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Quantum Operations in CFTs and Holography

Tadashi Takayanagi (高柳　匡)

Yukawa Institute for Theoretical Physics (YITP),
Kyoto University

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Collaborators: Tokiro Numasawa (YITP)
Noburo Shiba (YITP)
Kento Watanabe (YITP)

① Introduction

The main purpose of this talk is to study operational aspects of quantum entanglement in CFTs, which have not been well investigated so far.

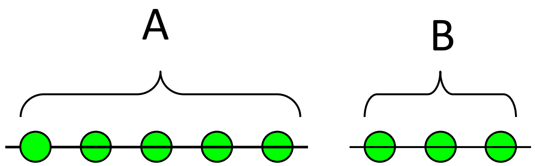
Indeed, quantum information theory is originally formulated in an operational way.

In this talk, we will describe three operations in 2d CFTs:

(i) Projecting States Locally
(ii) Adding Entanglement locally between Two CFTs
(iii) Swapping locally between Two CFTs
→ A CFT model of quantum teleportation

Example of an operational aspect of QI: LOCC

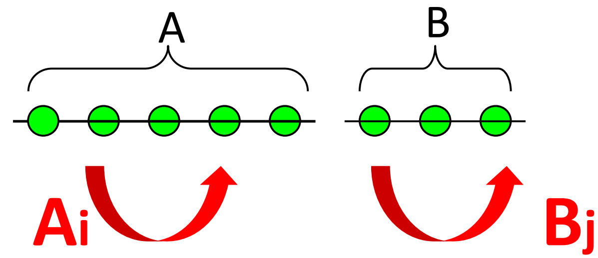
Setup



$$\Rightarrow H_{tot} = H_A \otimes H_B$$

LO (=Local Operations)

$$\rho \rightarrow \sum_{i,j} (A_i \otimes B_j) \cdot \rho \cdot (A_i^+ \otimes B_j^+)$$



$$\sum_i A_i^+ A_i = 1$$

$$\sum_j B_j^+ B_j = 1$$

This includes projections and unitary trfs.

CC (=Classical Communications between A and B)

⇒These operations are combined and called **LOCC**.

Operational Meaning of Entanglement entropy (EE)

The unit of entanglement ⇒EPR pair

$$|\mathrm{EPR}\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B\right).$$

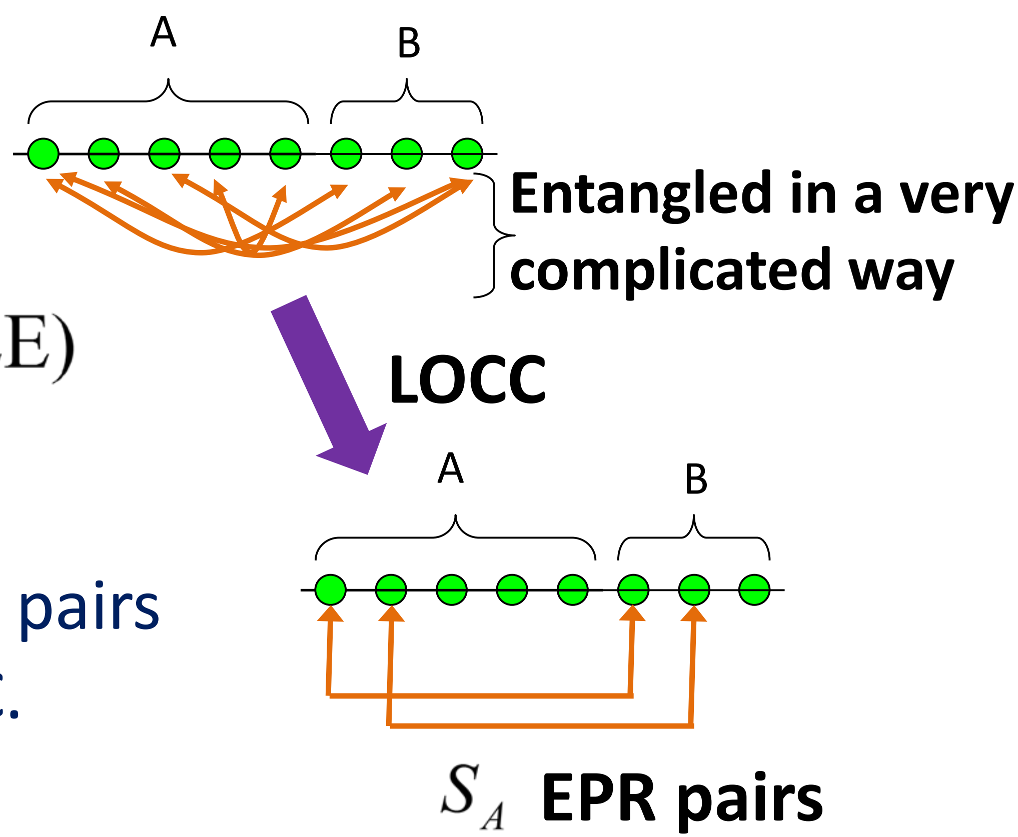
Reduced density matrix

$$\Rightarrow \quad \rho_A = \mathrm{Tr}_B\left[|\Psi\rangle\langle\Psi|\right],$$

Entanglement Entropy (EE)

$$\Rightarrow S_A = -\mathrm{Tr}_A\, \rho_A \log \rho_A$$

≈ Maximal # of EPR pairs obtained by LOCC.



Quantum Teleportation

[Bennett et.al 1993]

Initial state:

$$|\psi\rangle_V \otimes |\mathrm{EPR}\rangle_{AB} = \frac{1}{2}\sum_{k=1}^{4} |\Psi_k\rangle_{VA} \otimes |\psi_k\rangle_B,$$

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle,$$

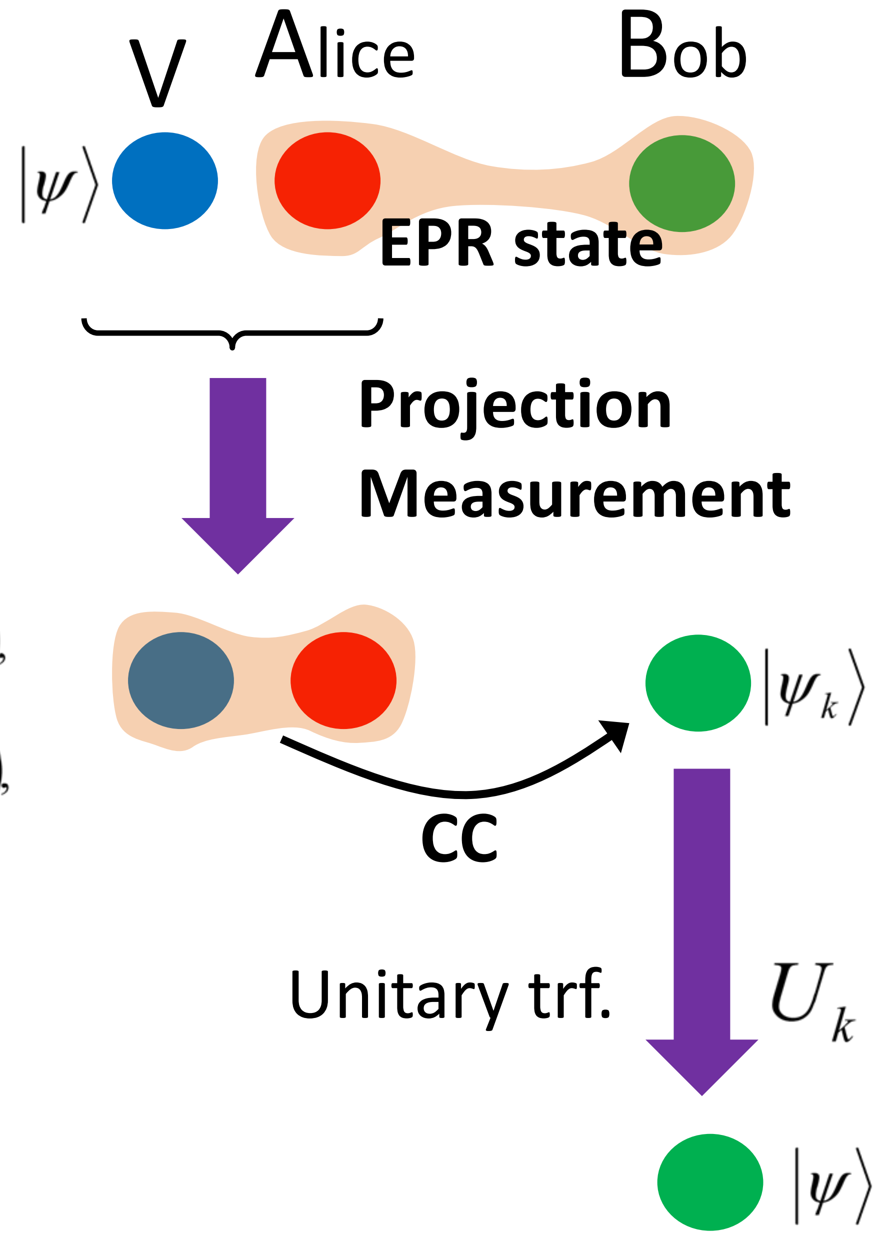
$$|\Psi_1\rangle = \frac{1}{\sqrt{2}}(|0\rangle|1\rangle - |1\rangle|0\rangle), \quad |\Psi_2\rangle = \frac{1}{\sqrt{2}}(|0\rangle|1\rangle + |1\rangle|0\rangle),$$

$$|\Psi_3\rangle = \frac{1}{\sqrt{2}}(|0\rangle|0\rangle - |1\rangle|1\rangle), \quad |\Psi_4\rangle = \frac{1}{\sqrt{2}}(|0\rangle|0\rangle + |1\rangle|1\rangle),$$

$$|\psi_1\rangle = \alpha|1\rangle - \beta|0\rangle, \quad |\psi_2\rangle = \alpha|1\rangle + \beta|0\rangle,$$

$$|\psi_3\rangle = \alpha|0\rangle - \beta|1\rangle, \quad |\psi_4\rangle = \alpha|0\rangle + \beta|1\rangle.$$

Projection: $\sum_{i=1}^{4} |\psi_k\rangle\langle\psi_k| = 1$

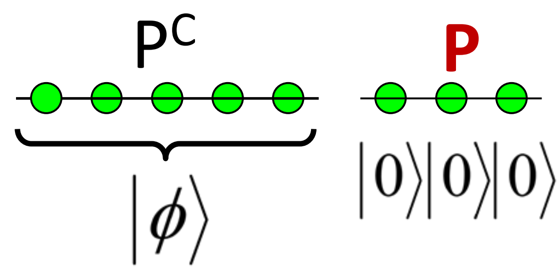


Contents

② Local Projection in a CFT

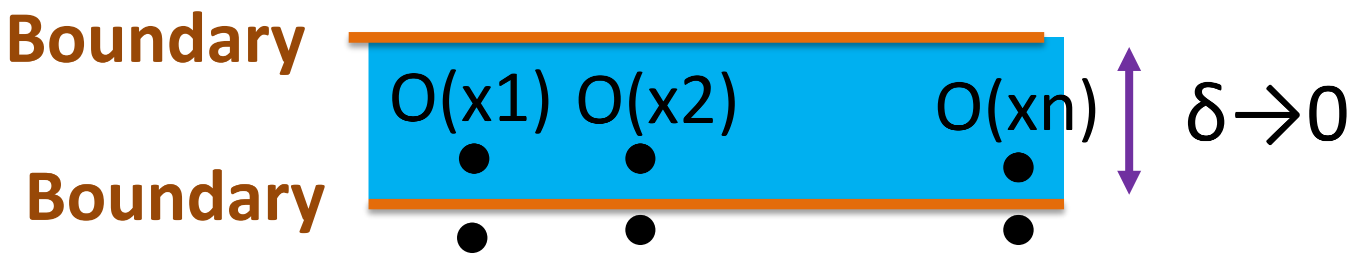
In QFTs, we can consider a local projection measurement where the state at each point in a region **P** is projected:

$$P = \left(\prod_{x \in P} |\psi_x\rangle\langle\psi_x| \right) \otimes \left(\prod_{x \in P^C} I_x \right)$$



Note that after the projection, there is no real space entanglement in the region **P**.

In CFTs, an important class of such states with no real space entanglement is given by boundary state $|B\rangle$ (Cardy state). [Miyaji-Ryu-Wen-TT 2014]



$$\frac{\langle B|e^{-\delta \cdot H}O(x_1)O(x_2)\cdots O(x_n)e^{-\delta \cdot H}|B\rangle}{\langle B|e^{-2\delta \cdot H}B\rangle} \approx \prod_{i=1}^{n}\langle O(x_i)\rangle.$$

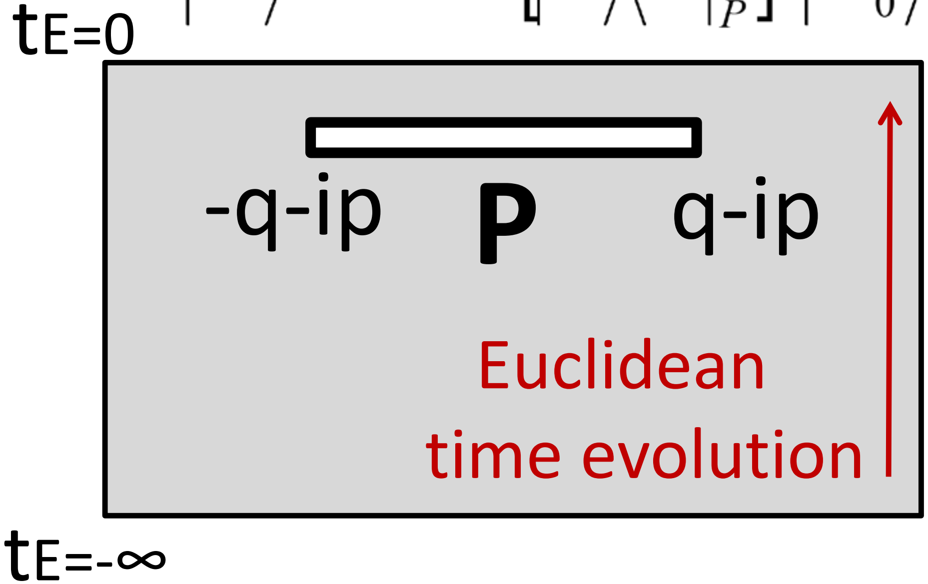
Thus a local projection measurement can be described by a boundary state $|B\rangle$. [Rajabpour 2015]

(More generally, we have $\prod_x U_x|B\rangle$.)

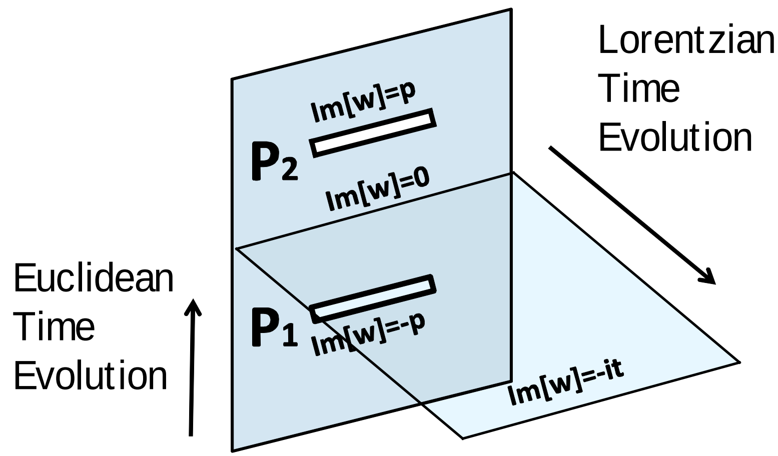
Consider a local projection of a 2d CFT on an interval P.

This is described by an Euclidean path-integral as follows:

$$\left|\Psi\right\rangle = e^{-pH}\cdot\left[\left|B\right\rangle\left\langle B\right|_P\right]\cdot\left|\Psi_0\right\rangle$$



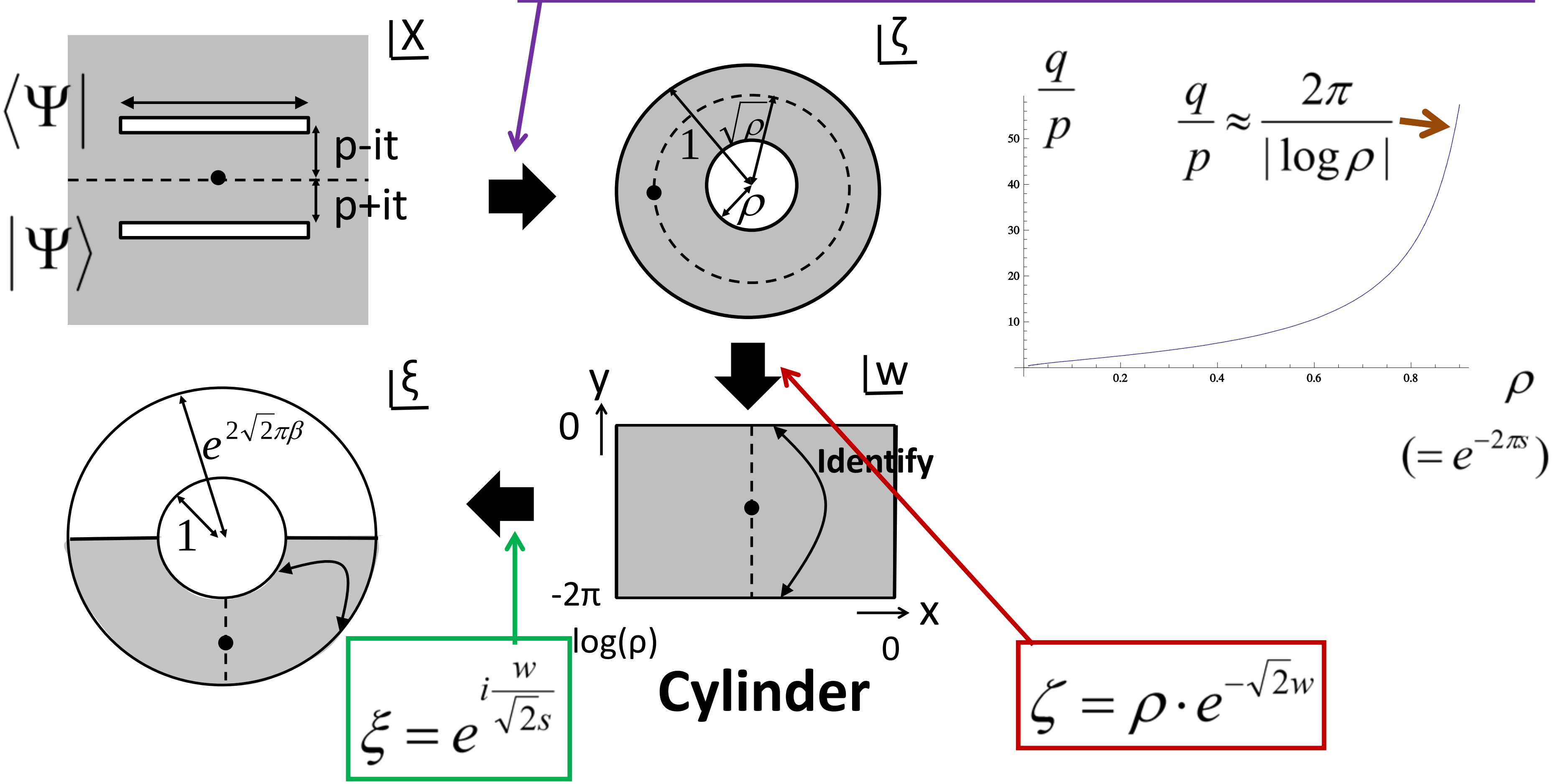
Real time evolution



Conformal Maps

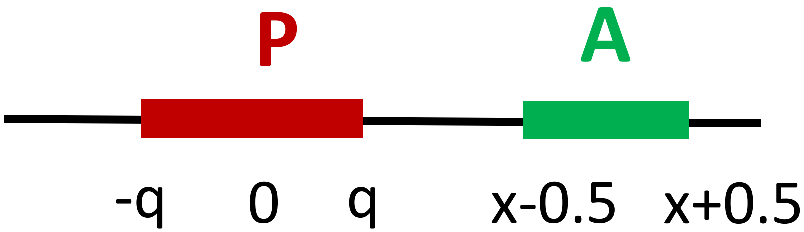
$$X(\zeta) = 2ip\left(K\left(\zeta/\sqrt{\rho}\right) + K\left(\zeta\sqrt{\rho}\right) - \frac{1}{2} \right) - t,$$

$$K(\zeta) = \frac{\zeta}{\zeta - 1} + \sum_{k=1}^{\infty} \left(\frac{\rho^{2k}}{\zeta - \rho^{2k}} - \frac{\zeta\rho^{2k}}{1 - \zeta\rho^{2k}} \right).$$



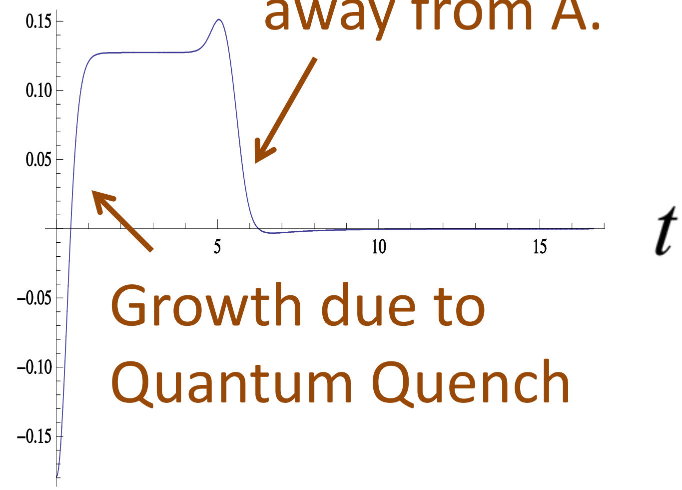
EE in Free Dirac Fermion CFT

$\rho = 0.6, \quad p = 0.5, \quad q = 5.3$



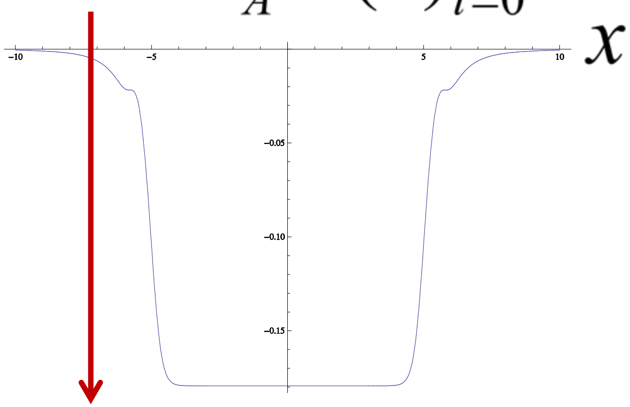
$\Delta S_A^{CFT}(t)_{x=0}$

Entangled pairs goes away from A.



Growth due to Quantum Quench

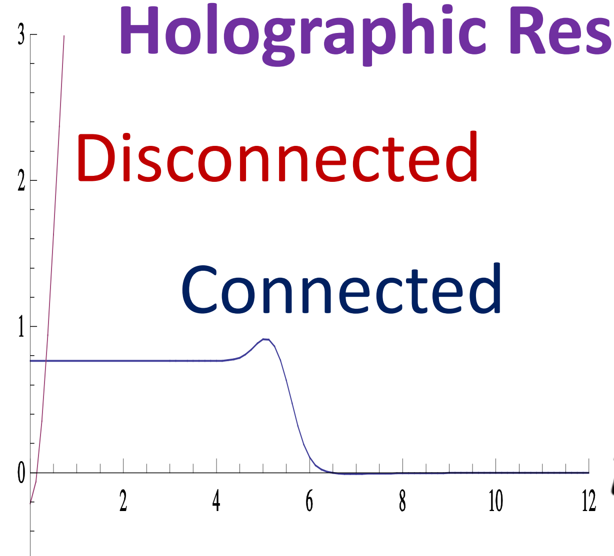
$\Delta S_A^{CFT}(x)_{t=0}$



Reduction of EE due to the projection

$\Delta S_A(t)_{x=0}^{\mathrm{AdS}}$

Holographic Result



Disconnected

Connected

③ Partially Entangling and Swapping in two CFTs

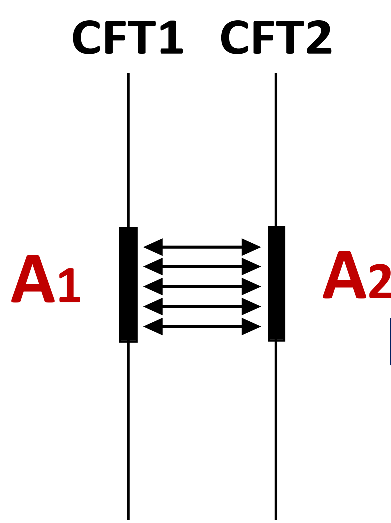
(3-1) Partially Entangling of Two CFTs

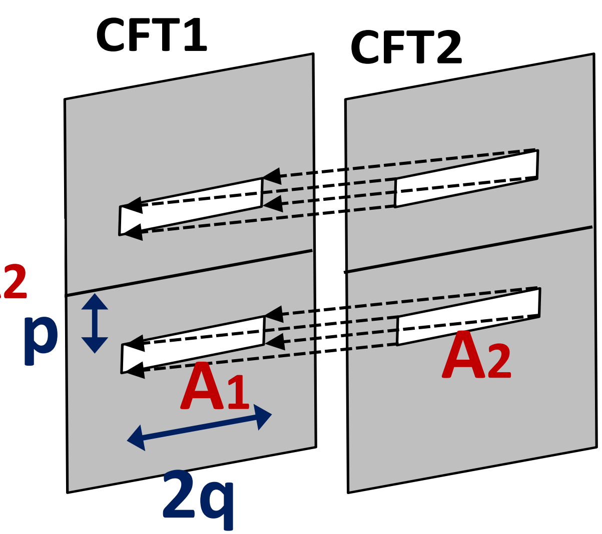
In QI operations, it is also very important to prepare **EPR states** (maximally entangled states).

Thus we would like to create EPR states between two identical CFTs (CFT1 and CFT2) on an interval **A**.

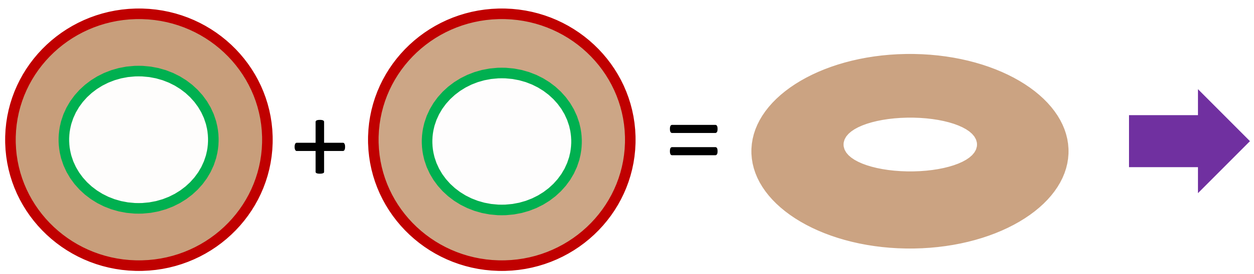
This is realized in the Euclidean path-integral:

$$P=\left(\prod_{x\in A}\sum_{n_x}|n_x\rangle_1|n_x\rangle_2\right)\left(\prod_{y\in A}\sum_{m_y}\langle m_y|_1\langle m_y|_2\right)\otimes\left(\prod_{x\in A^C}I_x\right)$$





Since each plane with two cuts is conformal to a cylinder, our doubled geometry is conformal to a torus.



Torus moduli

$$\tau = \underbrace{\tau_1}_{=0} + i\tau_2 \quad .$$

The entanglement entropy between two CFTs is given by (assume $\tau_2 >> 1$. In AdS/CFT, only need $\tau_2 > 1$.) :

$$S_{ent} = \frac{\pi c}{3} \tau_2 \quad .$$

In our setup (q>>p), this leads to

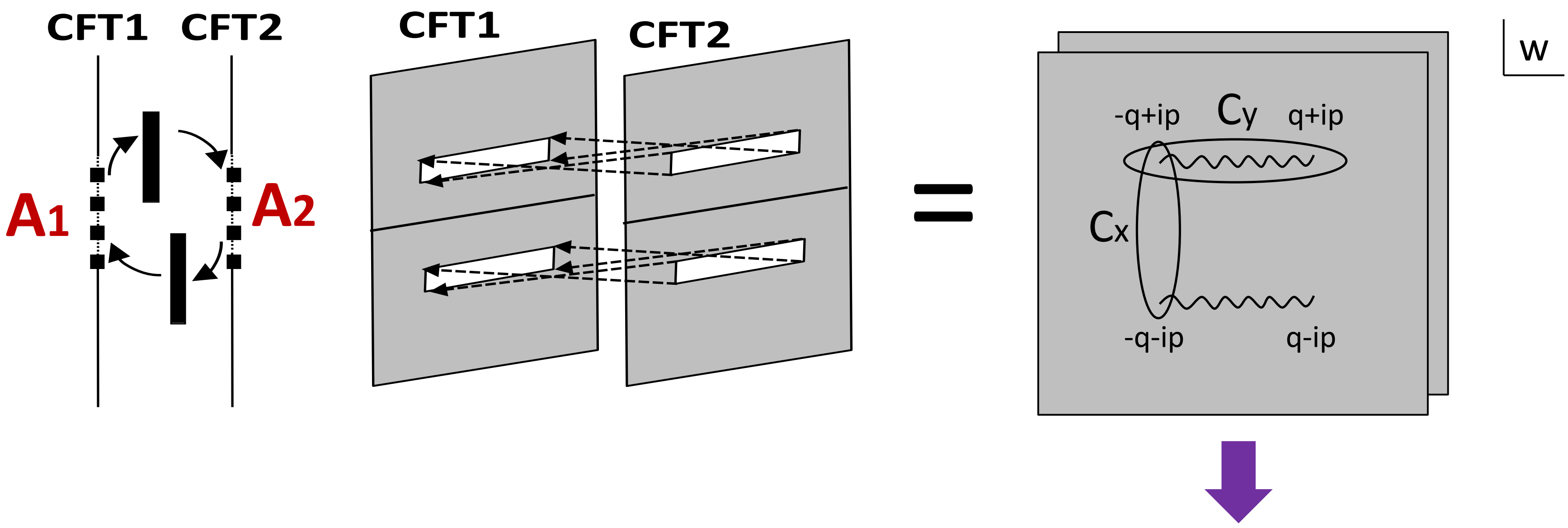
→ **EE is extensive as expected.**

(i.e. Volume law)

$$S_{ent} \cong \frac{\pi c}{6} \cdot \frac{q}{p} \quad .$$

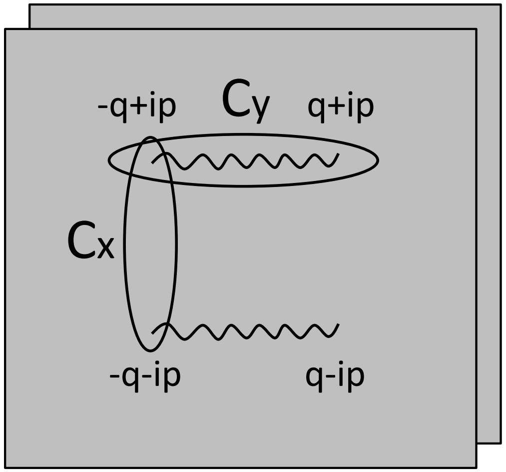
(3-2) Partial Swapping Two CFTs

We cut out A_1 and A_2 from CFT1 and CFT2. After we exchange them, we glue them again.



A torus with a different period than the previous one

EE after partial swapping

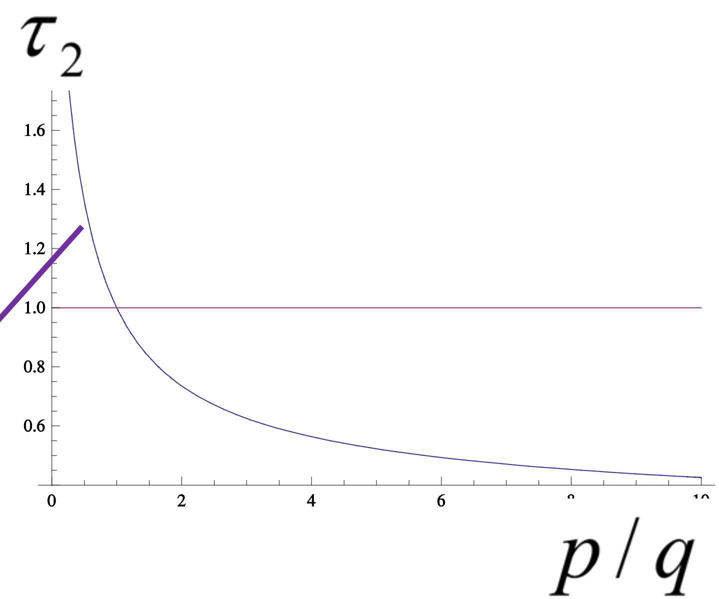


Elliptic Curve:

$$y^2 = (x - ip - q)(x - ip + q)(x + ip - q)(x + ip + q)$$

Period of the torus:

$$\tau_2 = \frac{\int_{Cy} \frac{dx}{y}}{\int_{Cx} \frac{dx}{y}}.$$



When $p/q << 1$, we find

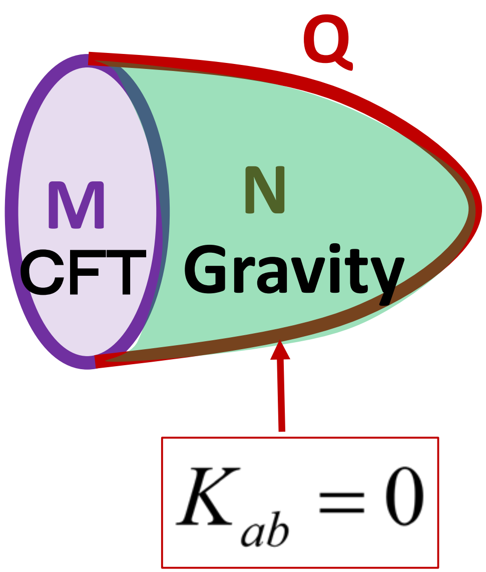
$$S_{ent} \approx \frac{2c}{3}\log\left(\frac{q}{p}\right). \Rightarrow 2 \cdot S_{\text{Interval}}$$

Reproduce the formula $S_{\text{Interval}} = \frac{c}{3}\log\left(\frac{l}{\varepsilon}\right)$ **!** [Holzhey-Larsen-Wilczek 1994]

④ Holographic Descriptions and Time evolutions

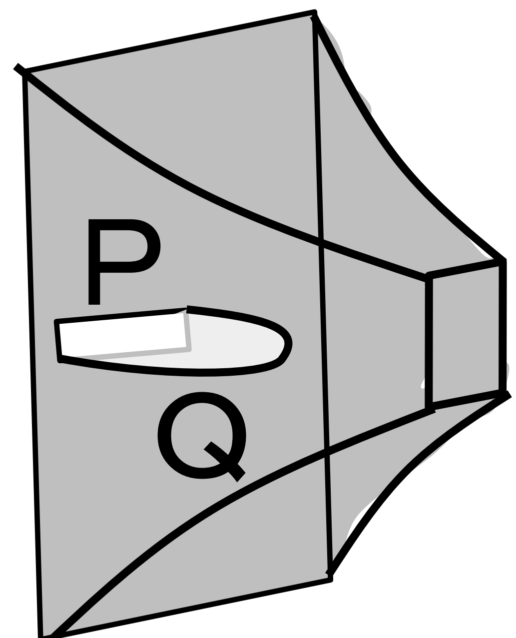
(4-1) Holographic dual of BCFT [TT 11, Fujita-Tonni-TT 11]

A gravity dual of a CFT on a space **M** with a boundary:



Note: The bdy Q backreacts in general, as opposed to HEE.

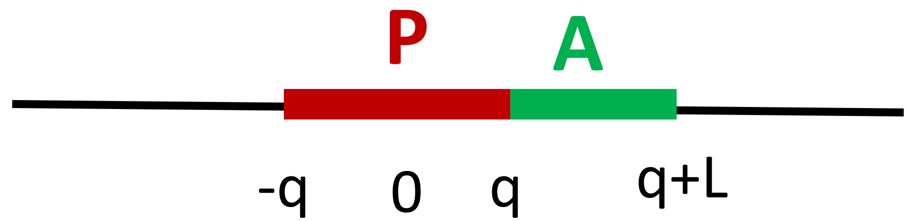
Our model of local projection



Projection → Reduce Entanglement
→ Making a hole in holographic spacetime

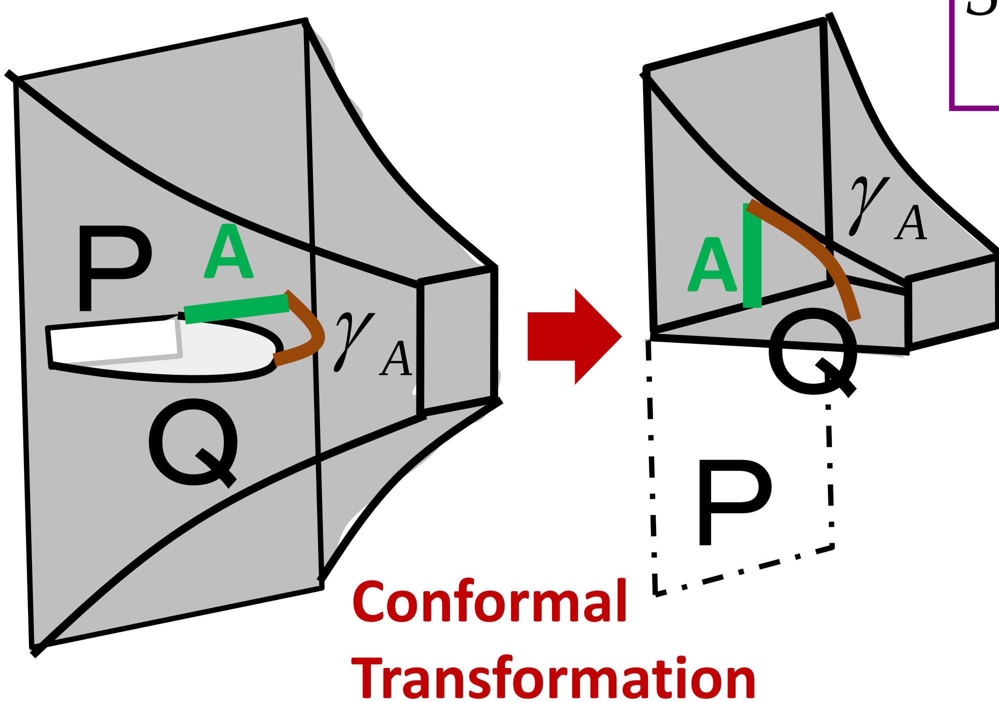
(4-2) Holographic Local Projection

Simple example:



Holographic Computation:

$$S_A = \frac{|\gamma_A|}{4G_N} = \frac{c}{6} \log \frac{2(L+2q)L}{\varepsilon q}.$$



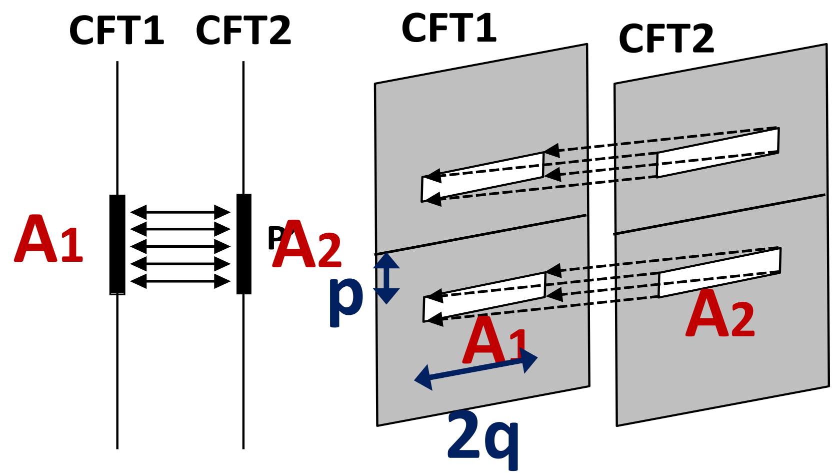
This agrees with the general CFT result in [Rajabpour 2015].

Note also if $q \approx \varepsilon \to 0$, this is reduced to the familiar formula

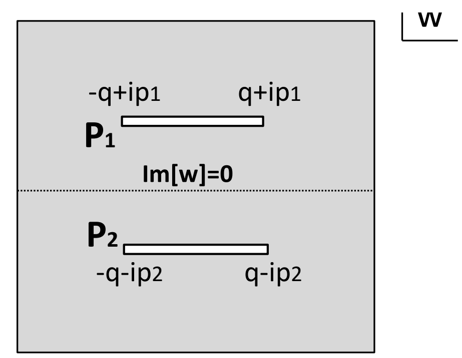
$$S_A = \frac{c}{3} \log\left(\frac{L}{\varepsilon}\right).$$

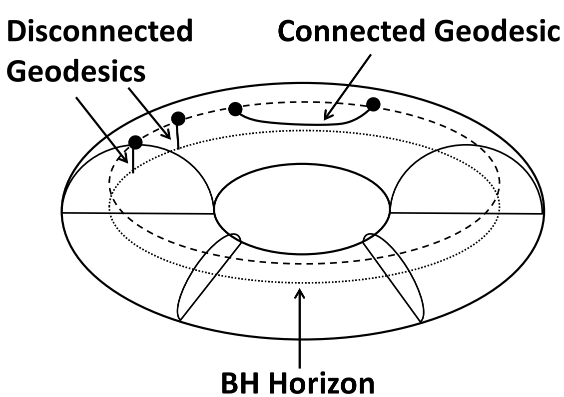
(4-3) Holographic Partial Entangling of Two CFTs

- **Partial Entangling of Two CFTs**
  - ⇒ Torus geometry
    - ⇔ BTZ black hole



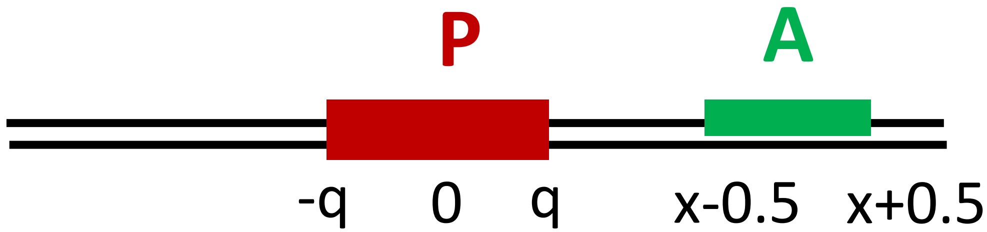
- **A local projection**
  - ⇒ Cylinder geometry
    - ⇔ A half of BTZ black hole

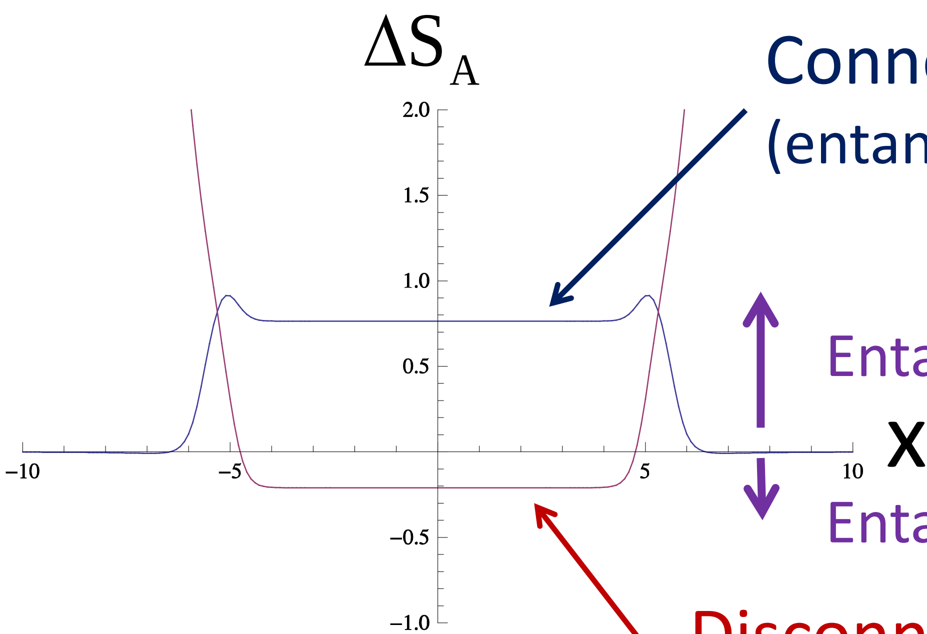




HEE for a single interval at t=0

$\rho = 0.6, \quad p = 0.5, \quad q = 5.3$





Connected Geodesic
(entangling and local projection)

Entanglement is generated

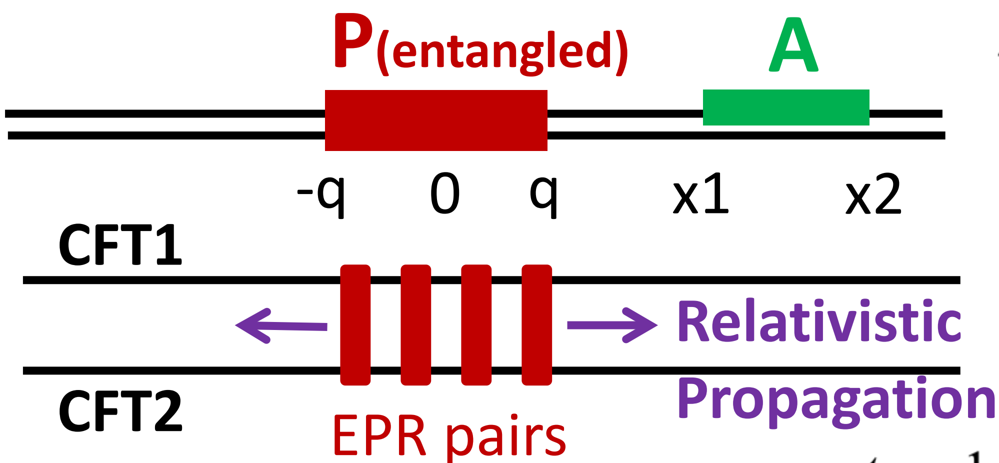
Entanglement is reduced

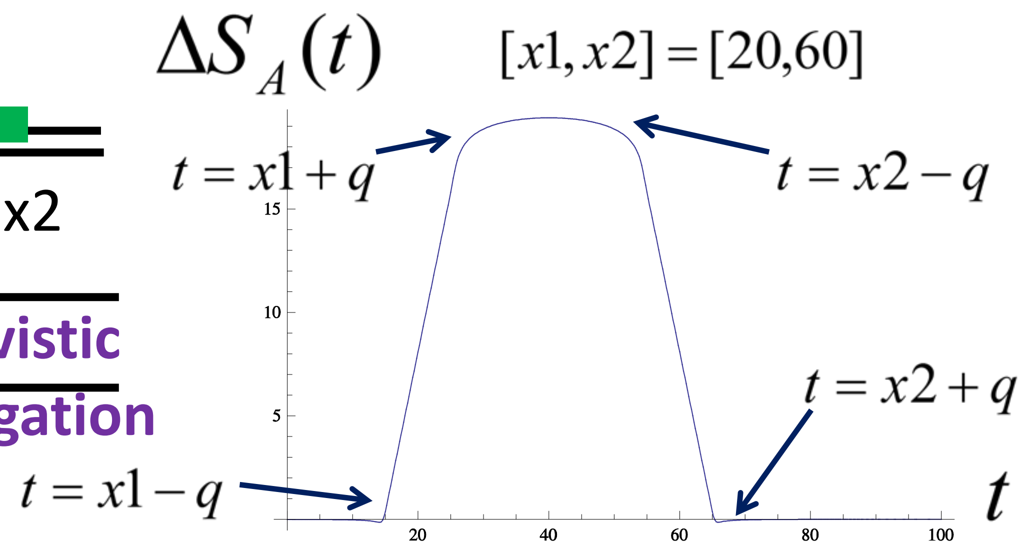
Disconnected Geodesic
(local projection only)

Time Evolutions of HEE for a single interval

$\rho = 0.6,$
$p = 0.5, \quad q = 5.3$

After we fix x1 and x2, we study the time evolutions.





In the limit x2>>t>>β and x1=0, we find a log growth:

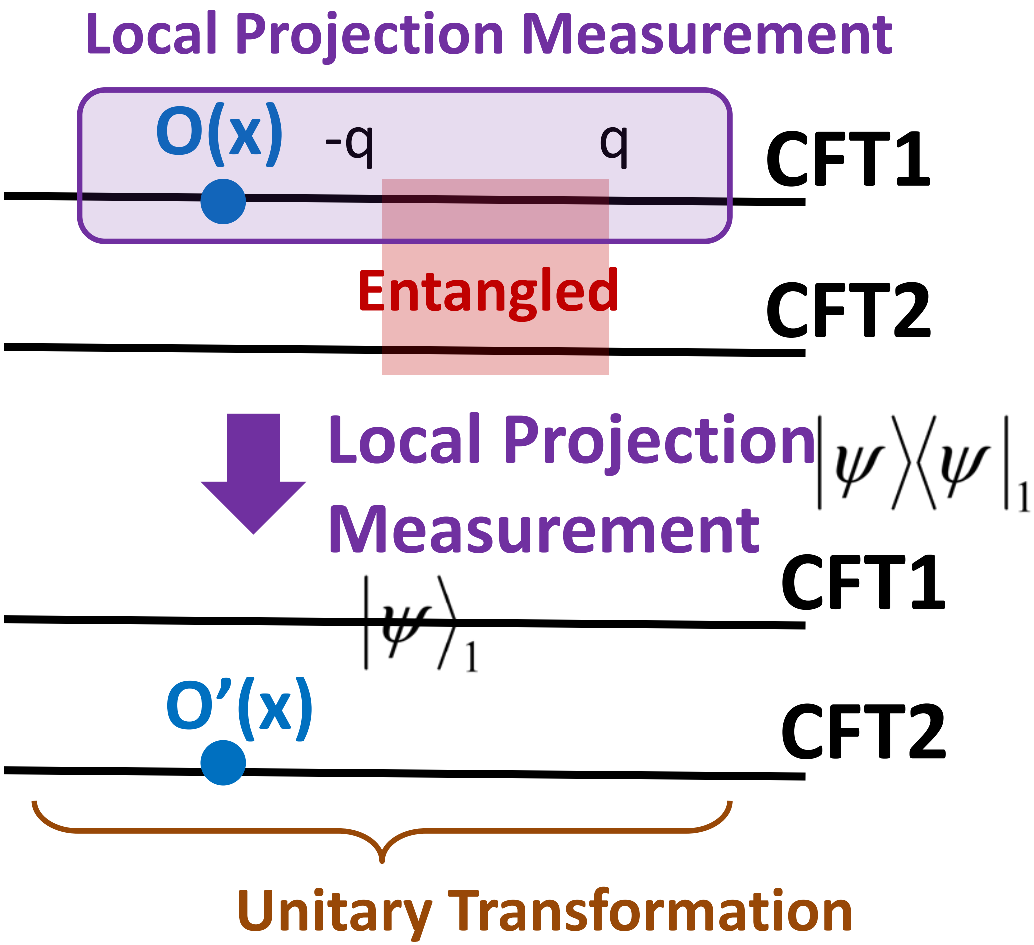
$$\Delta S_A \approx \frac{c}{6}\log\left[\frac{\sqrt{2}t}{\beta}\sinh\left(\sqrt{2}\pi\beta\right)\right].$$

⇒ Why log t growth ?
No quasi-particle picture ?

Similar to locally excited states in hol. CFTs [Caputa-Nozaki-Numasawa-TT 13,14].
Cf. Integrable CFTs show only a finite growth of EE [He-Numasawa-Watanabe-TT 14].

⑤ Holographic Quantum Teleportation

(5-1) Quantum Teleportation in CFTs



We teleport the operator O(x) from CFT1 to CFT2:

$$O(x) = \alpha_1 O_1(x) + \alpha_2 O_2(x),$$
$$\text{where } |\alpha_1|^2 + |\alpha_2|^2 = 1.$$

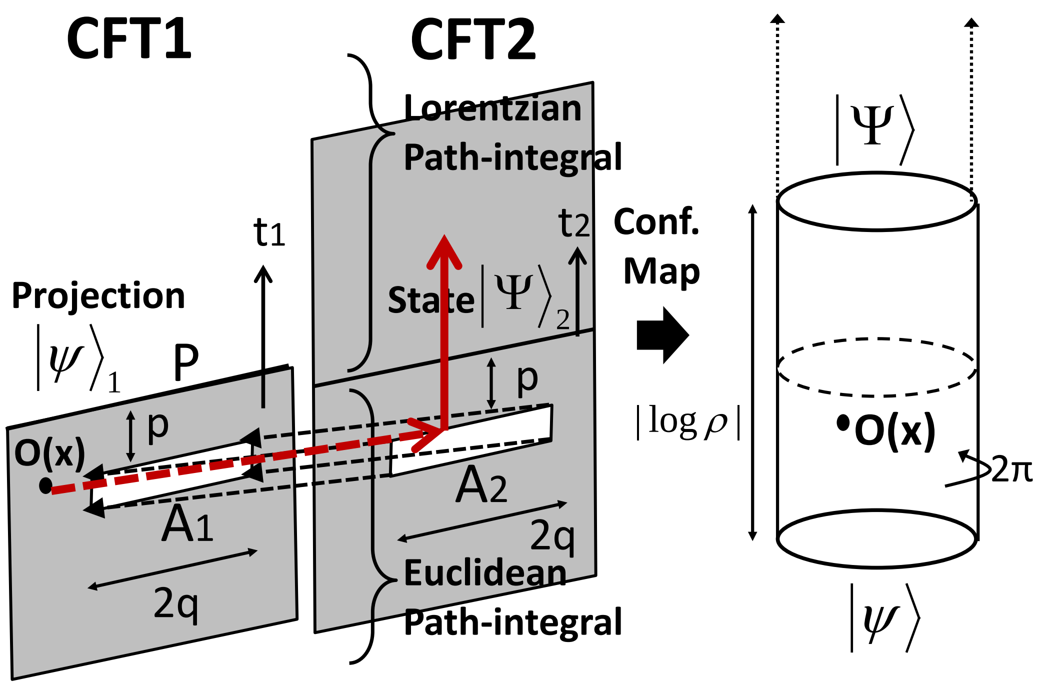
$$\left.\begin{aligned} O_1(x)|0\rangle_1 &\Leftrightarrow |0\rangle \\ O_2(x)|0\rangle_1 &\Leftrightarrow |1\rangle \end{aligned}\right\} \text{1 Qubit}$$

Path-integral formulation and Conformal transformation

After the projection measurement, we obtain the state:

$$\left|\Psi\right\rangle_2 = N \cdot e^{-\frac{\beta}{4}H} \cdot \left(\alpha_1 O_1 + \alpha_2 O_2\right) \cdot e^{-\frac{\beta}{4}H} \left|\psi\right\rangle_2,$$

$$(\beta \equiv 2\,|\log\rho\,|).$$



This satisfies the **linearity** w.r.t. α1 and α2 if

$$\left\langle\psi\right|O_1^{+}O_2\left|\psi\right\rangle = \left\langle\psi\right|O_2^{+}O_1\left|\psi\right\rangle = 0.$$

This is satisfied by assuming a **U(1) charge** such that $Q(O_1)=1$

$Q(O_2)=-1$.

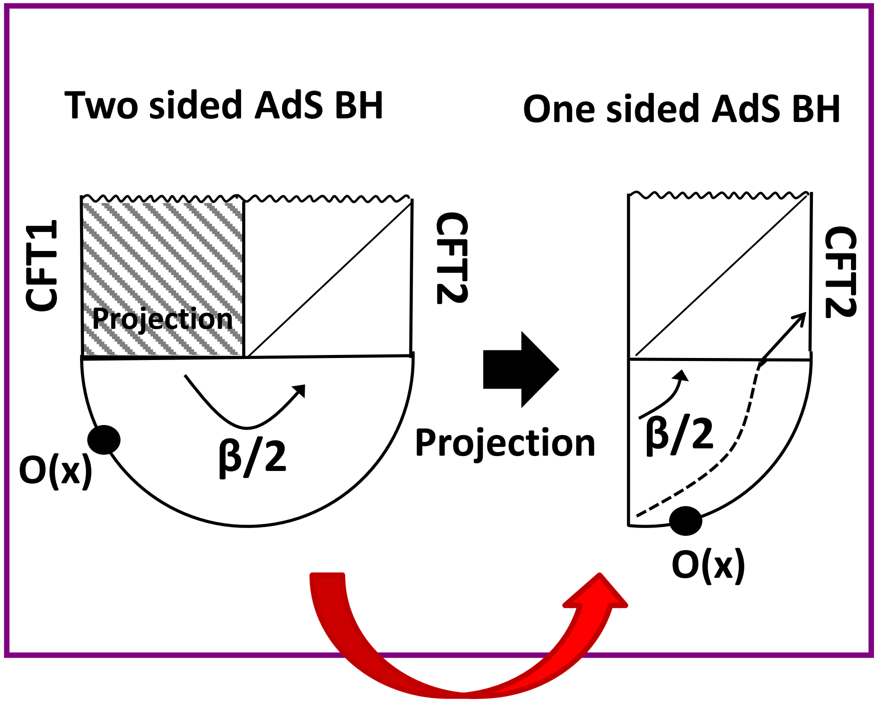
(5-2) Holographic Quantum Teleportation

Partially entangled CFT1+CFT2

⇔ An eternal BTZ black hole [Maldacena 2001 + argument here] .

⇒ The two AdS boundaries are causally disconnected.

After the local projection, a boundary (CFT1) is removed.
⇒ CFT2 can access to O(x) via the Einstein-Rosen bridge.
[See also Susskind 2014 for earlier work]



Collapse of wave functions
⇒Collapse of hol. spacetimes
(thus can change topology)

Eff. temperature reduced by a half

⑥ Conclusions

- We introduced quantum information theoretic operations in CFTs and their holographic duals:

  (i) Local projection
  (ii) Partial Entangling of two CFTs
  (iii) Swapping of two CFTs

  } Lab. for Thought Experiments of QI in CFTs

- We presented a CFT and Hol. model of quantum teleportation. A projection measurement eliminates a part of spacetime. The information is teleported through the Einstein-Rosen bridge.

Future problems

A candidate: $2(S_A - S_A^{\mathrm{Prj}(B)}) - I(A:B)$

♦ Higher dim. generalizations

♦ Multi-partite entanglement measures using projections ?

♦ Explicit analysis of quantum teleportations in CFTs

Thank you very much！

多謝！