A Duality Web in 2 + 1 Dimensions and the Unity of Physics

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IAS
Based on:
NS and E. Witten, arXiv:1602.04251;
NS, T. Senthil, C. Wang, and E. Witten, arXiv:1606.01989;
P.-S. Hsin, NS, arXiv:1607.07457

Recent related papers:
A. Karch and D. Tong, arXiv:1606.01893;
Three (almost) independent lines of development – the unity of physics

• The condensed matter, $3d$ quantum field theory route
• The supersymmetric route
• The AdS/CFT, large $N$ route
The condensed matter, $3d$ QFT route

- Statistical transmutation: a massive particle coupled to a dynamical (statistical, emergent) gauge field with a Chern-Simons term can change its spin and statistics [Wilczek; Polyakov; Jain].
  - Many applications (FQHE, composite fermions, flux attachment, ...)


The condensed matter, $3d$ QFT route

• Statistical transmutation: a massive particle coupled to a dynamical (statistical, emergent) gauge field with a Chern-Simons term can change its spin and statistics [Wilczek; Polyakov; Jain].
  – Many applications (FQHE, composite fermions, flux attachment, ...)

• This does not mean that a second-quantized theory of massless interacting bosons coupled to a gauge field with a Chern-Simons term is dual to a theory of fermions (or the other way around).
The condensed matter, 3d QFT route

\[ |D_B \Phi|^2 - |\Phi|^4 \iff |D_b \hat{\Phi}|^2 - |\hat{\Phi}|^4 + \frac{1}{2\pi} B db \]
The condensed matter, 3d QFT route

Particle/vortex duality [Peskin; Dasgupta and Halperin]

\[ |D_B \Phi|^2 - |\Phi|^4 \iff |D_b \hat{\Phi}|^2 - |\hat{\Phi}|^4 + \frac{1}{2\pi} B db \]

- LHS is XY, \(O(2)\) Wilson-Fisher.
- \(B\) is a background field coupled to a global \(U(1)_B\) symmetry.
- RHS is a gauged version of this theory. \(b\) is a dynamical field.
- IR duality – two different theories flowing to the same IR fixed point.
- \(\Phi \leftrightarrow \mathcal{M}_b\) is a monopole operator of \(b\) (charged under \(U(1)_B\)).
- \(|\Phi|^2 \iff -|\hat{\Phi}|^2\). Upon deformation: unbroken \(U(1)_B\) phase is Higgs phase in the RHS; broken \(U(1)_B\) phase massless \(b\).
The condensed matter, $3d$ QFT route

\[ |D_B \Phi|^2 - |\Phi|^4 \iff i \bar{\Psi} D_a \Psi + \frac{1}{2\pi} B da \]
The condensed matter, 3d QFT route

Boson/fermion duality [Chen, Fisher, Wu; Barkeshli, McGreevy]

\[ |D_B \Phi|^2 - |\Phi|^4 \leftrightarrow i \bar{\Psi} D_a \Psi + \frac{1}{2\pi} B da \]

LHS Wilson-Fisher fixed point (\(B\) is a background gauge field)

RHS QED with gauge field \(a\) with a single fermion, a.k.a \(U(1)_{1/2}\)

- Arguments involve elementary fields with fractional charges and fractional level Chern-Simons terms.
- LHS is \(T\)-reversal invariant, while RHS seems like it is not.
- LHS does not need a spin structure, while RHS does. Violating gravitational \('t Hooft matching conditions?\)
- The IR behavior of the RHS is debated.
The condensed matter, 3$d$ QFT route

\[ i \bar{\psi} D_A \psi \leftrightarrow i \bar{\chi} D_a \chi + \frac{1}{4\pi} A da \]
The condensed matter, 3d QFT route

Fermion/fermion duality [Son; Wang, Senthil; Metlitski, Vishwanath]

\[ i \bar{\Psi} \mathcal{D}_A \Psi \leftrightarrow i \bar{\chi} \mathcal{D}_a \chi + \frac{1}{4\pi} A da \]

- Motivated by
  - physics of the lowest Landau level at half-filling [Halperin, Lee, Read]
  - T-Pfaffian state of topological insulators [Chen, Fidkowski, Vishwanath].

- Improperly quantized Chern-Simons term

- LHS is \( T \)-reversal invariant (with anomaly) and RHS seems like it is not. Its IR behavior is debated.
The supersymmetric route

- Many dualities of $4d \, \mathcal{N} = 1$ theories (IR dualities) [NS; ...]
- They motivated many dualities in $3d$
  - $\mathcal{N} = 2$ [Aharony, Hanany, Intriligator, NS, Strassler; Aharony; Giveon, Kutasov; ... Benini, Closset, Cremonesi; Intriligator, NS; Aharony, Razamat, NS, Willett; Park, Park; ...]
  - $\mathcal{N} = 4$ $3d$ mirror symmetry [Intriligator, NS; ...]
- These use particle/vortex duality
- Later derived by compactification of $4d \, \mathcal{N} = 1$ dualities on a circle and then flow with relevant operators [Aharony, Razamat, NS, Willett].
  - More checks
  - Leads to many new dualities
The supersymmetric route
The supersymmetric route

- Many checks using supersymmetry and localization
- Related to string duality
- Connected to level/rank duality of 3d topological quantum field theory and 2d RCFT (rigorous [...; Hsin, NS]).

- Can flow from them to non-supersymmetric theories [Jain, Minwalla, Yokoyama; Gur-Ari, Yacoby; ...].
  - This motivates non-supersymmetric dualities.
  - But the flow might not be smooth.
The AdS/CFT, large $N$ route

• Same 4$d$ Vasiliev theory is dual to two different 3$d$ field theories [Vasiliev; Sezgin, Sundell; Klebanov, Polyakov; Giombi, Yin; Aharony, Gur-Ari, Yacoby; Giombi, Minwalla, Prakash, Trivedi, Wadia, Yin].
  – Scalars coupled to a Chern-Simons gauge theory
  – Fermions coupled to a Chern-Simons gauge theory
• Hence, a purely field theoretic duality between them
• Many explicit checks of this duality at large $N$ [Maldacena, Zhiboedov; Aharony, Giombi, Gur-Ari, Maldacena, Yacoby; Jain, Minwalla, Sharma, Takimi, Wadia, Yokoyama; Minwalla, Yokoyama; Yokoyama; Jain, Mandlik, Minwalla, Takimi, Wadia, Yokoyama; Yokoyama; Jain, Inbasekar, Jain, Mazumdar, Minwalla, Umesh, Yokoyama; ...]
Synthesis [Aharony]
3 [Aharony] + 1 [Hsin, NS] conjectures

$N_f$ scalars at $|\Phi|^4$ point coupled to $N_f$ fermions coupled to

- $SU(N)_k \leftrightarrow U(k)_{-N+N_f/2,-N+N_f/2}$
- $U(N)_{k,k} \leftrightarrow SU(k)_{-N+N_f/2,-N+N_f/2}$
- $U(N)_{k,k+N} \leftrightarrow U(k)_{-N+N_f/2,-N-k+N_f/2}$
- $U(N)_{k,k-N} \leftrightarrow U(k)_{-N+N_f/2,-N+k+N_f/2}$

Fits the large $N$ picture ($N, k \to \infty$ with finite $N/k$)

Fits the supersymmetric picture

Related to level/rank duality

Baryon and monopole operators match [Radicevic]
Puzzles
Puzzles

• Is it true for all $N, k, N_f$?
• How can a theory of bosons, which does not need a spin structure be dual to a theory of fermions, which needs it?
• What is the relation to the dualities in the condensed matter literature (with puzzles about quantization of coefficients, $T$-reversal invariance, etc.)?
• What is the precise statement of the dualities (including the coupling to background gauge fields and their Chern-Simons counterterms)?
• Are the assumptions independent? Can we assume some of these dualities and derive others?
• Are there other such dualities?
Examine $N = k = N_f = 1$

\[ i\bar{\Psi} \not{D}_A \Psi \leftrightarrow |D_b \Phi|^2 - |\Phi|^4 + \frac{1}{4\pi} bdb + \frac{1}{2\pi} Adb \]
Examine $N = k = N_f = 1$

Assume: a free fermion coupled to a background $A$ is dual to a gauged Wilson-Fisher fixed point with Chern-Simons interaction for the dynamical field $b$

$$i \bar{\Psi} \not{D}_A \Psi \leftrightarrow |D_b \Phi|^2 - |\Phi|^4 + \frac{1}{4\pi} bdb + \frac{1}{2\pi} Adb$$

• Same symmetries
  – $U(1)_A$
  – $T$-reversal invariance (with anomaly) of the RHS follows from particle/vortex duality. $T$-reversal is a quantum symmetry there. (More below.)

• Conversely, assuming this duality we derive the known particle/vortex duality.
\[ i \bar{\Psi} \not{\partial}_A \Psi \leftrightarrow |D_b \Phi|^2 - |\Phi|^4 + \frac{1}{4\pi} bdb + \frac{1}{2\pi} A db \]

- Mapping of operators
  - \( \Psi \leftrightarrow \Phi^+ M_b \) with \( M_b \) a monopole operator (charged under \( U(1)_A \))
  - Mass term \( \bar{\Psi} \Psi \leftrightarrow |\Phi|^2 \)
  - Mass deformation leads to two phases depending on the sign...
\[ i \bar{\Psi} \not{D}_A \Psi \leftrightarrow |D_b \Phi|^2 - |\Phi|^4 + \frac{1}{4\pi} bdb + \frac{1}{2\pi} Adb \]

- Mass deformation depends on the sign:
  - \( \Phi \) is massive with spin \( \frac{1}{2} \). It is charged under \( U(1)_b \) and \( U(1)_A \). It is mapped to the massive \( \Psi \). (Statistical transmutation of massive particles.)
  - \( U(1)_b \) is Higgsed. Vortex of spin \( -\frac{1}{2} \) is charged under \( U(1)_A \). It is mapped to the massive \( \Psi \).
- \( T \) changes the sign of the fermion mass (\( = \) boson mass square) and maps \( \Phi \) particles to vortices.
\[ i \bar{\Psi} D_A \Psi \quad \leftrightarrow \quad |D_b \Phi|^2 - |\Phi|^4 + \frac{1}{4\pi} bdb + \frac{1}{2\pi} A db \]

- Here both sides of the duality need a spin structure (spinors in the LHS and odd Chern-Simons level in the RHS)
  - But if \( A \) is a spin\(_c\) connection,
    \[ \int \frac{dA}{2\pi} = \frac{1}{2} \int w_2 \mod Z, \]
    there is no need for a spin structure on either side.
Derive many other dualities

• Starting with

\[ i {\bar{\Psi}} \slashed{D}_A \Psi \leftrightarrow |D_b \Phi|^2 - |\Phi|^4 + \frac{1}{4\pi} bdb + \frac{1}{2\pi} Adb \]

we can derive other dualities by changing the two sides:

• Add a Chern-Simons counterterm for the classical field \( A \)
• Gauge it by turning \( A \) into a dynamical field \( a \) and adding a new classical field.
• Use other dualities.
• Repeat.
Another boson/fermion duality

- For example, derive:

\[
|D_B \Phi|^2 - |\Phi|^4 \iff i \bar{\Psi} \not D_a \Psi + \frac{1}{2\pi} B da - \frac{1}{4\pi} B dB
\]

LHS Wilson-Fisher fixed point
RHS QED with a single fermion, a.k.a \( U(1)_{1/2} \)

- Derived from the other duality
- Neither side needs a spin structure when \( a \) is a spin\(_c\) connection
- Need a Chern-Simons counterterm for \( B \)
- Can map the operators and check the phases
- RHS is \( T \)-reversal invariant (quantum symmetry)
A fermion/fermion duality

- Derive $i \bar{\Psi} \not{D}_A \Psi \iff i \bar{\chi} \not{D}_a \chi - \frac{2}{4\pi} bdb + \frac{1}{2\pi} adb + \frac{1}{2\pi} Adb - \frac{1}{4\pi} AdA$

LHS free fermion

RHS QED with a single fermion, coupled to $U(1)_{-2}$ of $b$.

- If we incorrectly integrate out $b$, we find the previously mentioned version with improperly quantized Chern-Simons terms.
- No need for a spin structure when $a$ and $A$ are spin$_c$ connections.
- Can map the operators and check the phases
- $T$-reversal invariance (with anomaly) is manifest in LHS. It acts non-trivially in the RHS (quantum symmetry).
More
• ✓ Many more dualities and relations between them
  ✓ Add gravitational Chern-Simons counterterms (more checks)
  ✓ Relation to 4d S-duality in half-space with these 3d theories on the boundary (Witten’s S and T operations on 3d field theories)
  ✓ Generalization to arbitrary $N$ and $k$
    – Using a precise version of level/rank duality
    – Problem with large $N_f$
    – Leads to many more dualities
  ❏ Much more can be done