The Sachdev-Ye-Kitaev model and AdS_2

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IAS

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based on 1604.07818 with Maldacena; drawing on talks by Kitaev

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SYK	"$\ell_s \sim \ell_{AdS}$"	$O(1)$	maximal	yes

Naive expectations for QM system dual to AdS_2

- **thermodynamics:** large entropy even at low temperature
- **dynamics:** four point function should include gravitational scattering and reveal whatever spectrum of bulk fields we have
- **symmetry:** approximate conformal invariance; expect e.g. two point function on circle to be

$$\langle O(\tau)O(0)\rangle = \left(\frac{1}{\sin\frac{\pi\tau}{\beta}}\right)^{2\Delta}$$

the relationship with conformal symmetry is subtle!

The Sachdev-Ye-Kitaev model

QM of N majorana fermions $\psi_1, ..., \psi_N$, which satisfy

$$\{\psi_a, \psi_b\} = \delta_{ab}.$$

The Hamiltonian consists of all four-body interactions

$$H = \sum_{a<b<c<d} j_{abcd}\, \psi_a \psi_b \psi_c \psi_d$$

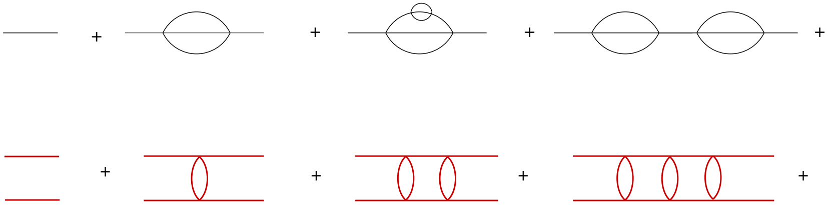
with random coefficients

$$\langle j_{abcd}^2 \rangle = \frac{J^2}{N^3} \quad \text{(no sum)}$$

- Dimensionless coupling is βJ. Interesting behavior at $\beta J \gg 1$.
- Can also consider a version with fermions interacting in groups of q, instead of four. $q \to \infty$ and $q \to 2$ are simpler limits.
- System "self-averages."

Feynman diagrams

Diagrams for the two point and four point functions at leading order in $1/N$:



[Kitaev 2015]

The disorder average

After integrating over j_{abcd}, can introduce new fields G, Σ. Σ is a Lagrange multiplier that sets $G(\tau_1, \tau_2) = \frac{1}{N} \sum_a \psi_a(\tau_1)\psi_a(\tau_2)$.

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$$\langle Z(\beta) \rangle_J = \int DGD\Sigma \, e^{-N \, I(G,\Sigma)}$$

$$I(G, \Sigma) = -\frac{1}{2} \log \det(\partial_\tau - \Sigma) + \frac{1}{2} \int_0^\beta d\tau_1 d\tau_2 \Big[\Sigma(\tau_1, \tau_2) G(\tau_1, \tau_2) - \frac{J^2}{q} G(\tau_1, \tau_2)^q \Big]$$

[Parcollet/Georges/Sachdev, Kitaev]

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[Parcollet/Georges/Sachdev, Kitaev]

- this is an exact rewrite of the theory
- G, Σ are the master fields, should be the "bulk theory"

Plan for the rest of the talk

1. Saddle point
2. Quadratic $1/N$ fluctuations about saddle

Saddle point

Saddle point equations $\Sigma_* = J^2 G_*^{q-1}$, and $G_* = (\partial_\tau - \Sigma_*)^{-1}$ can be solved numerically, or exactly at large q.

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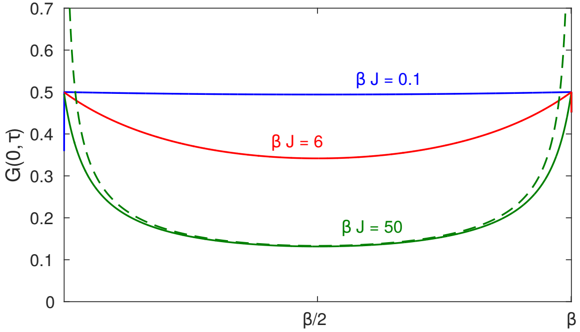

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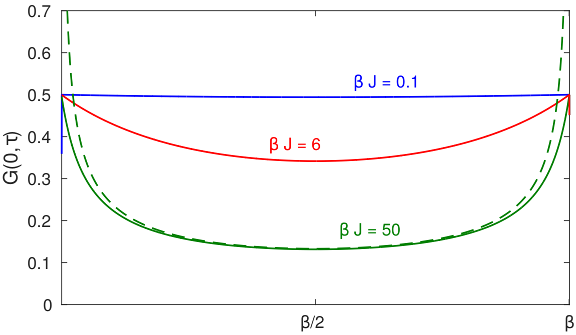
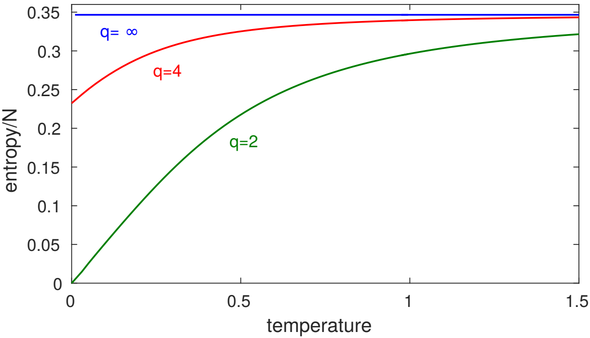


Conformal behavior at low temp $\beta J \gg 1$, [Sachdev/Ye, Parcollet/Georges]

$$G_*(0,\tau) \to \frac{const.}{(\sin \frac{\pi\tau}{\beta})^{2\Delta}}, \qquad \Delta = \frac{1}{q}.$$

Saddle point

To get large N thermodynamics, plug G_*, Σ_* back into the action, $Z(\beta) \approx e^{-N\, I(G_*,\Sigma_*)}$.



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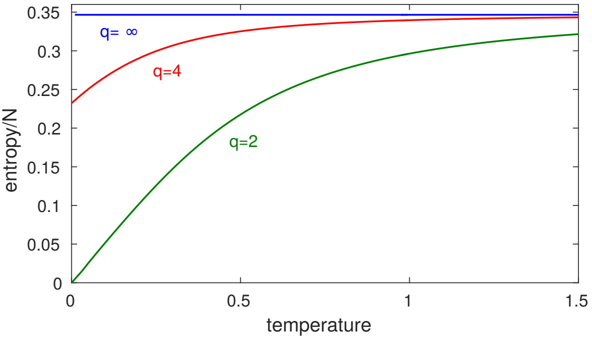

Large entropy even at very low temperature, provided $q > 2$.
(Striking agreement with exact diagonalization numerics, [Gur-Ari et. al. in progress])

Fluctuations

$$I(G) = I(G_*) + \int \delta G(\tau_1, \tau_2) Q(\tau_1 \tau_2; \tau_3 \tau_4) \delta G(\tau_3, \tau_4) + \dots$$

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The quadratic form Q gives the $1/N$ term in the fermion 4pt fn:

$$\frac{1}{N^2} \sum_{a,b} \langle \psi_a(\tau_1) \psi_a(\tau_2) \psi_b(\tau_3) \psi_b(\tau_4) \rangle = \langle G(\tau_1, \tau_2) G(\tau_3, \tau_4) \rangle$$

$$= G_*(\tau_1, \tau_2) G_*(\tau_3, \tau_4) + \frac{1}{N} Q^{-1}(\tau_1 \tau_2; \tau_3 \tau_4) + \dots$$

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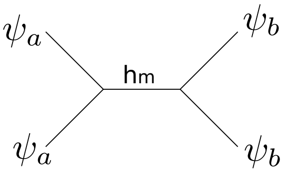
For $\beta J \gg 1$ the quadratic form Q is conformally invariant. Can diagonalize and find (for $\chi < 1$)

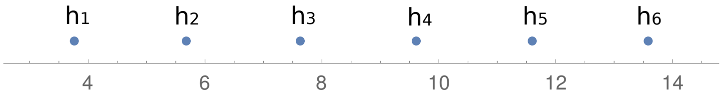
$$\frac{\langle 4pt \rangle_{conn.}}{\langle 4pt \rangle_{disc.}} = \frac{\text{``}\infty\text{''}}{N} + \frac{1}{N} \sum_{m=1}^{\infty} c_{h_m}^2 \, \chi^{h_m} F(h_m, h_m, 2h_m, \chi) + \ldots$$

Fluctuations: nonzero modes

The values h_m represent the conformal dimensions of the operators appearing in the OPE.



The dimensions are roughly evenly spaced



They correspond to $O_m = \psi_a \partial_\tau^{2m+1} \psi_a$ with $O(1)$ anomalous dimensions. They demand a tower of bulk fields in the dual, reminiscent of a string spectrum in two dimensions.

Fluctuations: zero modes

- At large βJ, the action is invariant under general reparameterizations $\tau \to f(\tau)$ [Kitaev]

$$G_* \to G_f = \left(f'(\tau_1)f'(\tau_2)\right)^{\Delta} G_*(f(\tau_1) - f(\tau_2))$$

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- This leads to a finite correlator

$$\langle 4pt \rangle_{conn.} = \frac{1}{N}\left[(\beta J)\mathcal{F}_{big}(\tau_1 \ldots \tau_4) + \sum_{n=1}^{\infty} c_{h_n}^2 \chi^{h_n} F(h_n, h_n, 2h_n, \chi)\right]$$

Fluctuations: zero modes (II)

One can show that the effective action for the zero modes is the Schwarzian derivative,

$$I_{eff} = -\frac{\#}{J} \int \text{Sch}(f, \tau) d\tau.$$

This exactly matches what you get from dilaton gravity [Almheiri/Polchinski, Jensen, Maldacena/DS/Yang, Engelsoy/Mertens/Verlinde].
(See Gong Show talk by Zhenbin Yang later today!)

The $\mathcal{F}_{big}$ contribution is in some ways similar to the contribution of the stress tensor in a 2d CFT,

- both are associated to reparameterizations or “boundary gravitons”

There are key differences:

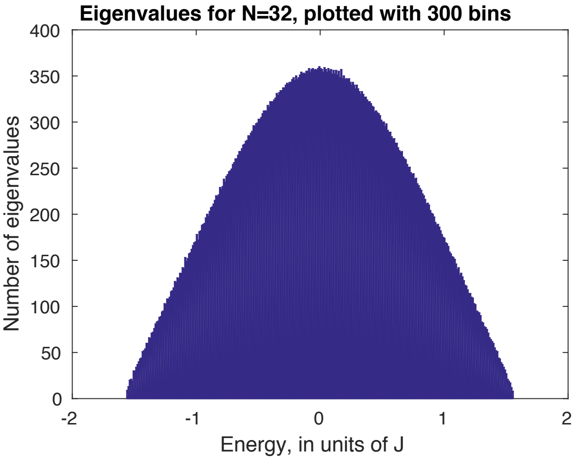
- in 2d, the contribution is conformal, here it is not
- stress tensor (gravity) dominance in 2d requires sparseness, here it happens automatically because of the βJ enhancement

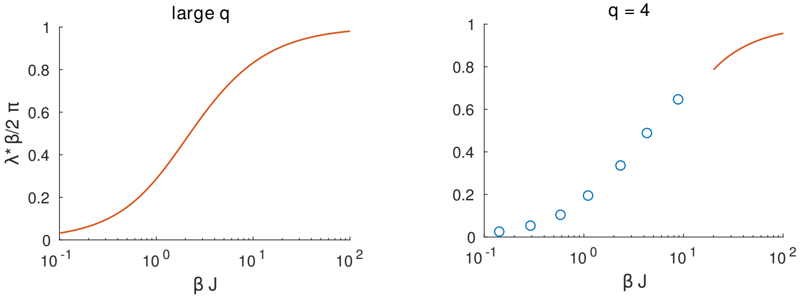
Summary

- SYK is an interesting solvable QM system
- the dominant low-energy physics is determined by spontaneously and (weakly) explicitly broken conformal symmetry
- this aspect is shared with dilaton gravity in AdS_2
- in addition, the model has states reminiscent of a stringy dual with $\ell_s \sim \ell_{AdS}$

NEXT: find the black hole interior in this model...

Thanks!


Eigenvalues for N=32, plotted with 300 bins
Number of eigenvalues
0
50
100
150
200
250
300
350
400
-2
-1
0
1
2
Energy, in units of J


large q
q = 4
λ* β/2 π
β J
β J
1
0.8
0.6
0.4
0.2
0
10^{-1}
10^{0}
10^{1}
10^{2}

Eigenvalues of the kernel

$$k(h) = -(q-1)\frac{\Gamma(\frac{3}{2}-\frac{1}{q})\Gamma(1-\frac{1}{q})}{\Gamma(\frac{1}{2}+\frac{1}{q})\Gamma(\frac{1}{q})}\frac{\Gamma(\frac{1}{q}+\frac{h}{2})}{\Gamma(\frac{3}{2}-\frac{1}{q}-\frac{h}{2})}\frac{\Gamma(\frac{1}{2}+\frac{1}{q}-\frac{h}{2})}{\Gamma(1-\frac{1}{q}+\frac{h}{2})}.$$