Hamiltonian truncation methods in strongly coupled QFT

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Strings 2016, Tsinghua University, Beijing

QFT is an unfinished business

Need: general method to solve* strongly coupled QFTs

- No extra assumptions (SUSY, large N, integr.)

*) solve ≡ compute

What will it take?

- Building such a method needs a combination of formal background & an eye towards practical applications
- Unlikely that a fully analytic approach will work.
- Probably need a combination of numerics & analytics

General QFT = RG flow starting from a UV CFT

$\mathrm{CFT}_{UV} \bullet \xrightarrow{\text{RG flow}}$

$\boxed{1}$ Solve CFT_{UV}

[conformal bootstrap]

$\boxed{2}$ Solve RG flow

Today's talk

Allowed UV CFT perturbations

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$\boxed{G}$ Gauge a global symmetry:

$$\Delta S = -\frac{1}{4g^2}\int F_{\mu\nu}^2 + \int J_\mu^a A^{a\mu} + \text{seagulls}$$

- relevant if $d<4$
- marginally relevant in 4d depending on $\langle JJ \rangle$

What happens?

- At high energies, can use conformal perturbation theory

- At low energies, typically strong coupling $\Longrightarrow$ need nonperturbative technique

Established NP techniques

- Lattice Monte-Carlo

⊕ works in any d

⊖ need to put CFT_{UV} on the lattice
⇒ extra work unless UV is free
(and may be impossible for some CFTs)

⊖ expensive
Years of supercomputer time for
4d QCD with dynamical fermions

▶ DMRG / Matrix Product States / Tensor Networks

efficient for low-lying states of spin systems with finite ξ

⊖ so far computationally tractable only in 1+1 d

⊖ need to realize CFT_{UV} as a spin chain

▸ Light-cone quantization / DLCQ

Especially suited to $(1+1)d$ gauge theories w/ matter [Gross, Klebanov et al 90's]

Recent progress in basis optimization, extending to $d>2$, and to general CFT_{UV}

[Katz et al '13 '14]

[Katz, Khandker, Walters '16]

Truncated Conformal Space Approach (TCSA)

[Yurov, Al. Zamolodchikov '89]

Used mostly in cond-mat, but deserves hep-th attention

⊕ Works directly in terms of UV CFT data

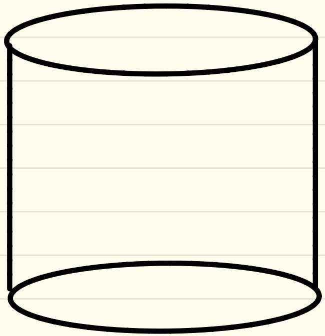
⊖ Currently only for $\boxed{R}$ perturbations (no gauging)

⊖ Works best for strongly relevant perturbations

⊖ Most work in $d=2$.
$d>2$ straightforward but poorly explored
[Hogervorst, S.R., van Rees '14]

TCSA algorithm

$\boxed{1}$ Put CFT_{UV} on the "cylinder" $S^{d-1}_R \times \mathbb{R}$



State-operator correspondence $\Longrightarrow$

H_{CFT} diagonal with spectrum

$$E_i = \frac{\Delta_i}{R}$$

$\boxed{2}$ Perturb the Hamiltonian:

$$H = H_{CFT} + \mu^{d-\Delta} \int_{S^{d-1}_R} \mathcal{V}(x)$$

diagonal $E_i \delta_{ij}$

off-diagonal

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Perturbation matrix elements:

$$\langle i | \int_{S_R^{d-1}} \mathcal{V} | j \rangle \propto R^{d-\Delta-1} \underbrace{\langle \mathcal{O}_i(0)\, \mathcal{V}(1)\, \mathcal{O}_j(\infty) \rangle}_{\text{OPE coeff. } C_{i\mathcal{V}j}}$$

Full Hamiltonian :

$$H = \frac{1}{R}\left[\Delta_i \delta_{ij} + (\mu R)^{d-\Delta} C_{i\sigma_j}\right]$$

- $\mu R \ll 1$ small correction
- $\mu R \gg 1$ strong coupling

$\boxed{3}$ Truncate Hilbert space to

$$\Delta \leq \Delta_{max}$$

Diagonalize truncated Hamiltonian on a computer

$\Rightarrow$ finite volume spectrum

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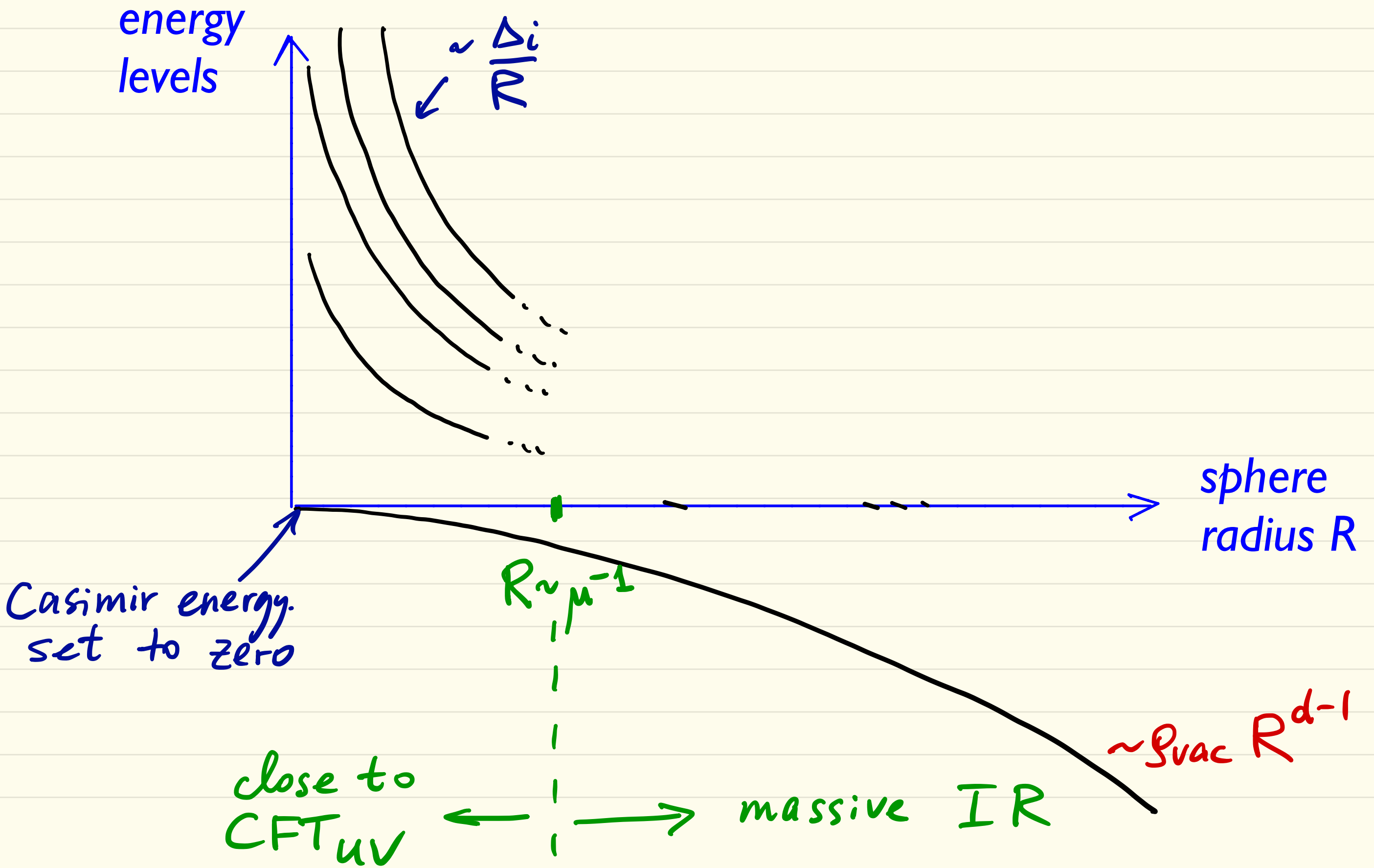
Expansion parameter $\frac{\mu}{\Lambda_{UV}}$

$\Rightarrow$ can go to $R \sim \Delta_{max}\, \mu^{-1}$

Can access IR physics if $\boxed{\Delta_{max} \gg 1}$

more about rate of conv., UV div's ...

What do we expect to see?



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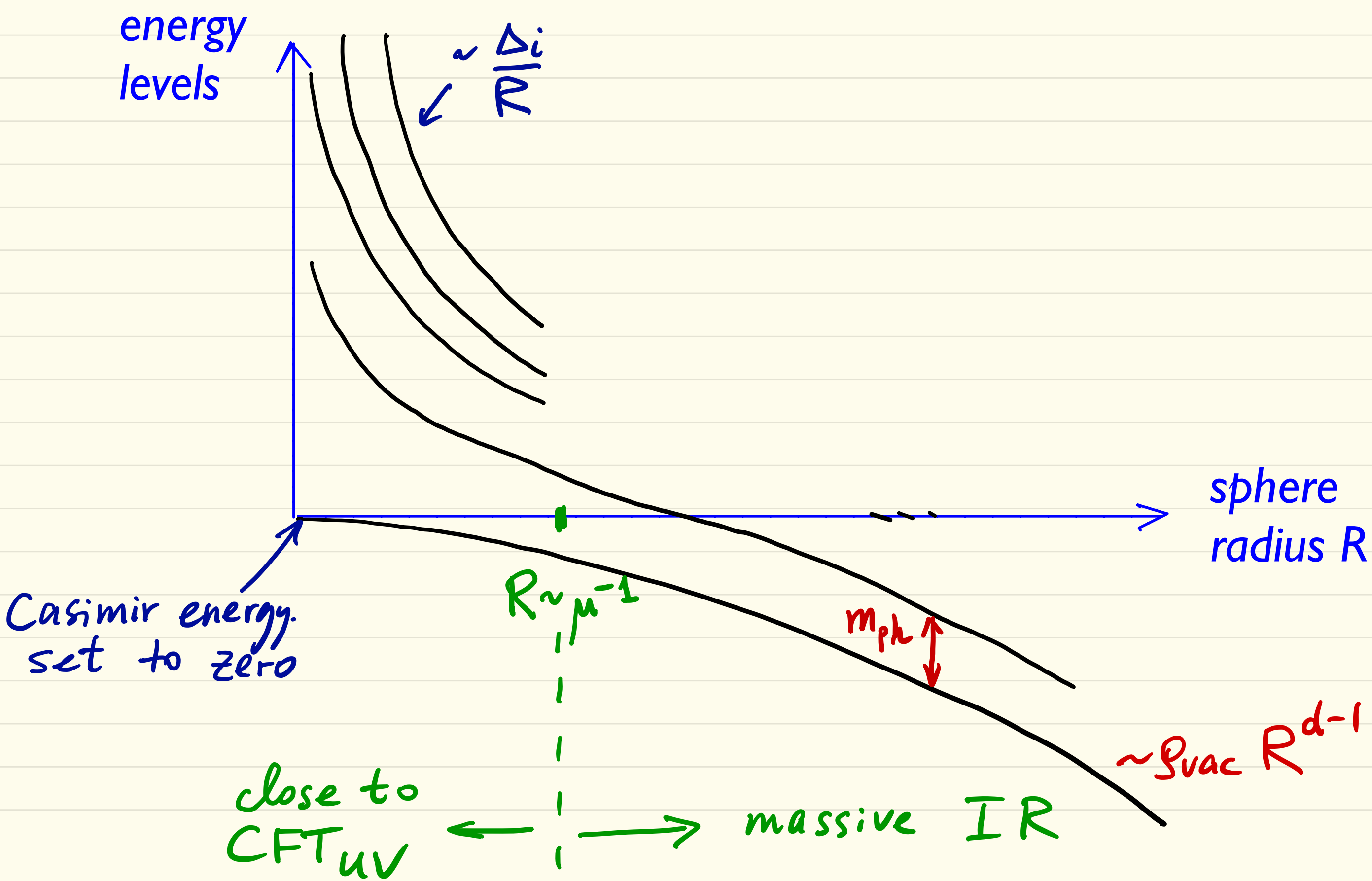

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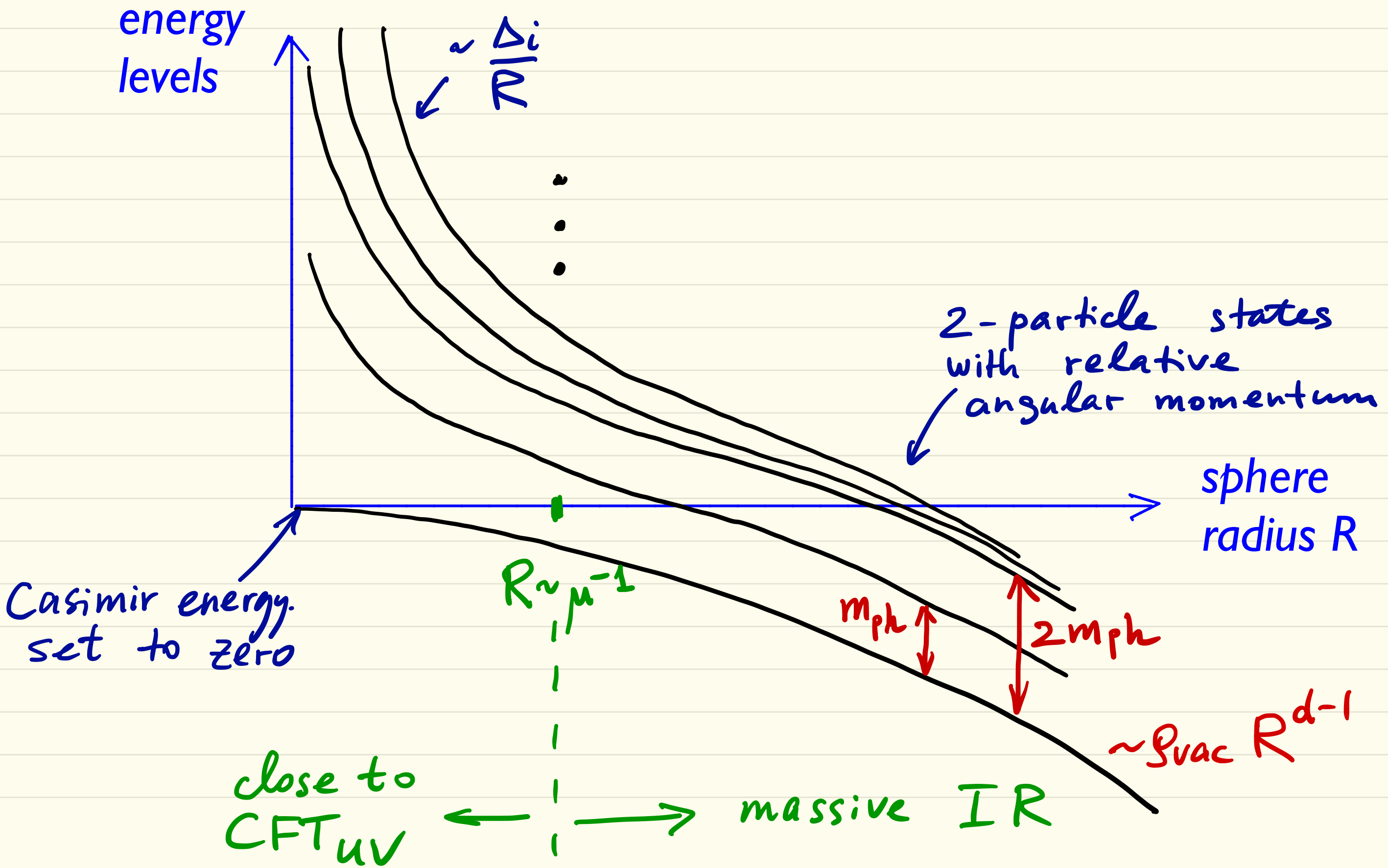
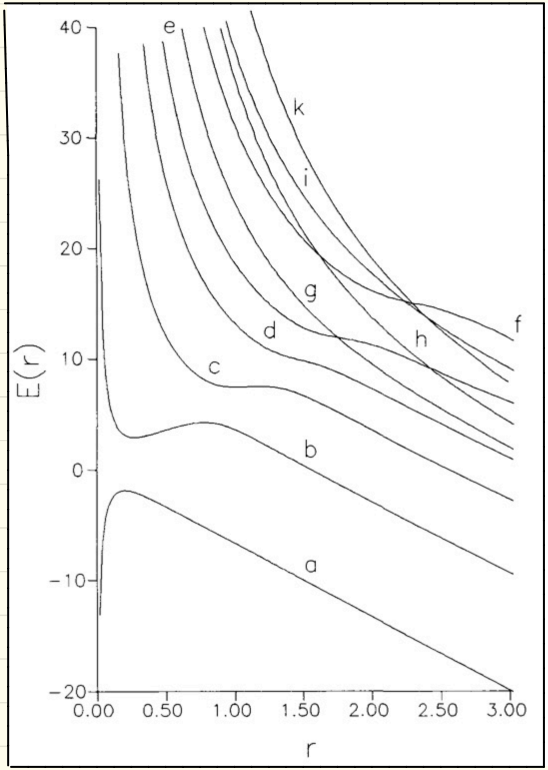


Real data example [Yurov, Al. Zamolodchikov '91]



2d Ising + $h \int \sigma(x)\, d^2x$

Integrable RG flow ; 8 particles

$\frac{m_2}{m_1} = 1.61803\ldots \approx 1.61(1)$

$\frac{m_3}{m_1} = 1.98904\ldots \approx 1.98(2)$

$\frac{m_4}{m_1} = 2.40486\ldots \approx 2.43(3)$

Exact — TCSA

...

Obtained with $\Delta_{max} = 10$ (34 states)

Truncation error

- Caused by mixing with high-energy states

$\Delta_\ell \le \Delta_{max}$ $\qquad\qquad$ $\Delta_h > \Delta_{max}$

mixing $\quad \langle \ell | V | h \rangle \neq 0$

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mixing $\langle \ell | V | h \rangle \neq 0$

- Mixing can be estimated from correlators

$$\langle \ell | V(1)\, V(2) | \ell' \rangle$$

in the OPE limit $1 \to 2$

[Hogervorst, S.R., van Rees '14]
[S.R., Vitale '14]

- Convergence rate $\left(\frac{\mu}{\Lambda_{UV}}\right)^{2d-\Delta_V} \sim \left(\frac{\mu R}{\Delta_{max}}\right)^{2d-\Delta_V}$

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- For $\Delta_V > d/2$ naive TCSA will <u>not</u> converge [Klassen, Melzer '92]

  Related to UV divergences

Vacuum energy divergence

- 1^{st} divergence appears in ρ_{vac}

Visible already in 2^{nd} order pert. theory:

$$\delta\rho_{vac} \propto \int d^d x \, \langle \mathcal{V}(0)\mathcal{V}(x)\rangle$$

UV-divergent if $\Delta_{\mathcal{V}} \geq \frac{d}{2}$

This divergence cancels in energy differences (particle masses)

Renormalization

- Further divergences for larger Δ_σ

These renormalize couplings & affect masses

Counterterms need to be added.

This is as usual in QFT except for an unusual regulator (cutoff in Δ_{max})

RG improvement

- Even for $\Delta_V < \frac{d}{2}$, when naive TCSA converges, it makes sense to compute cutoff effects & compensate for them

- I.e. instead of truncation one tries to integrate out high-energy states

- This is similar to using RG-improved actions in lattice QCD

See [Hogervorst, S.R., van Rees] for details

Conclusions

- Hamiltonian truncation methods, such as TCSA, give a fascinating window on strongly coupled RG flows

  Generalize Rayleigh-Ritz method from QM to QFT

- Large body of cond-mat literature

  see our papers for refs

- Would be nice to
  - explore further
  - develop RG-improvement
  - extend to $d>2$
  - extend to gauge theories

Basis optimization

Number of states grows exponentially:

$$N(\Delta) \sim \exp\left[\text{const.}\, \Delta^{\alpha}\right], \quad \alpha = 1 - \frac{1}{d}$$

But empirically, only a small fraction turns out important.

Important to understand theoretically.