac{g^2}{N} \langle W_\Box(\gamma) \delta S_{YM}[A] \rangle =$$

$$= \lambda \delta_{\gamma = \gamma_1 \star \gamma_2} \mathcal{W}(\gamma_1) \mathcal{W}(\gamma_2) + \frac{1}{N^2} \textit{ correctons}$$

MAKEENKO-MIGDAL LOOP EQUATIONS

MATRIX MODEL as a TOY GAUGE THEORY

$$\Phi \in \mathrm{Lie}\, U(N)$$

$$\frac{1}{\hbar} S[\Phi] = \frac{1}{\hbar} \mathrm{tr}\, V(\Phi)$$

$$V(X) = v_p X^p + v_{p-1} X^{p-1} + \ldots + v_1 X + v_0$$

$$\mathcal{O}(x) = \frac{1}{N} \mathrm{tr}_{\square} \left(\frac{1}{x - \Phi} \right)$$

MATRIX MODEL

PLANAR LIMIT: $\lambda = \hbar N$ FIXED

$$\hbar \to 0, \;\; N \to \infty$$

DS EQUATIONS $\Longrightarrow$ LOOP EQUATIONS

$$y(x)^2 = V'(x)^2 + g_{p-2}(x)$$

$$y(x) = \langle \mathcal{O}(x) \rangle + V'(x)$$

$g_{p-2}(x) =$ DEGREE $p-2$ POLYNOMIAL IN x

QFT PATH INTEGRAL INVOLVES SUMMATION

OVER TOPOLOGICAL SECTORS

FOR EXAMPLE, IN GAUGE THEORY

$$Z = \sum_{n\in\mathbb{Z}} e^{in\vartheta} \int_{\mathcal{A}_n} \left[\frac{DA}{Vol(\mathcal{G}_n)}\right] e^{-S_{YM}[A]}$$

$$-\frac{1}{8\pi^2}\int \mathrm{tr} F_A \wedge F_A = n, \qquad A \in \mathcal{A}_n$$

NON-PERTURBATIVE DS EQUATIONS

IDENTITIES DERIVED BY

LARGE “DEFORMATIONS” OF THE PATH INTEGRAL CONTOUR

$$A \in \mathcal{A}_n \longrightarrow A + \delta A \in \mathcal{A}_{n+1}$$

GRAFTING A POINT-LIKE INSTANTON

EXPLICIT REALIZATION

SUPERSYMMETRIC

GAUGE THEORIES

SIGMA MODELS

$\mathcal{N} = 2$ **SUPERSYMMETRIC**

FOUR DIMENSIONAL GAUGE THEORIES

TWO DIMENSIONAL SIGMA MODELS

Also, $\mathcal{N} = 1$ SUPERSYMMETRIC

Five and Six DIMENSIONAL GAUGE THEORIES

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OBSERVABLES FOR DS EQUATIONS

$\mathbf{Y}(\mathbf{x})$

IN FOUR DIMENSIONAL $U(N)$ GAUGE THEORY

$$\mathbf{Y}(\mathbf{x}) \sim \mathbf{det}_{\mathbb{C}^N}(x - \Phi) \sim \prod_{\alpha=1}^{N}(x - a_\alpha)$$

NAIVELY

◇

OBSERVABLES FOR DS EQUATIONS

$\mathbf{Y}(\mathbf{x})$ IN FOUR DIMENSIONS

MORE PRECISELY

$$\mathbf{Y}(\mathbf{x}) = \mathbf{x}^N \exp -\sum_{k=1}^{\infty} \frac{1}{k\mathbf{x}^k} \mathrm{Tr}\Phi^k$$

$$\mathbf{Y}(\mathbf{x}) = \mathbf{x}^N \exp - \sum_{k=1}^{\infty} \frac{1}{k\mathbf{x}^k} \mathrm{Tr}\Phi^k$$

Non-perturbatively, e.g. in an instanton background
$\mathbf{Y}(\mathbf{x})$ acquires poles in $\mathbf{x}$

FOR GAUGE GROUP WITH SEVERAL FACTORS

$$G = U(N_1) \times \ldots \times U(N_r)$$

$$\mathbf{Y}(\mathbf{x}) \longrightarrow (\mathbf{Y}_1(\mathbf{x}), \mathbf{Y}_2(\mathbf{x}), \ldots, \mathbf{Y}_r(\mathbf{x}))$$

$\mathbf{Y}_i(\mathbf{x})$ degree N_i meromorphic functions of $\mathbf{x}$

CLAIM

for quiver gauge theories

THERE EXIST

LAURENT POLYNOMIALS or SERIES

$$\mathcal{X}_i(x) = Y_i(x) + \ldots$$

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LAURENT POLYNOMIALS or SERIES

$$\mathcal{X}_i(x) = Y_i(x) + \ldots$$

in

$Y_j(x+$ linear combinations of bi-fundamental masses)

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LAURENT POLYNOMIALS or SERIES

$$\mathcal{X}_i(x) = Y_i(x) + \dots$$

coefficients depend on gauge couplings

and masses of fundamental hypermultiplets

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such that

$$\langle \; \mathcal{X}_i(x) \; \rangle = \text{POLYNOMIAL IN} \;\; x$$

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WE CALL $\mathcal{X}_i(x)$

THE FUNDAMENTAL GAUGE CHARACTERS

THE FUNDAMENTAL GAUGE CHARACTERS $\mathcal{X}_i(x)$

are four dimensional analogues of the matrix model's

$$\left(\frac{1}{N} \mathrm{tr}_{\square} \frac{1}{x - \Phi} + V'(x) \right)^2$$

MORE GENERAL LOCAL OBSERVABLES $\mathcal{X}_{\mathbf{w}}(x)$

THE GAUGE CHARACTERS

$$\mathcal{X}_{\mathbf{w}}(x) \;=\; \mathcal{X}_{w_1}(x-\nu_1)\mathcal{X}_{w_2}(x-\nu_2)\ldots\mathcal{X}_{w_p}(x-\nu_p) \;+\; \text{corrections}$$

THE CLAIM = SEIBERG-WITTEN GEOMETRY

of low-energy effective theory

NN, V.Pestun, 2012

(DOUBLE) QUANTUM SEIBERG-WITTEN GEOMETRY

when theory is subject to Ω-deformation

$\mathcal{X}_{\mathbf{w}}(x) \longrightarrow \chi_{\mathbf{w}}(x)$ – qq-characters

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For example: $SU(N)$ theory

$$\tau = \frac{\vartheta}{2\pi} + \frac{4\pi i}{g^2}$$

A_1 case: $N_c = N$, $N_f = 2N$

fundamental qq-character

$$Y(x+\epsilon_1+\epsilon_2) + e^{2\pi i\tau} \prod_{i=1}^{N_f} (x - m_i) Y(x)^{-1}$$

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THE ORIGIN OF *qq*-CHARACTERS

$\mathcal{X}_{\mathbf{w}}(x) =$ PARTITION FUNCTION

OF A POINT-LIKE DEFECT $\mathscr{D}_{\mathbf{w}}(x)$

$\mathscr{D}_{\mathbf{w}}(x)$ CAN BE ENGINEERED

USING INTERSECTING BRANES

Brane-world scenarios

propose that the Standard Model is confined to a brane while gravity propagates in the bulk

Brane-world scenarios

propose that the Standard Model is confined to a brane

which could originate from the string theory D-branes

with closed strings propagating in the bulk

Brane-world scenarios

propose that the Standard Model is confined to a brane

which could originate from the string theory D-branes

spanning a nearly flat, or a nearly AdS space

What if there is more to the world then meets the eye?

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What if there is more then one stack of branes?

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What if there is more then one stack of branes?
Branes that intersect?

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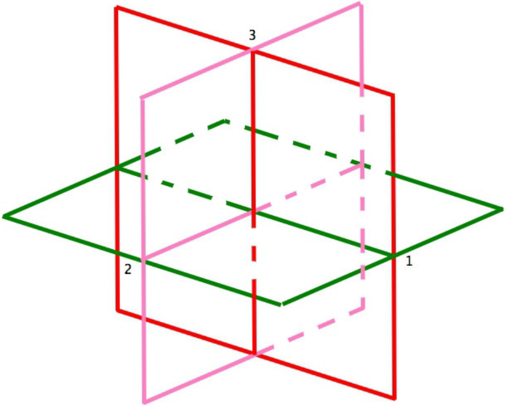
The intersections could be either
the defects in the worldvolume or
our braneworld could be an intersection

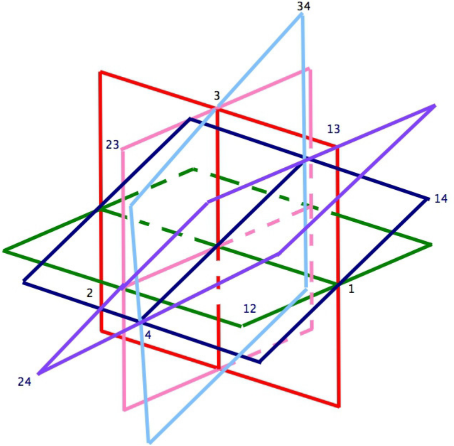
Local model: $\cup_{a<b} \mathbb{C}^2_{ab} \subset \mathbb{C}^4$

For example, when $1 \leq a, b \leq 3$



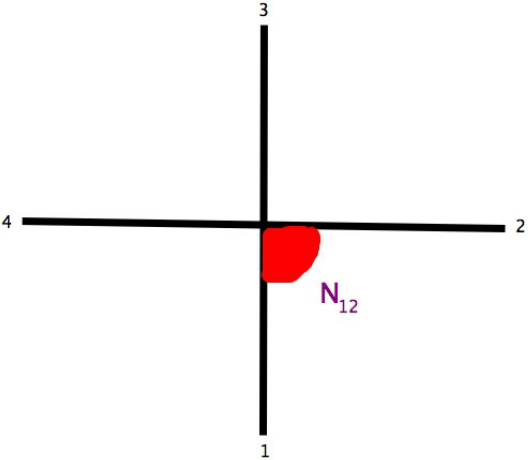
Local string model: $\left(\cup_{a<b}\, \mathbb{C}^2_{ab}\right) \times \mathbb{R}^2 \subset \mathbb{R}^{10}$

When $1 \le a, b \le 4$ one preserves $(0,2)$ susy in $\mathbb{R}^{1,1}$



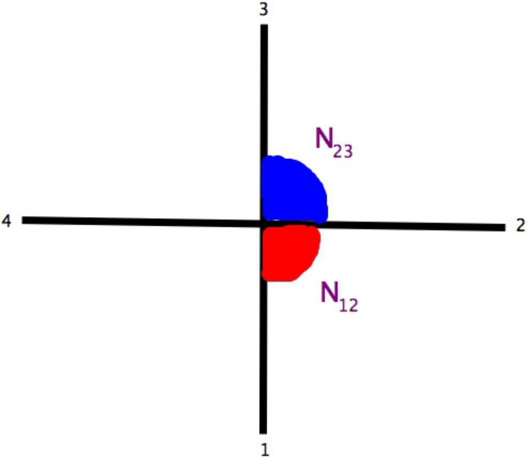
Useful pictures

$n_{12} = \dim N_{12}$ branes along $\mathbb{C}^2_{12}$

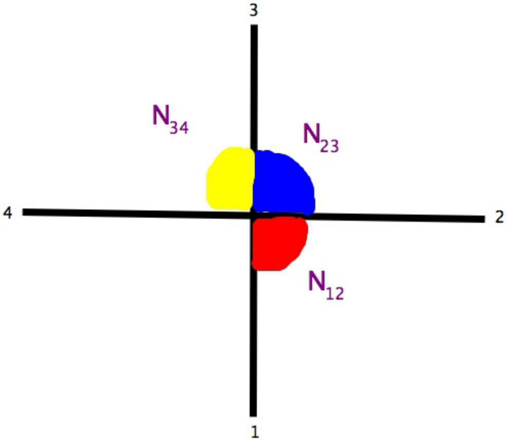


Useful pictures

$n_{12} = \dim N_{12}$ branes along $\mathbb{C}^2_{12}$ and $n_{23} = \dim N_{23}$ branes alond $\mathbb{C}^2_{23}$



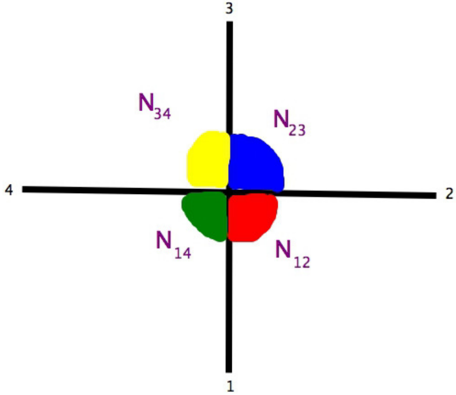
$n_{12} = \dim N_{12}$ branes along $\mathbb{C}^2_{12}$, $\quad n_{23} = \dim N_{23}$ branes along $\mathbb{C}^2_{23}$,
$n_{34} = \dim N_{34}$ branes along $\mathbb{C}^2_{34}$



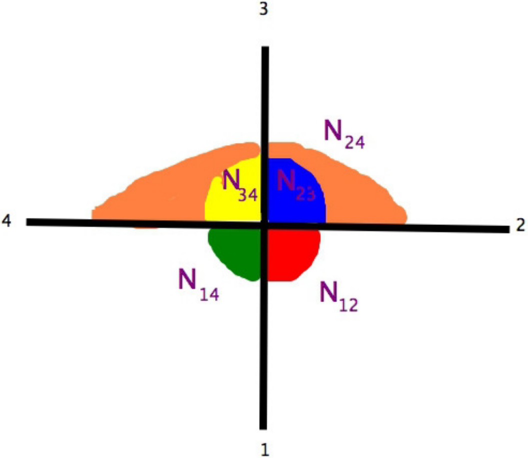
Useful pictures

$n_{12} = \dim N_{12}$ branes along $\mathbb{C}^2_{12}$, $\quad n_{23} = \dim N_{23}$ branes along $\mathbb{C}^2_{23}$,

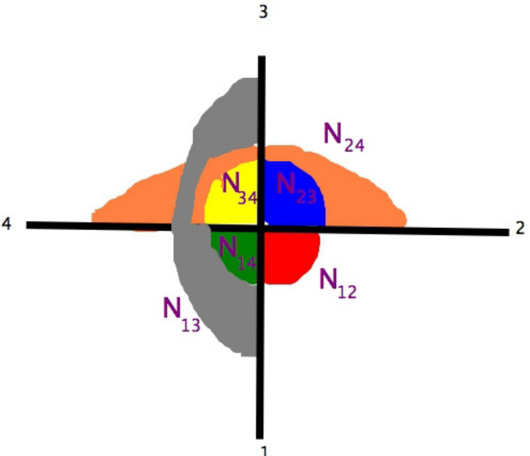
$n_{34} = \dim N_{34}$ branes along $\mathbb{C}^2_{34}$, $\quad n_{14} = \dim N_{14}$ branes along $\mathbb{C}^2_{14}$

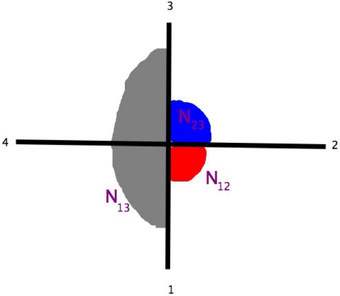


$n_{12} = \dim N_{12}$ branes along $\mathbb{C}^2_{12}$, $n_{23} = \dim N_{23}$ branes along $\mathbb{C}^2_{23}$,
$n_{34} = \dim N_{34}$ branes along $\mathbb{C}^2_{34}$, $n_{14} = \dim N_{14}$ branes along $\mathbb{C}^2_{14}$
$n_{24} = \dim N_{24}$ branes along $\mathbb{C}^2_{24}$

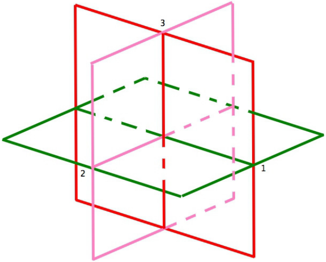


$$
\begin{aligned}
n_{12} &= \dim N_{12} \text{ branes along } \mathbb{C}^2_{12}, & n_{23} &= \dim N_{23} \text{ branes along } \mathbb{C}^2_{23},\\
n_{13} &= \dim N_{13} \text{ branes along } \mathbb{C}^2_{13}, & n_{24} &= \dim N_{23} \text{ branes along } \mathbb{C}^2_{24},\\
n_{14} &= \dim N_{14} \text{ branes along } \mathbb{C}^2_{14}, & n_{34} &= \dim N_{34} \text{ branes along } \mathbb{C}^2_{34}
\end{aligned}
$$




3
4
2
1
N23
N12
N13

=


3
2
1

Integrate out the degrees of freedom

on all but one of the stacks

To produce observables on the remaining stack of branes

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Degrees of freedom associated with k instantons

Chan-Paton spaces

$$K = \mathbb{C}^{k = \#instanton\ charge}, \qquad N_{ab} = \mathbb{C}^{n_{ab}}$$

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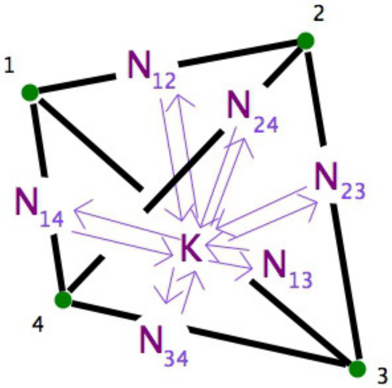
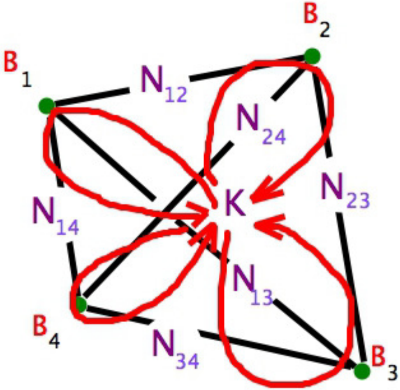
Degrees of freedom associated with instantons:

Rectangular complex I_{ab}, J_{ab} matrices, $1 \leq a < b \leq 4$

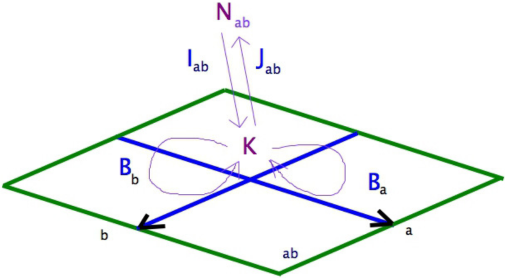


Degrees of freedom associated with instantons

Square complex matrices B_a, $a = 1, \ldots, 4$



Local ADHM data



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Surface operators from intersecting braneworlds

Sewn quiver gauge theories, using orbifolds

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Partition Z-function of gauge origami

Partition Z-function of gauge origami

Equivariant integral over the space of solutions

of generalized ADHM equations

Generalized ADHM equations

$$\mu_{ab} + \varepsilon_{abcd}\mu^{\dagger}_{cd} = 0$$

$$\mu_{ab} = [B_a, B_b] + I_{ab}J_{ab}$$

Generalized ADHM equations I.

$$\mu_{ab} + \varepsilon_{abcd}\mu^{\dagger}_{cd} = 0$$

$$\mu_{ab} = [B_a, B_b] + I_{ab}J_{ab}$$

$$\sum_a [B_a, B_a^{\dagger}] + \sum_{a<b} I_{ab}I^{\dagger}_{ab} - J^{\dagger}_{ab}J_{ab} = \zeta \cdot \mathbf{1}_K$$

7 hermitian $k \times k$ matrix equations

Generalized ADHM equations I.

$$\mu_{ab} + \varepsilon_{abcd}\mu^{\dagger}_{cd} = 0,$$

for all $1 \leq a < b \leq 4$, where

$$\mu_{ab} = [B_a, B_b] + I_{ab}J_{ab}$$

and

$$\mu \equiv \sum_{a} [B_a, B_a^{\dagger}] + \sum_{a<b} I_{ab}I^{\dagger}_{ab} - J^{\dagger}_{ab}J_{ab} = \zeta \cdot \mathbf{1}_K$$

7 hermitian $k \times k$ matrix equations

Divide by the $U(k)$ action

Generalized ADHM equations II.

$$B_a I_{bc} + \varepsilon_{abcd} B_d^\dagger J_{bc}^\dagger = 0$$

for all a, b, c, d, s.t. $\varepsilon_{abcd} \neq 0$

$2k \sum_{a<b} n_{ab}$ complex equations

Generalized ADHM equations III.

$$J_{ab} I_{cd} - I^{\dagger}_{ab} J^{\dagger}_{cd} = 0$$

for all a, b, c, d, s.t. $\varepsilon_{abcd} \neq 0$

$$J_{ab} B_b^{p-1} I_{bc} = 0$$

for all $1 \leq a < b < c \leq 4$, and $p \geq 1$

THE MAIN CLAIM

with a suitably defined perturbative factors

the partition **Z**-function

of gauge origami

is an entire function of all $\sum_{a<b} n_{ab}$ Coulomb parameters

APPLICATIONS

THE NONPERTURBATIVE DS EQUATIONS

APPLICATIONS

BPS/CFT CORRESPONDENCE

NN, 2002-2004

Correlators of chiral observables
in four dimensional supersymmetric theories
are **holomorphic blocks (form-factors)**
of some **conformal field theory**
(or a massive integrable deformation thereof)
in two dimensions

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Z-FUNCTIONS

OF A-TYPE QUIVER THEORIES, WITH OR WITHOUT DEFECTS

OBEY THE BPZ/KZ-TYPE EQUATIONS

of chiral algebras for 2d CFT and SCFTs

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THANK YOU

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