

Some Applications of String Field Theory: Dealing with the Infrared Issues

Ashoke Sen

Harish-Chandra Research Institute, Allahabad, India

Roji Pius, A.S., arXiv:1604.01783

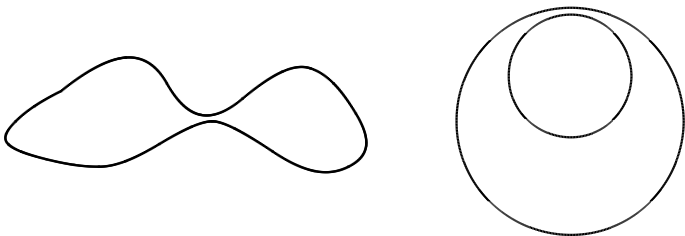
A.S., arXiv:arXiv:1508.05387; 1607.06500; 1607.08244

Beijing, August 2016

'Infrared' divergences in string theory

String theory is free from ultraviolet divergences but suffers from the usual infrared divergences

– associated with degenerate Riemann surfaces



In the degeneration limit the string theory amplitudes resemble field theory amplitudes in Schwinger parameter representation

$$(k^2 + m^2)^{-1} = \int_0^\infty ds e^{-s(k^2+m^2)}$$

e^{-s} : size of the degenerating cycle

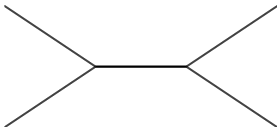
$s \rightarrow \infty$ in the degeneration limit.

Sources of divergence in string theory can be understood from the divergences in field theory amplitudes in large s limit

$$(k^2 + m^2)^{-1} = \int_0^\infty ds e^{-s(k^2+m^2)}$$

– has two types of divergence

1. For $k^2 + m^2 < 0$, l.h.s. is finite but r.h.s. diverges



– can be dealt with in quantum field theory by working directly with l.h.s.

– in conventional string perturbation theory these divergences have to be circumvented via ‘analytic continuation’

$$(k^2 + m^2)^{-1} = \int_0^\infty ds e^{-s(k^2+m^2)}$$

2. For $(k^2 + m^2) = 0$, l.h.s. and r.h.s. both diverge.

– present in quantum field theories e.g. in external state mass renormalization and massless tadpole diagrams



– have to be dealt with using renormalized mass and correct vacuum.

In standard superstring perturbation theory these divergences have no remedy.

Superstring field theory is a quantum field theory whose amplitudes, computed with Feynman diagrams, have the following properties:

1. They agree with standard superstring amplitudes when the latter are finite

2. They agree with analytic continuation of standard superstring amplitudes when the latter are finite

3. They formally agree with standard superstring amplitudes when the latter have genuine divergences, but ...



... in superstring field theory we can deal with these divergences using standard field theory techniques like mass renormalization and shift of vacuum.

Such a field theory can be constructed for heterotic, type IIA and type IIB string theories

AS

– follows closely the construction of closed bosonic SFT with some twists

Zwiebach

Structure of the action

Two sets of string fields, ψ and ϕ

Each is an infinite component field

Action takes the form

$$\mathbf{S} = \left[-\frac{1}{2}(\phi, \mathbf{QX}\phi) + (\phi, \mathbf{Q}\psi) + \mathbf{f}(\psi) \right]$$

\mathbf{Q} , \mathbf{X} : commuting linear operators, $\mathbf{Q}^2 = 0$

$(,)$: Appropriate Lorentz invariant inner product

$\mathbf{f}(\psi)$: a functional of ψ describing interaction term.

Some details (for heterotic string)

$$S = \left[-\frac{1}{2}(\phi, \mathbf{QX}\phi) + (\phi, \mathbf{Q}\psi) + \mathbf{f}(\psi) \right]$$

ψ has picture numbers $(-1, -1/2)$ in (NS, R) sectors

ϕ has picture numbers $(-1, -3/2)$ in (NS, R) sector

Q: BRST operator

X: (Identity, zero mode of PCO) in the (NS, R) sectors.

$\mathbf{f}(\psi)$: given by an integral over subspace of moduli space of Riemann surfaces

Integrand: correlation function of ψ states, PCO's, ghosts etc.

The subspace never includes degenerate Riemann surfaces.

$$\mathbf{S} = \left[-\frac{1}{2}(\phi, \mathbf{QX}\phi) + (\phi, \mathbf{Q}\psi) + \mathbf{f}(\psi) \right]$$

Equations of motion:

$$\mathbf{Q}(\psi - \mathbf{X}\phi) = \mathbf{0}$$

$$\mathbf{Q}\phi + \mathbf{f}'(\psi) = \mathbf{0}$$

first + \mathbf{X} × second equation gives

$$\mathbf{Q}\psi + \mathbf{X}\mathbf{f}'(\psi) = \mathbf{0}$$

ψ describes interacting fields

Rest of the independent degrees of freedom describe decoupled free fields.

This action has infinite dimensional gauge invariance

– can be quantized using Batalin-Vilkovisky formalism

1. Gauge fix

2. Derive Feynman rules

3. Compute amplitudes

Amplitudes of ψ give the scattering amplitudes with desired properties.

Rest of the degrees of freedom decouple and will be irrelevant for our analysis.

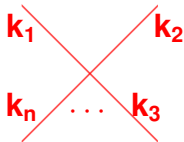
The tree level propagators have standard form in the 'Siegel gauge'

$$(\mathbf{L}_0 + \bar{\mathbf{L}}_0)^{-1} \mathbf{X} \mathbf{b}_0 \bar{\mathbf{b}}_0 \delta_{\mathbf{L}_0, \bar{\mathbf{L}}_0}$$

In momentum space

$$(\mathbf{k}^2 + \mathbf{M}^2)^{-1} \times \text{polynomial in momentum}$$

The polynomial comes from matrix element of $\mathbf{X} \mathbf{b}_0 \bar{\mathbf{b}}_0$.



Vertices are accompanied by a suppression factor of

$$\exp \left[-\frac{A}{2} \sum_i (k_i^2 + m_i^2) \right]$$

A: a positive constant that can be made large by a non-linear field redefinition (adding stubs).

Hata, Zwiebach

This makes

- momentum integrals UV finite (almost)
- sum over intermediate states converge

Momentum dependence of vertex includes

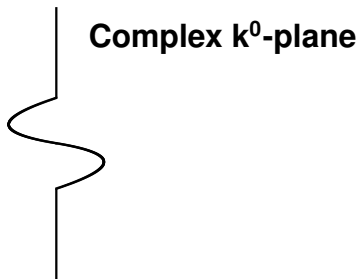
$$\exp \left[-\frac{A}{2} \sum_i (\mathbf{k}_i^2 + m_i^2) \right] = \exp \left[-\frac{A}{2} \sum_i (\vec{k}_i^2 + m_i^2) + \frac{A}{2} (k_i^0)^2 \right]$$

Integration over \vec{k}_i converges for large \vec{k}_i , but integration over k_i^0 diverges at large k_i^0 .

The spatial components of loop momenta can be integrated along the real axis, but we have to treat integration over loop energies more carefully.

Resolution: Need to have the ends of loop energy integrals approach $\pm i\infty$.

In the interior the contour has to be deformed away from the imaginary axis to avoid poles from the propagators.



We shall now describe in detail how to choose the loop energy integration contour.

General procedure:

Pius, A.S.

1. Multiply all external energies by a complex number u .
2. For $u=i$, all external energies are imaginary, and we can take all loop energy contours to lie along the imaginary axis without encountering any singularity.
3. Now deform u to 1 along the first quadrant.
4. If some pole of a propagator approaches the loop energy integration contours, deform the contours away from the poles, keeping its ends at $\pm i\infty$.

Result 1: Such deformations are always possible as long as u lies in the first quadrant

– the loop energy contours do not get pinched by poles from two sides.

Result 2: The amplitudes computed this way satisfy Cutkosky cutting rules

– relates $T - T^\dagger$ to $T^\dagger T$

$$S = 1 - iT$$

– proved by using contour deformation in complex loop energy plane

This is a step towards proof of unitarity but not a complete proof

In $T^\dagger T = T^\dagger |n\rangle \langle n| T$, the sum over intermediate states runs over all states in Siegel gauge.

Desired result: Only physical states should contribute to the sum.

This is shown using the quantum Ward identities of superstring field theory

A.S.

– requires cancellation between matter and ghost loops

The proof of unitarity takes into account

1. Mass and wave-function renormalization effects and lifting of degeneracy

2. The fact that some (most) of the string states become unstable under quantum corrections.

3. The possible shift in the vacuum due to quantum effects.

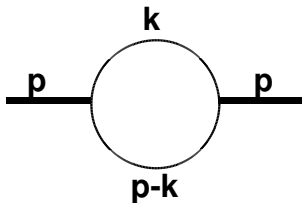
It does not take into account the infrared divergences from soft particles arising in $D \leq 4$.

(String field theory version of Kinoshita, Lee, Nauenberg theorem has not yet been proven.)

An example:

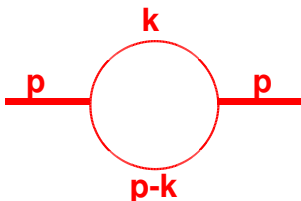
Consider two fields, one of mass M and another of mass m , with $M > 2m$.

Consider one loop mass renormalization of the heavy particle.



Thick line: heavy particle

Thin line: light particle.



$$\delta M^2 = i \int \frac{d^D k}{(2\pi)^D} \exp[-A\{k^2 + m^2\} - A\{(p - k)^2 + m^2\}] \\ \{k^2 + m^2\}^{-1} \{(p - k)^2 + m^2\}^{-1} B(k)$$

$B(k)$: a polynomial in momentum encoding additional contribution to the vertices and / or propagators.

We shall work in $\vec{p} = 0$ frame, and take $p^0 \rightarrow M$ limit from the first quadrant.

$$\delta M^2 = i \int \frac{d^D k}{(2\pi)^D} \exp[-A\{k^2 + m^2\} - A\{(p - k)^2 + m^2\}] \\ \{k^2 + m^2\}^{-1} \{(p - k)^2 + m^2\}^{-1} B(k)$$

Poles in the k^0 plane (for $\vec{p} = 0$):

$$Q_1 \equiv \sqrt{\vec{k}^2 + m^2}, \quad Q_2 \equiv -\sqrt{\vec{k}^2 + m^2},$$

$$Q_3 \equiv p^0 + \sqrt{\vec{k}^2 + m^2}, \quad Q_4 \equiv p^0 - \sqrt{\vec{k}^2 + m^2}$$

For p^0 imaginary, take k^0 contour along imaginary axis.

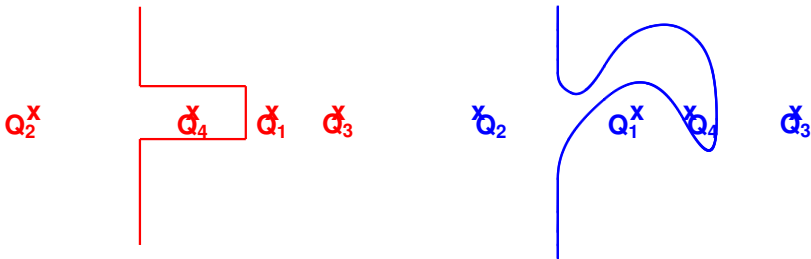
Q_1, Q_3 to the right and Q_2, Q_4 to the left of the imaginary axis.

$$Q_1 \equiv \sqrt{\vec{k}^{-2} + m^2}, \quad Q_2 \equiv -\sqrt{\vec{k}^{-2} + m^2},$$

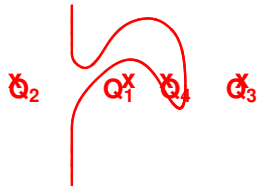
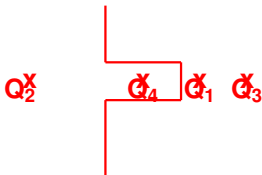
$$Q_3 \equiv p^0 + \sqrt{\vec{k}^{-2} + m^2}, \quad Q_4 \equiv p^0 - \sqrt{\vec{k}^{-2} + m^2}$$

As p^0 approaches real axis, the poles approach the real axis.

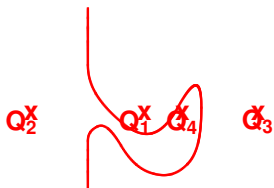
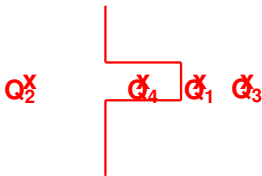
Two situations depending on the value of \vec{k} .



Note: Q_1, Q_3 to the right and Q_2, Q_4 to the left of the contour in both diagrams.



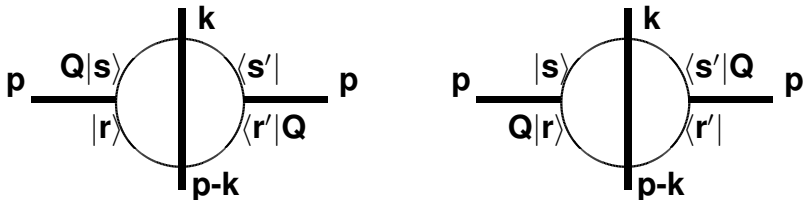
Complex conjugate contours giving $(\delta M^2)^*$



– can be deformed to each other without picking any residue unless $Q_4 \rightarrow Q_1$ putting both lines on-shell.

– residue given by Cutkosky rules.

The cut diagrams in string field theory will have some unwanted terms



These two diagrams cancel using Ward identity.

All order proof of unitarity involves generalization of this type of analysis

– takes into account quantum modification of the BRST operator Q computed from 1PI effective action.

String motivated approach: Evaluate the original integral using Schwinger parametrization

$$\exp[-A(k^2 + m^2)](k^2 + m^2)^{-1} = \int_A^\infty dt_1 \exp[-t_1(k^2 + m^2)]$$

$$\exp[-A((p - k)^2 + m^2)]((p - k)^2 + m^2)^{-1} = \int_A^\infty dt_2 \exp[-t_2((p - k)^2 + m^2)]$$

For constant B, after doing momentum integrals (formally)

$$\delta M^2 = -B (4\pi)^{-D/2} \int_A^\infty dt_1 \int_A^\infty dt_2 (t_1 + t_2)^{-D/2} \exp \left[\frac{t_1 t_2}{t_1 + t_2} M^2 - (t_1 + t_2) m^2 \right]$$

– diverges from the upper end for $M > 2m$.

– can be traced to the impossibility of choosing energy integration contour keeping $\text{Re}(k^2 + m^2) > 0$, $\text{Re}((p - k)^2 + m^2) > 0$.

$$i \mathbf{B} \int \frac{d^D \mathbf{k}}{(2\pi)^D} \exp[-\mathbf{A}\{\mathbf{k}^2 + \mathbf{m}^2\} - \mathbf{A}\{(\mathbf{p} - \mathbf{k})^2 + \mathbf{m}^2\}]$$

$$\{\mathbf{k}^2 + \mathbf{m}^2\}^{-1} \{(\mathbf{p} - \mathbf{k})^2 + \mathbf{m}^2\}^{-1} \quad \text{finite}$$

$$\text{'='} \quad -\mathbf{B} (4\pi)^{-D/2} \int_A^\infty dt_1 \int_A^\infty dt_2 (t_1 + t_2)^{-D/2}$$

$$\exp \left[\frac{t_1 t_2}{t_1 + t_2} \mathbf{M}^2 - (t_1 + t_2) \mathbf{m}^2 \right] \quad \text{divergent}$$

More generally for a polynomial \mathbf{B} , we have a polynomial \mathbf{P} s.t.

$$i \int \frac{d^D \mathbf{k}}{(2\pi)^D} \exp[-\mathbf{A}\{\mathbf{k}^2 + \mathbf{m}^2\} - \mathbf{A}\{(\mathbf{p} - \mathbf{k})^2 + \mathbf{m}^2\}]$$

$$\{\mathbf{k}^2 + \mathbf{m}^2\}^{-1} \{(\mathbf{p} - \mathbf{k})^2 + \mathbf{m}^2\}^{-1} \mathbf{B}(\mathbf{k})$$

$$\text{'='} \quad -(4\pi)^{-D/2} \int_A^\infty dt_1 \int_A^\infty dt_2 (t_1 + t_2)^{-D/2}$$

$$\exp \left[\frac{t_1 t_2}{t_1 + t_2} \mathbf{M}^2 - (t_1 + t_2) \mathbf{m}^2 \right] \mathbf{P}(1/(t_1 + t_2), t_2/(t_1 + t_2))$$

Such divergences arise in actual computation of one loop two point functions in heterotic and type II string theories.

Using these ‘identities’ we can convert these divergent expressions into finite expressions

– have both real and imaginary parts consistent with unitarity.

An example

One loop mass renormalization of the lowest massive state on the leading Regge trajectory in the heterotic string theory

Need to compute torus two point function of on-shell states

On shell two point function gives

$$\delta M^2 = -\frac{1}{32\pi} M^2 g^2 \int d^2\tau \int d^2z F(\mathbf{z}, \bar{\mathbf{z}}, \tau, \bar{\tau}),$$

$$F(\mathbf{z}, \bar{\mathbf{z}}, \tau, \bar{\tau}) \equiv \left\{ \sum_{\nu} \overline{\vartheta_{\nu}(\mathbf{0})}^{16} \right\} (\overline{\eta(\tau)})^{-18} (\eta(\tau))^{-6} (\overline{\vartheta_1'(\mathbf{0})})^{-4} \left(\vartheta_1(\mathbf{z}) \overline{\vartheta_1(\mathbf{z})} \right)^2 \\ \left[\left(\frac{\overline{\vartheta_1'(\mathbf{z})}}{\overline{\vartheta_1(\mathbf{z})}} \right)^2 - \frac{\overline{\vartheta_1''(\mathbf{z})}}{\overline{\vartheta_1(\mathbf{z})}} - \frac{\pi}{\tau_2} \right]^2 \exp[-4\pi \mathbf{z}_2^2 / \tau_2] (\tau_2)^{-5},$$

$\mathbf{z} = \mathbf{z}_1 + i\mathbf{z}_2 \in$ **torus**,

$\tau = \tau_1 + i\tau_2 \in$ **fundamental region**

$\vartheta_1, \dots, \vartheta_4$: **Jacobi theta functions**

η : **Dedekind function**

For large z_2 and $\tau_2 - z_2$, F has a growing part

$$2(2\pi)^{-4} \left(32\pi^4 - 32\frac{\pi^3}{\tau_2} + 512\frac{\pi^2}{\tau_2^2} \right) \exp[4\pi z_2 - 4\pi z_2^2/\tau_2] \tau_2^{-5}$$

\Rightarrow divergent integral.

Divergent part after using $t_1 = \pi z_2$, $t_2 = \pi(\tau_2 - z_2)$

$$\begin{aligned} \mathbf{J} = & -2^{-3}\pi^2 M^2 \int_A^\infty dt_1 \int_A^\infty dt_2 (t_1 + t_2)^{-5} \\ & \left(1 - \frac{1}{(t_1 + t_2)} + 16 \frac{1}{(t_1 + t_2)^2} \right) \exp \left[4 \frac{t_1 t_2}{t_1 + t_2} \right] \end{aligned}$$

A: arbitrary constant

J is divergent, but the integral matches the one we analyzed before for field theory with $m=0$, $M=2$

Strategy (can be justified using string field theory):

Replace J by momentum space integral.

$$\mathbf{J} = i(2\pi)^7 M^2 g^2 \int \frac{d^{10}k}{(2\pi)^{10}} \exp[-Ak^2 - A(\mathbf{p} - \mathbf{k})^2] \\ (\mathbf{k}^2)^{-1} \{(\mathbf{p} - \mathbf{k})^2\}^{-1} \{1 - 2(\mathbf{k}^1)^2 + 64(\mathbf{k}^1)^2(\mathbf{k}^2)^2\}$$

– finite integral once we choose the integration contour for energy integral as in the field theory example

Final result gives finite real and imaginary parts in accordance with unitarity.

Summary

Covariant superstring field theory gives a Lorentz invariant, ultraviolet finite and unitary theory.

Divergences associated with mass renormalization and shift of vacuum can be dealt with as in conventional quantum field theories.

It can also provide useful alternative to analytic continuation that is often needed in conventional superstring perturbation theory to make sense of divergent results.