Massive non-Gaussian Distribution

*Flauger, Mirbabayi, Senatore, ES '16 (cf MSS+Zaldarriaga '15)

+ Work in progress w/Munchmeyer; Peiris Bouchet.../Planck,Wenren, Roberts, ...

Outline

*Primordial Non-Gaussianity and sensitivity to heavy field production (periodic for axions)

*Shapes and amplitudes of N-point functions (orthogonal to previous; growth of Signal/Noise with N);

*Probability distribution

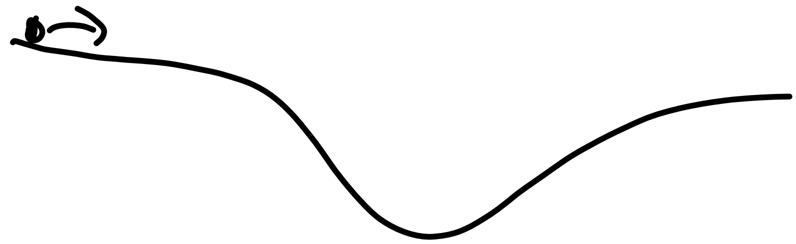
*Large-N behavior and analysis methods (work in progress)

Cosmological data, via Primordial Non-Gaussianity is remarkably sensitive to dynamics (field/string content, interactions, inflationary mechanism) 14 billion years ago. Large space of possibilities; some constrained.

String theory plays a significant role elucidating this, introducing dynamical mechanisms and signatures then incorporated in systematic treatments of EFT and data analysis. Today's talk: another basic example w/novel features. Leads to new tests of simplicity (or not) of early U.

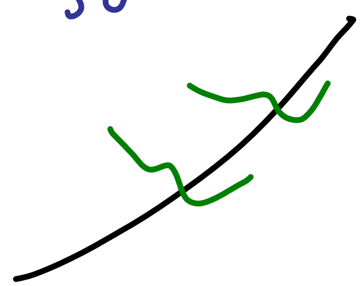
Brief Review of top-down Non-Gaussianity

- Slow roll
  V flat ⇒
  small interactions



(cf Maldacena '02)

- Additional fields not so constrained



(Bond Kofman Linde Mukhanov Zaldarriaga...)

- Even for single field, if self-interactions slow ϕ on steep potential
  → larger NG

(DBI '03 -> EFT Cheung et al, P(X) Chen et al, ..., Trapped Inflation'08...)

Smaller-amplitude* Resonant NG

Chen Easther Lim Kamionkowski McAllister
ES Westphal Flauger et al Efstathiou Peiris
Meerburg Spergel Wandelt Fergusson
Wallisch Munchmeyer Mirbabayi Senatore
*Behbahani Green...

$$V = V_0(\phi) + \Lambda^4 \cos\frac{\phi}{f}$$

$$\frac{(S/N)_3}{(S/N)_2} \sim \frac{\mathcal{L}_3}{\mathcal{L}_2} \sim \frac{\delta\phi}{f} \sim \frac{\omega}{f} < 1$$

perturbative control

Quasi-Single-Field

Chen Wang Baumann Green ...
Lewandowski Senatore ES
Zaldarriaga ... Arkani Hamed
Maldacena ...

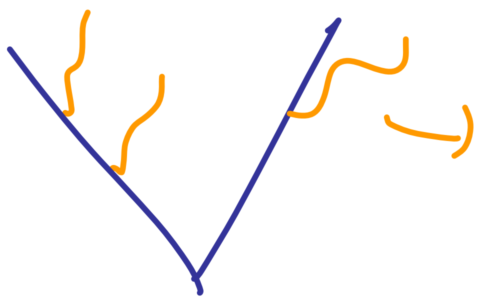
Vacuum production

$$e^{-m/H} \ll e^{-m/\dot\phi^{1/2}},\ e^{-\frac{m^2}{\dot\phi}}$$

Inflaton ϕ coupled to

heavy χ $\longrightarrow$ $m_\chi(\phi(t))$

time-dependent



$\delta\phi$ radiation

non-adiabatic χ production

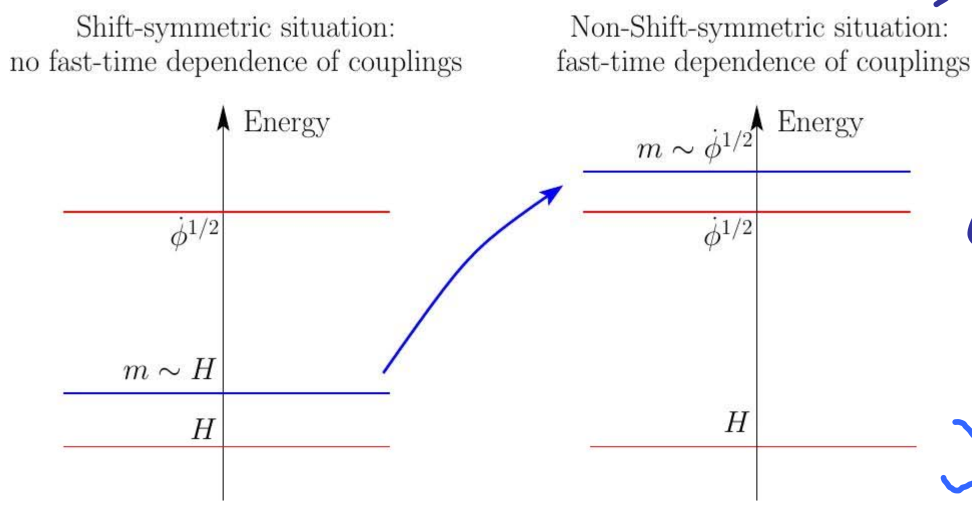
Kofman Linde
Starobinski Traschen
Brandenberger,
Chung et al, Green
Horn ES Senatore...

$$e^{-\frac{\pi \mu^2}{2\dot\phi}} \sim \frac{1}{\sqrt{N_{pixel}}}$$

sensitivity to heavy χ fields

Novel shape $\langle \delta\phi_{k_1} \cdots \delta\phi_{k_N} \rangle$

and $(S/N)_{N+1} / (S/N)_N > 1$ possible for a range of N



$\rightarrow$ O(.1-1) couplings to inflaton (preserving inflation overall)

Figure 1: Pictorial representation of our findings: in an inflationary theory with an approximate continuous shift symmetry for the inflaton, only particles that are not much heavier than the Hubble scale H are relevant for the dynamics of the fluctuations. However, as we will see, if the continuous shift symmetry is broken, e.g. to a discrete shift symmetry, heavier particles can become relevant as depicted on the right. In the scenarios studied in this work, the new scale is set by $\dot\phi$. The basic estimate $\exp(-\pi m^2/\dot\phi) \sim 1/\sqrt{N_{\rm modes}}$ suggests observational sensitivity to these massive particles, which we confirm in a detailed analysis.

EFT of inflationary perturbations contains arbitrary functions of t even at single-field level.

Precision of data means we can't safely integrate out heavy ($m_\chi \gtrsim \dot\phi^{\frac{1}{2}}$) fields

Competition between power-law and exponential effects:

NG from e.g. $\frac{(\partial\phi)^4}{m^4} \sim \dot{\phi}_0(t) \frac{(\partial\delta\phi)^3}{m^4}$

induces $\frac{\mathcal{L}_3}{\mathcal{L}_2} \sim \frac{\dot{\phi}_0^2}{m^4} \zeta \lesssim \zeta \sim 10^{-4}$

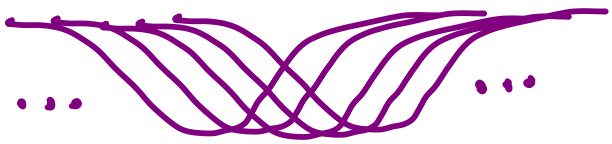
How can this lose to an exponentially small effect? Power-law prefactors:

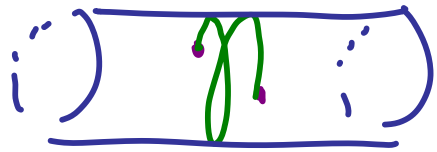
$$N_\chi \sim \left(\frac{\dot{\phi}^{\frac{1}{2}}}{H}\right)^3 e^{-m^2/\dot{\phi}}$$

number produced in Hubble patch

Large pre-factor

Two cases motivated by axion monodromy





both V and particle/string sectors

$$m^2_{\chi_n,(a)} = \mu_a^2 + \hat{\mu}_a^2 (a(\phi) - 2\pi n)^2 \simeq \mu_a^2 + g_a^2(\phi - 2\pi n f)^2$$

Coleman-Weinberg

$$V = V_0(\phi;\chi) + \Lambda^4(\chi)\cos\left(\frac{\phi}{f}+\gamma\right)$$

$\to$ sinusoidally varying heavy moduli masses

$$m_\chi^2 = \mu^2 + 2g^2 f^2 \cos\frac{\phi}{f}$$

$$\eta_n = -\frac{1}{H} e^{2\pi n \frac{H}{\omega}}$$

More general $m_\chi(\phi)$, periodic or not, also interesting. (opposite extreme: random $m_\chi(\phi)$ cf Green; Amin Baumann)

Radiative corrections < control / strength of NG

χ vacuum loops

depends on e.g. level of microscopic SUSY.

Bosons & Fermions partially cancel.

(as opposed to production)

Bosons & Fermions add

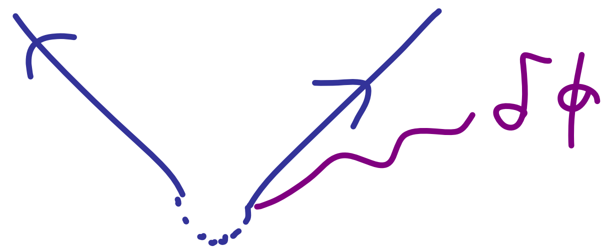
With some microscopic SUSY, the only general constraint is simply $g \ll 4\pi$

(derived including SUSY-breaking effects of time-dependence)

Also impose $\rho_\chi' = g\bar{n}_\chi \ll V'$, $\rho_\chi \ll \dot{\phi}^2$ to exclude backreaction of produced χ's

χ production

Sources ϕ



$$J = \chi^2 \frac{\delta m_\chi^2(\phi)}{\delta \phi}$$

Background $\phi(t)$ motion $\rightarrow$

$$|\psi\rangle = \mathcal{N} e^{\frac{\beta}{2g} a_\chi^{+2}} |0\rangle$$

$$\langle\psi|J_0|\psi\rangle \sim \underbrace{\bar{n}_\chi}_{=\int_{\vec k} |\beta_{\vec k}|^2} \sim (g\dot\phi)^{3/2} e^{\frac{-\pi\mu_0^2}{g\dot\phi}} \frac{a_n^3}{a(\eta)^3}$$

$$\langle\Psi|J_{\mathbf{k}_1}(\eta_1')J_{\mathbf{k}_2}(\eta_2')|\Psi\rangle \sim \frac{(2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2)}{a_n^3}\bar{n}_\chi \prod_{j=1}^{2} \tilde{\theta}((t_j'-t_n)/t_{pr}) 2\frac{\delta}{\delta\phi} m_\chi(\phi(\eta_j'))\frac{a_n^3}{a(\eta_j')^3} \quad (44)$$

$$\langle\psi|J_{\vec k_1}\cdots J_{\vec k_N}|\psi\rangle \sim \bar{n}_\chi (\cdots)$$

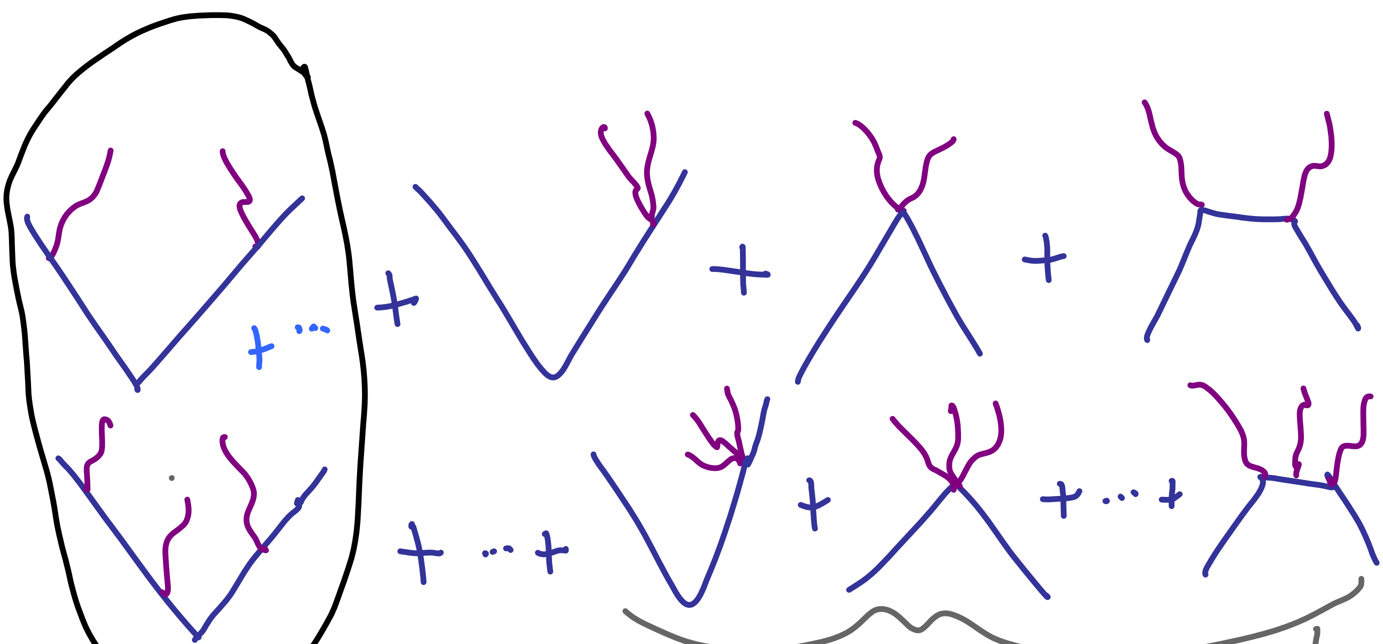
cf Poisson statistics

General Calculation

$$\langle in|\bar{T}\exp(i\int_{-\infty(1+i\epsilon)}^{t} dt_1 \mathcal{H}_{int})\delta\phi_{k_1}(t)\ldots\delta\phi_{k_N}(t) T\exp(-i\int_{-\infty(1-i\epsilon)}^{t} dt_2\mathcal{H}_{int})|in\rangle$$

$$\mathcal{H}_{int} = \chi^2 m_\chi^2(\phi_0(t)+\delta\phi)$$

$$|\mathcal{N}|^2\langle out|e^{\int_q \frac{\beta_q^*}{2\alpha_q^*}a_q^2}\bar{T}\exp(i\int_{-\infty(1+i\epsilon)}^{t} dt_1\mathcal{H}_{int})\delta\phi_{k_1}(t)\ldots\delta\phi_{k_N}(t)T\exp(-i\int_{-\infty(1-i\epsilon)}^{t} dt_2\mathcal{H}_{int})e^{\int_k \frac{\beta_k}{2\alpha_k}a_k^{\dagger 2}}|out\rangle$$



others can be subdominant

Novel shape for $\mathcal{L}_3 \sim \delta\phi\,\chi\chi\,\sin\omega t$

The Means of Production

The spontaneous forces of capitalism have been steadily growing in the countryside in recent years, with new rich peasants springing up everywhere and many well-to-do middle peasants striving to become rich peasants. On the other hand, many poor peasants are still living in poverty for lack of sufficient means of production, with some in debt and others selling or renting out their land. If this tendency goes unchecked, the polarization in the countryside will inevitably be aggravated day by day. Those peasants who

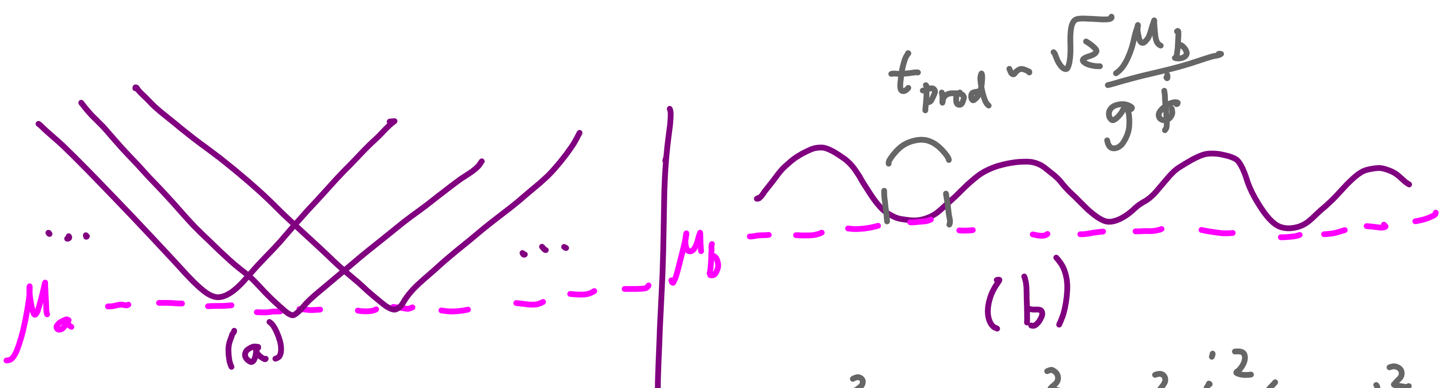
Between bursts of production

$$\chi(\eta, \mathbf{x}) = \int_{\mathbf{k}} a^{(in)}_{\chi,\mathbf{k}} v_k(\eta) e^{i\mathbf{k}\cdot\mathbf{x}} + h.c.\,,$$

$$a^{3/2} v_k(t) = \alpha_k^{(n)} \frac{\exp(-i\int_{t_n}^t dt'\omega_\chi(t'))}{\sqrt{2\omega_\chi(t')}} + \beta_k^{(n)*} \frac{\exp(i\int_{t_n}^t dt'\omega_\chi(t'))}{\sqrt{2\omega_\chi(t')}}, \qquad t_n + t_{pr} < t < t_{n+1} - t_{pr} \tag{3.25}$$

solves $-\ddot{v} - \omega_\chi^2(\phi_0(t))\, v = 0 \quad (\Delta t \ll \frac{1}{H})$

For appropriate $m_\chi(\phi_0(t))$, e.g. (a) & (b), we have well-defined bursts of χ production



$\omega_\chi^2 \to \mu_b^2 + g^2 \dot{\phi}^2 (t - t_n)^2$

$$\langle \delta\phi_{\vec{k}_1}(\eta) \quad \delta\phi_{\vec{k}_N}(\eta) \rangle =$$

$$\int d\eta_1' \cdots d\eta_N' \; G_{dS}(\eta,\eta_1') \cdots G_{dS}(\eta,\eta_N')$$

$$\langle \psi | J_{\vec{k}_1}(\eta_1') \cdots J_{\vec{k}_N}(\eta_N') | \psi \rangle$$

$$G(0,\eta') = \frac{H^2}{k^3}(\sin(k\eta') - k\eta'\cos(k\eta')) \equiv \frac{H^2}{k^3}\hat{g}(k\eta')$$

$$\hat{h}(k\eta_n) = \int_{\eta_n}^{0} \frac{d\eta'}{\eta'} (\sin k\eta' - k\eta' \cos(k\eta')) \frac{\delta}{\delta\phi} m_\chi(\phi(\eta'))$$

$$\langle \delta\phi_{\mathbf{k}_1} \delta\phi_{\mathbf{k}_2} \rangle_{pp} \sim \frac{(2\pi)^3 \delta(\mathbf{k}_1 + \mathbf{k}_2)}{k_1^3} \left(\frac{\bar{n}_\chi}{H^3} \right) H^2 \sum_n \frac{\hat{h}(k_1\eta_n)^2}{(-k\eta_n)^3}$$

$$\Rightarrow \zeta_{pp}^2 \sim \zeta_{vac}^2 \times \frac{\bar{n}_\chi}{H^3} \sum_n \frac{\hat{h}(k_1\eta_n)^2}{(-k_1\eta_n)^3}$$

Factorized for each n

$$\langle \delta\phi_{\mathbf{k}_1} \dots \delta\phi_{\mathbf{k}_N} \rangle \sim (2\pi)^3 \delta(\sum \mathbf{k}_i) \frac{\bar{n}_\chi}{H^3} H^{N+3} \sum_n (H\eta_n)^{-3} \Pi_{i=1}^N \frac{\hat{h}(k_i\eta_n)}{k_i^3}$$

Frequencies in the mass function can resonate with those in the Green's function, leading to NG that can be > power spectrum corrections

$$\frac{\zeta_{osc}^3/(\zeta_{vac}^2)^{3/2}}{\zeta_{osc}^2/\zeta_{vac}^2} \sim \frac{\sum_n (k\eta_n)^{-3} \hat{h}(k\eta_n)^3}{\sum_{n'} (k\eta_{n'})^{-3} \hat{h}(k\eta_{n'})^2}$$

Saddle:

case (b)

$c_b \equiv g^2 \frac{f}{\mu}$

$$\hat{h}(k\eta_n) = \int_{\eta_n}^{0} \frac{d\eta'}{\eta'}(\sin k\eta' - k\eta' \cos(k\eta'))\underbrace{\frac{\delta}{\delta\phi} m_\chi(\phi(\eta'))}_{c_b \sin\left(\frac{\phi}{f}+\gamma\right)}$$

Resonate

(not 'resonant NG')

$$c_b \, e^{i\frac{\omega}{H}\log\frac{\eta}{\eta_n}} \sim e^{-i(\cdots)}$$

$$k\eta_{\text{saddle}} = -\frac{\omega}{H}$$

using $\frac{\phi}{f} = \frac{\dot\phi t}{f} = \omega t$

$\eta = -\frac{1}{H}e^{-Ht}$

$\eta_n = -\frac{1}{H}e^{2\pi n\frac{H}{\omega}}$

$$\hat{h} \sim c_b \underbrace{\sqrt{\frac{\omega}{H}}}_{\text{enhancement}} \cos(\cdots)$$

(This saddle is also useful for data analyzing the shape at high frequency.)

$\Rightarrow$

$$\frac{\zeta_{osc}^3/(\zeta_{vac}^2)^{3/2}}{\zeta_{osc}^2/\zeta_{vac}^2} \sim \frac{\sum_n (k\eta_n)^{-3}\hat{h}(k\eta_n)^3}{\sum_{n'}(k\eta_{n'})^{-3}\hat{h}(k\eta_{n'})^2}$$

$\sim C_b\sqrt{\frac{\omega}{H}}$ can be $\gtrsim \mathcal{O}(1)$ for interesting range of ω/H

In contrast to 'Resonant NG': $\Lambda^4 \cos\left(\frac{\phi}{f} = \omega t\right)$

$\frac{\mathcal{L}}{\mathcal{L}_2} \sim \frac{(S/N)_3}{(S/N)_2} \sim \frac{\omega}{f} < 1$ using $\delta\phi\big|_{\text{freezeout}} \sim \omega$

$\Rightarrow$ • NG and joint power/bispectrum analysis well-motivated

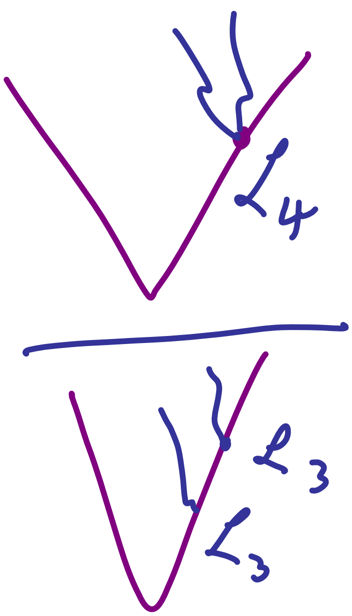
• $N \gg 1$ pt function can dominate $\rightarrow$ new search strategies

$$f_{NL}^{(b)} \equiv k^6 \frac{B(k,k,k)}{4P_\zeta^2} \simeq c_b^3 \frac{\bar{n}_\chi}{H^3}(4\times 10^3)\sum_n \frac{h(k\eta_n)^3}{(k\eta_n)^3} \simeq c_b^3 \frac{\bar{n}_\chi}{H^3}(4\times 10^3)\times\sqrt{\frac{H}{\omega}}$$

As with other f_NL parameters, a priori this could be >100 for all we know, data search could constrain to ~1 (at least

Other Diagrams Include:

$$\frac{1}{2}\chi^2 g^2 f^2 \cos\frac{\phi}{f} \to \mathcal{L}_{m+2} \sim \frac{g^2}{m!}\frac{\delta\phi^m}{f^{m-2}}\chi^2$$



$$\sim \frac{\mu H}{g^2 f^2}\sqrt{\frac{\omega}{\pi H}} \equiv \xi$$

can be small

(couplings + resonance structure)

But large combinatorial enhancement at large N (more later)

Probability Distribution

In general this is the functional

$$P[\delta\phi^0(\mathbf{x})] = \int D\chi^0 |\Psi[\delta\phi^0(\mathbf{x}), \chi^0(\mathbf{x})]|^2$$

$$\Psi[\delta\phi^0(\mathbf{x}), \chi^0(\mathbf{x})] = \int D\delta\phi(\mathbf{x}, t)|_{\delta\phi(t=t_C)=\delta\phi^0(\mathbf{x})} D\chi(\mathbf{x}, t)|_{\chi(t=t_C)=\chi^0(\mathbf{x})} e^{i\mathcal{S}[\delta\phi(\mathbf{x},t),\chi(\mathbf{x},t)]}$$

The histogram of temperature fluctuations in the map would in general be given by

$$N_{\delta\hat{\phi}} = \int d\mathbf{x}' \int D\delta\phi^0(\mathbf{x}) P[\delta\phi^0(\mathbf{x})] H\delta(\delta\phi(\mathbf{x}') - \delta\hat{\phi})$$

This can introduce a non-Gaussian tail that we may be able to constrain in a more model-independent way than searching for specific N-point functions.

For factorized contributions

$$N_{\delta\hat{\phi}} \sim \delta(0)\bar{n}_\chi \int d\mathbf{x}' \sum_n \eta_n^{-3}\delta(\tilde{H}_n(\mathbf{x}') - \frac{\delta\hat{\phi}}{H})$$

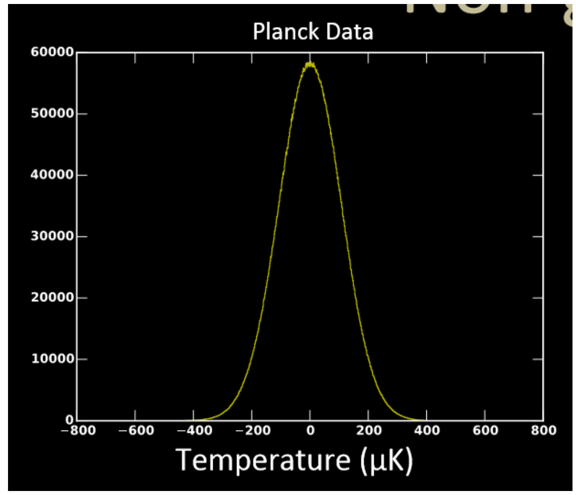
$$\tilde{H}_n(\mathbf{y}) = \int d\mathbf{k}_j e^{i\mathbf{k}_j\cdot\mathbf{y}} \frac{\hat{h}(k_j\eta_n)}{k_j^3}$$

For case (b) specifically,

$$N_{\delta\hat{\phi}} \simeq 4\pi\delta(0)\bar{n}_\chi N_e^{data} \frac{\omega}{2\pi H} \frac{1}{\sqrt{1-(\frac{\delta\hat{\phi}}{H})^2(\frac{\omega}{Hc_b})^2}}$$

Moreover, we can implement this for general $m_\chi(\phi)$ and get a much more model independent constraint.

(1) Basic tests of CMB statistics



(Fig from B. Racine talk, Aug

(2) Template based searches. Optimal use of the data for specific

$$< \zeta(K1)\, \zeta(K2)\, \zeta(K3) >$$

$$S/N \sim \frac{\langle \zeta_1 \cdots \zeta_N \rangle}{\sqrt{\langle (\zeta_1 \cdots \zeta_N)^2 \rangle}} \sqrt{N_{pix}} \qquad \frac{1}{\sqrt{N_{pixel}}} \sim 10^{-3}$$

General 3pf estimator Babich, Creminelli, Komatsu, Spergel, Wandelt, Senatore, Zaldarriaga,...

$$\mathcal{E} = \frac{1}{N} \cdot \sum_{l_i m_i} \int d^2\hat{n}\, Y_{l_1 m_1}(\hat{n}) Y_{l_2 m_2}(\hat{n}) Y_{l_3 m_3}(\hat{n}) \int_0^\infty r^2 dr\, j_{l_1}(k_1 r) j_{l_2}(k_2 r) j_{l_3}(k_3 r)\, C_{l_1}^{-1} C_{l_2}^{-1} C_{l_3}^{-1}$$
$$\int \frac{2k_1^2 dk_1}{\pi} \frac{2k_2^2 dk_2}{\pi} \frac{2k_3^2 dk_3}{\pi} F(k_1, k_2, k_3) \Delta_{l_1}^T(k_1) \Delta_{l_2}^T(k_2) \Delta_{l_3}^T(k_3)\, a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3}\,, \qquad (10)$$

Only tractable (currently) if factorizes:

assume that $F(k_1, k_2, k_3) = f_1(k_1) f_2(k_2) f_3(k_3)$, the estimator simplifies to

$$\mathcal{E} = \frac{1}{N} \cdot \int d^2\hat{n} \int_0^\infty r^2 dr \prod_{i=1}^{3} \sum_{l_i m_i} \int \frac{2k^2 dk}{\pi} j_{l_i}(kr) f_i(k) \Delta_{l_i}^T(k) C_{l_i}^{-1} a_{l_i m_i} Y_{l_i m_i}(\hat{n})\,. \qquad (12)$$

Future direction/question: Machine learning for non-factorizable cases? e.g. for hybrid components of our shape, which can get enhanced at large N (see below)

Shapes and overlaps

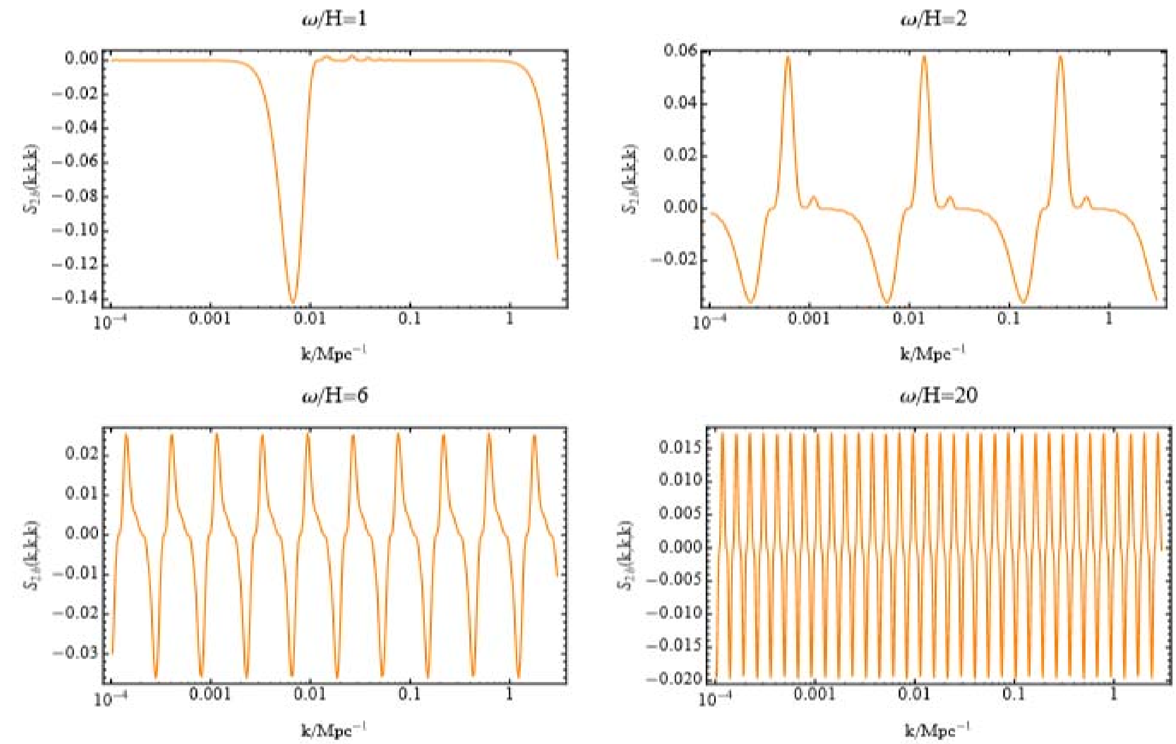


Figure 4: Shape for case (b) plotted along the equilateral axis for a range of frequencies.

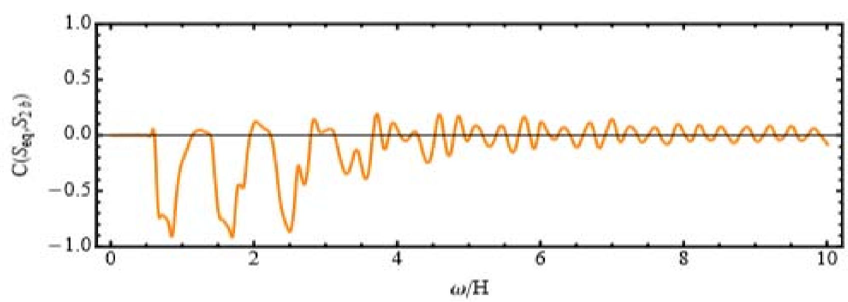


Figure 5: Overlap of shape (b) with the equilateral template using the prescription developed in [2]

Case (b) also orthogonal to the resonance shape, so new data search to constrain it.

Large N Theory & data analysis

in progress w/ M. Munchmeyer

$$S/N \sim \frac{\langle \zeta(k_1)\cdots\zeta(k_N)\rangle}{\sqrt{\langle(\zeta(k_1)\cdots\zeta(k_N))^2\rangle}}\sqrt{N_{pix}}$$

for sufficiently low N this $\sim \langle \zeta^2\rangle^{N/2}$

$$S/N \sim \Sigma\, X^N \qquad N < N_c$$

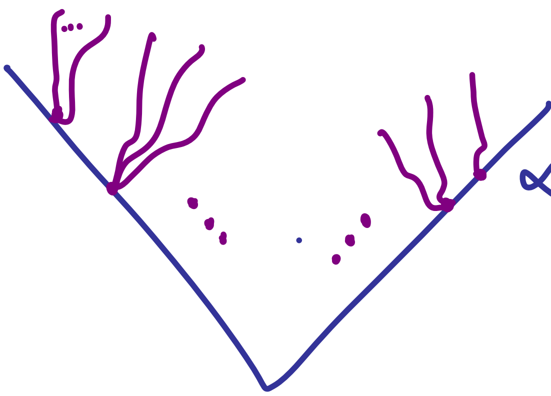
$$X \sim C_\phi \sqrt{\frac{\omega}{H}} \sim \frac{g^2 f}{\mu}\sqrt{\frac{\omega}{H}}$$

$$\Sigma \propto \mathrm{Exp}\left[-\frac{\pi\mu^2}{g\dot\phi}\right] << 1$$

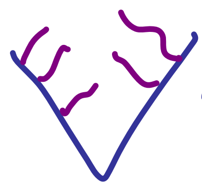
× combinatoric factors :

Combinatorics: Tree Diagrams are given by partitions of $N = \sum_m N_m m$

$$\mathcal{L}_{m+2} = g^2 \frac{f^{2-m}}{m!} \delta\phi^m \chi^2$$

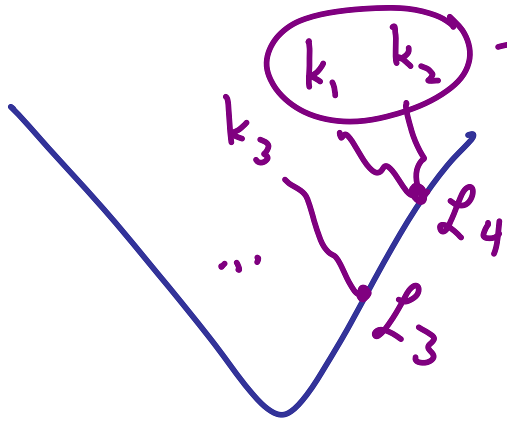


$$\propto \left(\prod_m \frac{1}{(m!)^{N_m} N_m!} \right) \times N!$$

- Fully factorized shape:  $\propto \frac{N!}{N!} = 1$

all $\mathcal{L}_3$'s $\quad m=1,\ N_m = N$

- Other partitions of N give hybrid resonant/factorized shape, with a combinatorial enhancement:



$\to \cos\left(\frac{\omega}{H}\log\frac{k_1+k_2}{H}\right)$

Let us spell this out a little more explicitly for the bispectrum, which is schematically of the form

$$
\begin{aligned}
&\frac{A}{k_1^2k_2^2k_3^2}\sum_{n=n_{min}}^{\infty}\left(\prod_{J=1}^{3}\frac{1}{-\eta_n k_J}\right)\left\{\prod_{I=1}^{3}\cos\left(\tilde{\gamma}_I+\frac{\omega}{H}\log(-k_I\eta_n)\right)\right.\\
&+C_{34}\frac{k_2k_3}{(k_2+k_3)^2}\cos\left(\gamma_{34}+\frac{\omega}{H}\log(-(k_2+k_3)\eta_n)\right)\cos\left(\tilde{\gamma}_{34}+\frac{\omega}{H}\log(-k_1\eta_n)\right)+\text{permutations}\\
&\left.+C_5\frac{k_1k_2k_3}{k_T^3}\cos\left(\gamma_5+\frac{\omega}{H}\log(-(k_1+k_2+k_3)\eta_n)\right)\right\}. \qquad (3.65)
\end{aligned}
$$

Ratio of partition $N=\sum_m m N_m$ to fully factorized term is

$$N!\prod_m\left(\frac{\xi^{m-1}}{m!}\right)^{N_m}\frac{1}{N_m!}$$

$$\xi = \frac{H\mu}{g^2f^2}\sqrt{\frac{\omega}{\pi H}}$$

Can favor hybrid shapes for $\xi N > 1$

Regardless, we have for factorized shape

$$S(k_1, \dots k_N) \sim \sum_n \prod_{i=1}^{N} S_n(k_i)$$

$$\text{Signal/Noise}\Big|_N \sim \epsilon x^N$$

For regime x>1, can test factorized shape with N>3 version of a standard estimator. If each factor has overlap

Cos[S(k), S(k)_true]=y

we can constrain

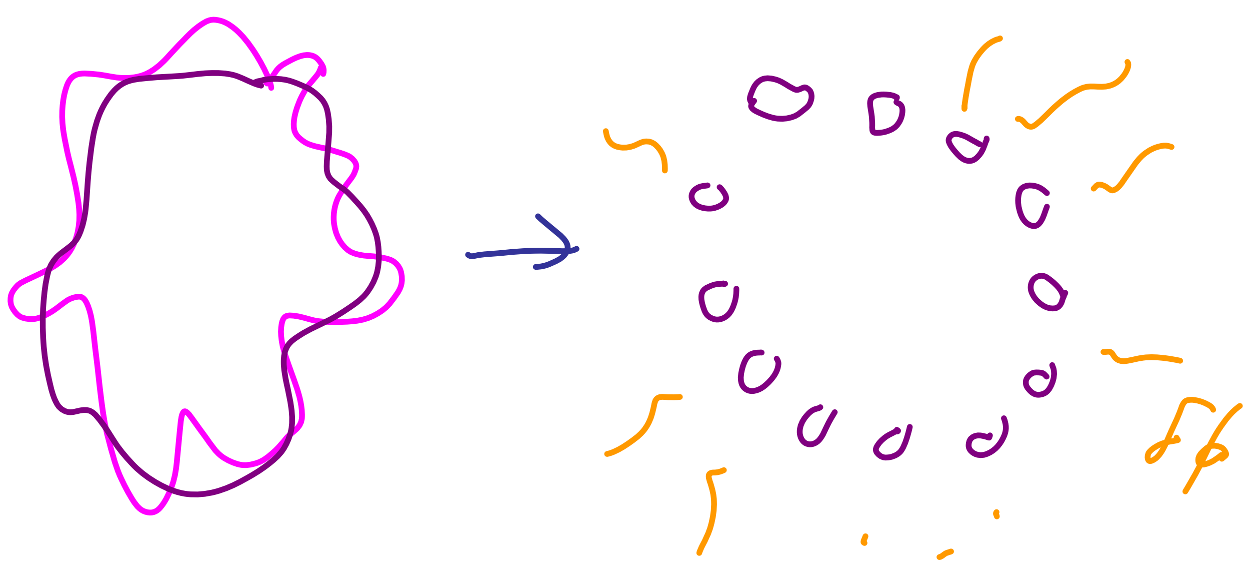
$$x < \frac{1}{y} \frac{1}{\epsilon^{1/N}}$$

String Production

Bachas, McAllister Mitra, Senatore ES Zaldarriaga, D'Amico
Kleban Schillo et al, J. Polchinski ES, ...

Shape and search sensitive to microscopic details. Many interesting subtleties (and enhancements with string production (either between D-branes, or varying-tension wrapped branes).

$$\mu(t)^2 = a^2 + b^2 t^2. \Rightarrow \langle N_k \rangle \sim e^{\frac{k^2}{4br} + \frac{\pi^2 a^2 r}{b}}.$$



Summary+Ongoing directions:

Non-adiabatic dynamics ->

--Relatively large signal/noise in non-Gaussianity, including novel regime growing with N

--Sensitive to heavy fields.

(Q:) String production, F vs B statistics?

--Part of a complete treatment of the phenomenology of axion monodromy (in addition to original oscillatory features)

--Templates (a) and (b) under analysis, as well as ideas for large-N analysis

(Q:) Npf estimator, non-factorizable shape components (machine learning?), histogram for model-independent constraints

**EFT must be supplemented by even such heavy fields; at truly single-field level could analyze multifrequency Fourier expansion of arbitrary couplings M(t)

Extra Slides

Cheung et al '08

$$S = \int d^4x \sqrt{-g} \left[M_{\rm Pl}^2 \dot{H}(t+\pi) \left(\dot{\pi}^2 - \frac{(\partial_i \pi)^2}{a^2} \right) + \right.$$
$$\left. + M_1^4(t+\pi) \left(\dot{\pi}^2 + \dot{\pi}^3 + \ldots \right) + M_2^4(t+\pi) \left(\dot{\pi}^3 + \ldots \right) + M_3^4(t+\pi) \left(\dot{\pi}^4 + \ldots \right) \right] .$$

$$M_i^4(t) = \int d\omega \, e^{i\omega t} \, \tilde{M}_i^4(\omega) \simeq \sum_{j=-j_{\rm max}}^{j_{\rm max}} e^{i \Delta\omega j t} \, \tilde{M}_{i,j}^4$$

↳ Must include particles of mass $\gtrsim \omega_{max}$ rather than integrating them out, or can miss substantial effects from their production.

Suppose we integrated out some field with mass $m_\chi(\phi(t)) \sim \sqrt{\mu^2 + \dot{\phi}^2 t^2}$

$$\Rightarrow M_i^4(t), H(t) \propto \frac{1}{m(\phi(t))^2} \sim \frac{1}{\mu^2 + \dot{\phi}^2 t^2}$$

Fourier Transform →

$\omega_{max} \sim \sqrt{\dot{\phi}}$ for $\mu \sim \dot{\phi}^{\frac{1}{2}}$

In DBI inflation we have a sequence of power-law effects from integrating out $\sim N_3$ particles of mass $g\phi$

$$S = -\frac{1}{g^2(2\pi\alpha')}\int d^4x \frac{r^4}{R^4}\sqrt{1-\frac{R^4\dot{r}^2}{r^4}}$$
$$\simeq \int d^4x \left\{\frac{1}{2}\dot{\phi}^2 + g^4\frac{N_3\dot{\phi}^4}{4\pi^2 m_\chi^4} + \ldots\right\}.$$

$$c_s = \frac{1}{\gamma}, \qquad \gamma = \frac{1}{\sqrt{1-\frac{g^4 N_3\dot{\phi}^2}{2\pi^2\mu^4}}} \equiv \frac{1}{\sqrt{1-v^2}},$$

$$v^2 = \frac{g^2}{2}N_3\left(\frac{g\dot{\phi}}{\pi\mu^2}\right)^2 \ll 1,$$

Only wins over particle production effects for $g^2N \gg 1$.

(2)

$\langle J \cdots J \rangle$ Also has interference terms at $O(\beta)$, giving different shape (more similar to resonant)

$$\underbrace{\chi\chi}\frac{\delta m_\chi}{\delta\phi}$$

$$\sim a^{+2}\exp\left(-2i\int^t dt'\mu\right) + c.c.$$

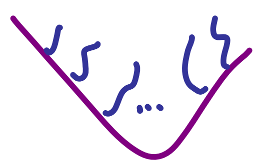
rapid oscillation when integrated against Green's ftn

Contribution to $\langle J \cdots J \rangle$ above came from a^+a terms and can dominate in a range of parameters (again based on details of power law & exponential hierarchies).

Parameter Windows

$$z \equiv \frac{\pi \mu_b^2}{g\dot{\phi}}\,, \quad g\,, \quad \frac{\omega}{H}$$

$$\mu^2 = \mu_b^2 + 2g^2 f^2; \quad f = \frac{\dot{\phi}}{\omega}\,, \quad \dot{\phi} = \frac{\pi \mu_b^2}{g z}\,, \quad \frac{\dot{\phi}}{H^2} \simeq 58^2$$

Impose $\frac{\delta\langle \zeta^2\rangle}{\langle \zeta^2\rangle} \lesssim 10^{-2}$ and consistency conditions for control and dominance of the  factorized shape.

Solved for reasonable range of coupling

$$.01 < g << 4\pi$$

for $\frac{\omega}{H} \leq O(100)$

Back to generalities/EFT:

$$S=\int d^4x\,\sqrt{-g}\,\left[M_{\rm Pl}^2\dot H(t+\pi)\left(\dot\pi^2-\frac{(\partial_i\pi)^2}{a^2}\right)+\right.$$
$$\left.+M_1^4(t+\pi)\left(\dot\pi^2+\dot\pi^3+\ldots\right)+M_2^4(t+\pi)\left(\dot\pi^3+\ldots\right)+M_3^4(t+\pi)\left(\dot\pi^4+\ldots\right)\right]\,.$$

$$M_i^4(t)=\int d\omega\,e^{i\omega t}\,\tilde M_i^4(\omega)\simeq\sum_{j=-j_{\rm max}}^{j_{\rm max}}e^{i\,\Delta\omega\,j\,t}\,\tilde M_{i,j}^4$$

At truly single-field level, could try to analyze all shapes given finite resolution, but lots of parameters.

Just saw that even heavy fields can be important -- must add to EFT, so not strictly single-field.

Is there a useful systematic approach, beyond testing specific mechanisms? Good role for UV completion, but hit or miss...