6d Superconformal Field Theories

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The progress is based on works of many physicists from `early days':

Witten, Seiberg, Ganor, Strominger, Intriligator, Blum, Hanany, Zaffaroni, Karch, Brunner, Aspinwall, Morrison,…

As well as more recent years (past 3-4 years):

Aganagic, Beem, Bhardwaj, Bolognesi, Cordova, Dumitrescu, Gadde, Haghighat, Gaiotto, Hayashi, Heckman, Hohenegger, Intriligator, Iqbal, H-C Kim, S Kim, S-S Kim, Klemm, Koh, Kozcaz, Lemos, Lockhart, K Lee, S Lee, Morrison, Ohmori, D Park, J Park, Rastelli, Rey, Rudelius, Shimizu, Tachikawa, Taki, Taylor, Tomassiello, van Rees, Xie, Yagi, Yin, Yonekura and Del Zotto…

The plan for this talk is:

Motivation for 6d SCFT's and their basic properties
(2,0) SCFT's and ADE classification
(1,0) SCFT's and their classification via F-theory
Anomaly polynomial and a-theorem
Properties of tensionless strings
Superconformal index
Extension to Little String Theories in 6d
Compactifications to lower dimensions
Future directions

Conformal Theories:

Proven to be important for a deeper understanding of strongly coupled systems. Standalone quantum systems.

Applications: Condensed matter physics
Particle physics
Quantum gravity via holography

Can we classify them?

Do they exist in arbitrary dimensions?

Difficult to prove existence and classification of non-trivial CFT’s in higher dimensions.

However, the assumption of supersymmetry becomes more powerful as we go up in dimension:

The highest dimension for a conformal field theory which enjoys supersymmetry is 6 dimensions.

(related to the fact that conformal group is SO(6,2) and triality of so(8) plays a key role for the existence of superconformal algebra OSp(6,2|N)).

There are two types of superconformal theories in 6d depending on the amount of supersymmetry: N=(2,0) or (1,0) SCFT's.

What we know:

1-They exist! (based on string theoretic arguments)

2-They involve tensionless strings as their basic ingredients.

3-We have no Lagrangian formulation for any of them!

4-(2,0) case is classified by ADE.

5-The (1,0) case has also been classified recently in terms of generalized quivers—more intricate.

6-Many interesting CFT's in lower dimensions can be obtained by compactifying these theories on manifolds of various dimensions.

Conjecturally all lower dimensional CFT's can be obtained from these 6 dimensional theories. Moreover, in this way dualities of lower dimensional theories become geometric symmetries.

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6d SCFT's: Mother of all CFT's and dualities!

Basic algebraic results based on realization of unitary superconformal group [CDI]:

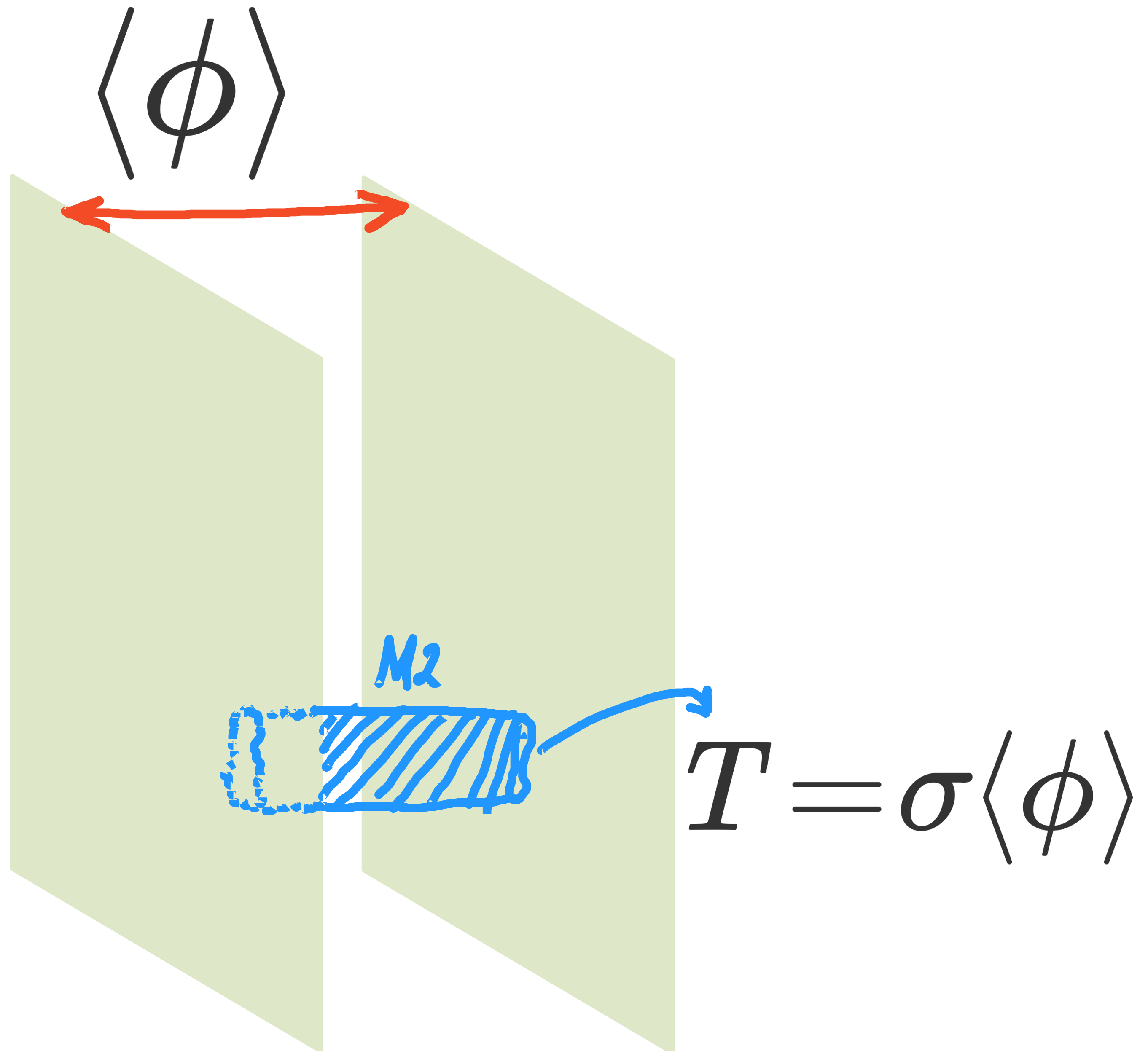
0- OSp(6,2|N) can be realized in QFT for N=1,2 only (existence of stress tensor).
1- The conformal fixed point has no moduli which preserves superconformal invariance.
2- To go away from conformal fixed points we can only give vev to fields— adding operators is not an option.
3- There are two ways to go away: Tensor branch (analog of Coulomb branch in 4d) and Higgs branch.
4- (2,0) theory has SO(5) R-symmetry. No global symmetries.
5- (1,0) theory has SU(2) R-symmetry (which continues to hold on tensor branch). It can also have additional global symmetries.

The (2,0) Case

These are believed to be classified by ADE. The simplest string construction involves the A-case:

The A-type (2,0) SCFT=
IR limit of the 6d-theory living on N-coincident M5 branes. The CFT has no adjustable parameters as expected.

There is a deformation away from the Conformal point:



Further evidence for their existence:

Holographic realization [M] via $AdS^7 \times S^4$

More recent evidence based on bootstrap [BLRvR]:

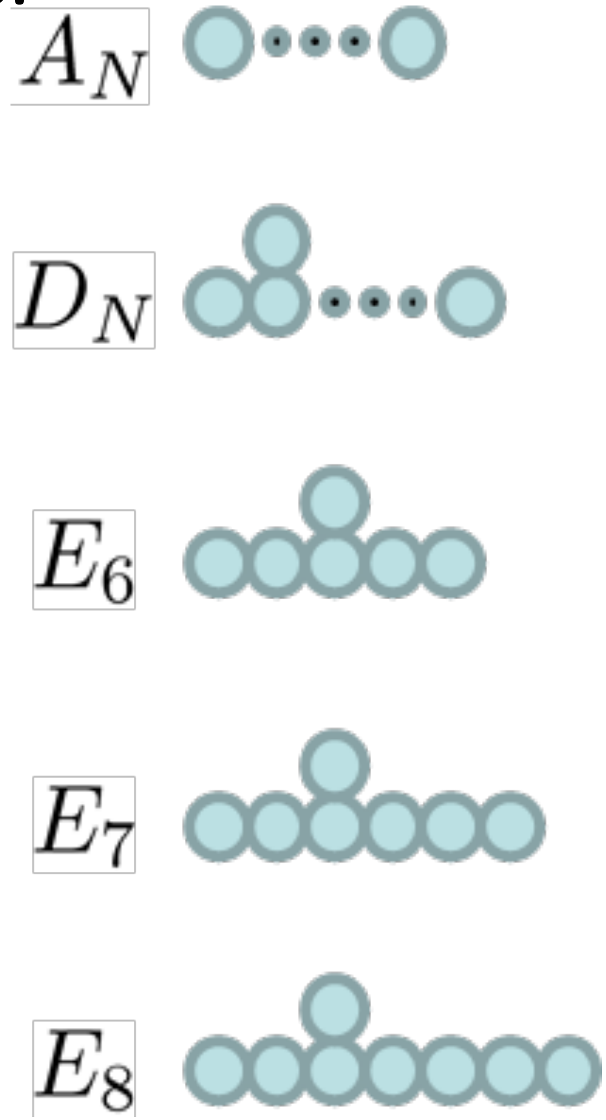
The 2,3 pt function of energy momentum tensor are uniquely fixed by a known parameter c(ADE), which they find using bootstrap methods for the A1 case; strong direct evidence for the power of bootstrap methods to lead to an effective study of these models.

The D and E cases cannot be realized naturally in M-theory. Type IIB is more powerful for this. The IR limit of Type IIB on the singular locus:

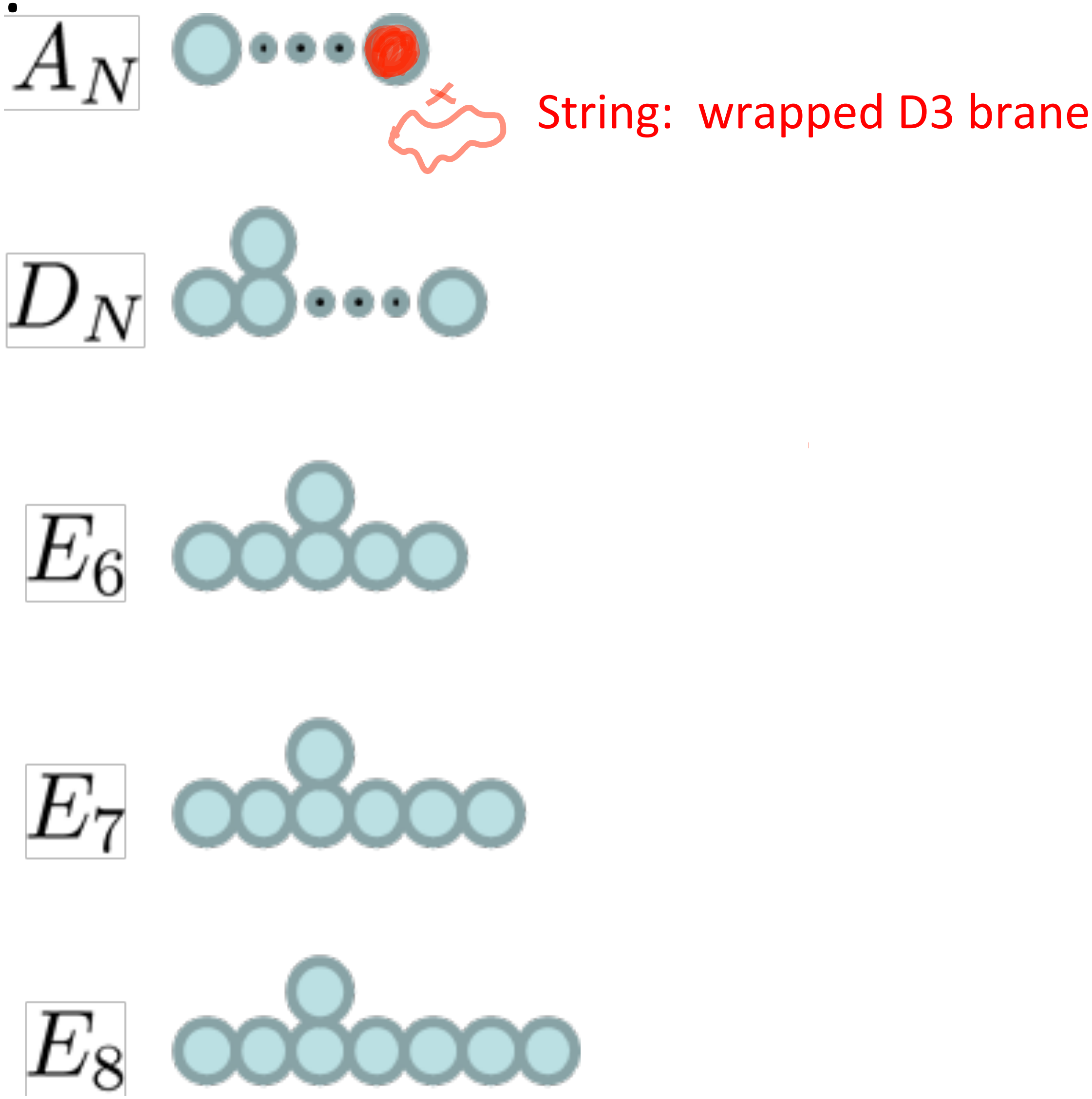
$$\mathbb{C}^2/\Gamma_{ADE}$$

$$\Gamma_{ADE} \subset SU(2)$$

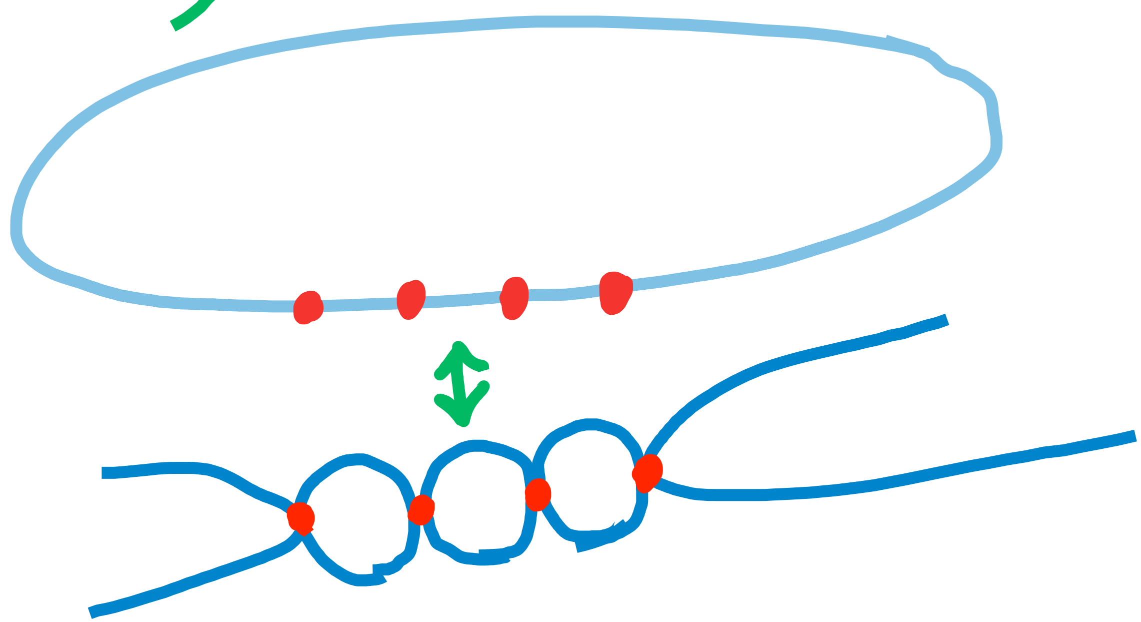
Blow up of ADE-singularity. The sizes of spheres dual to scalar vevs.



Blow up of ADE-singularity: The sizes of spheres dual to scalar vevs.



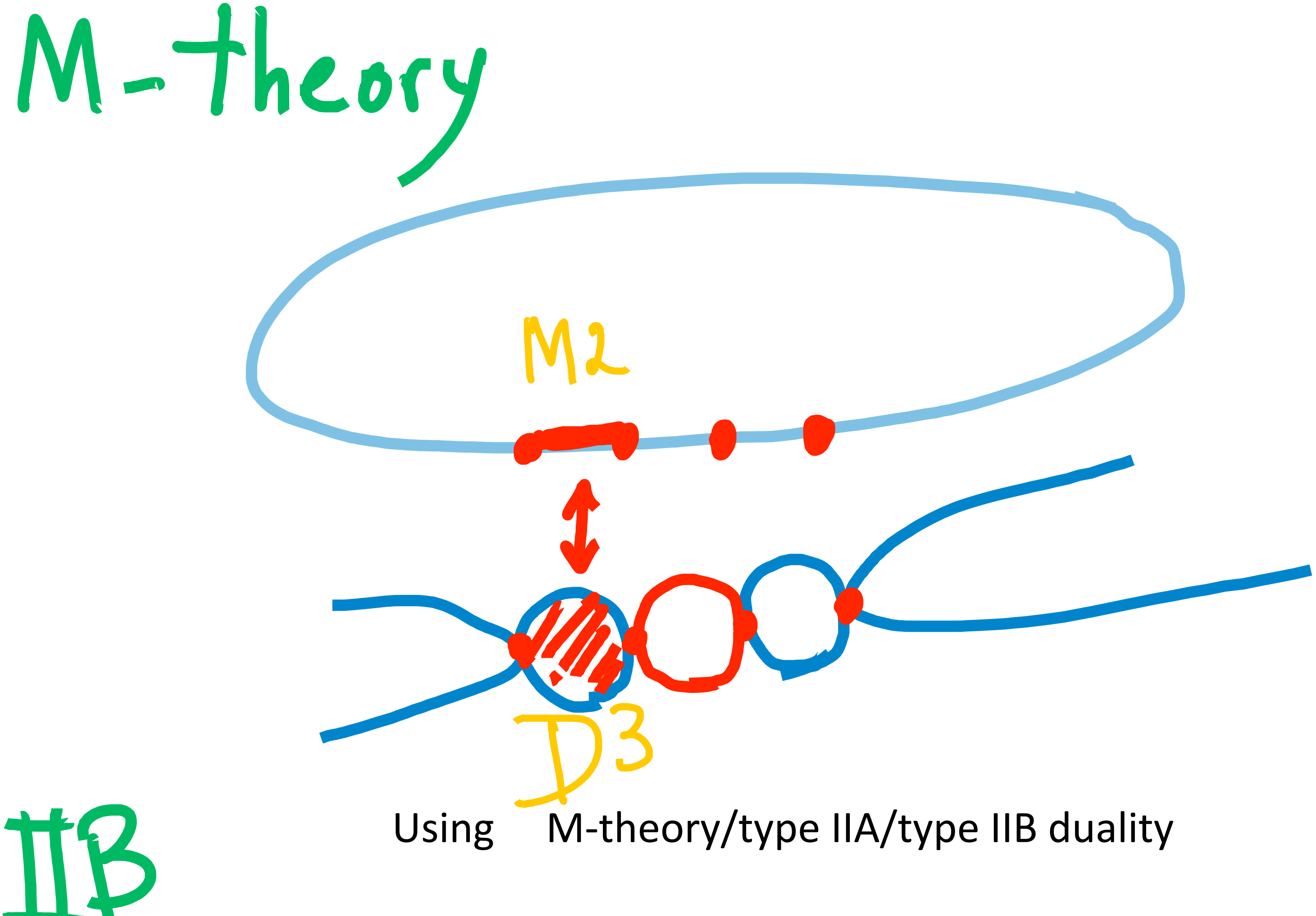
M-theory



IIB

Using M-theory/type IIA/type IIB duality

M5--->NS5--->TN



Using M-theory/type IIA/type IIB duality

M2--->D2--->D3

(1,0) SCFT's

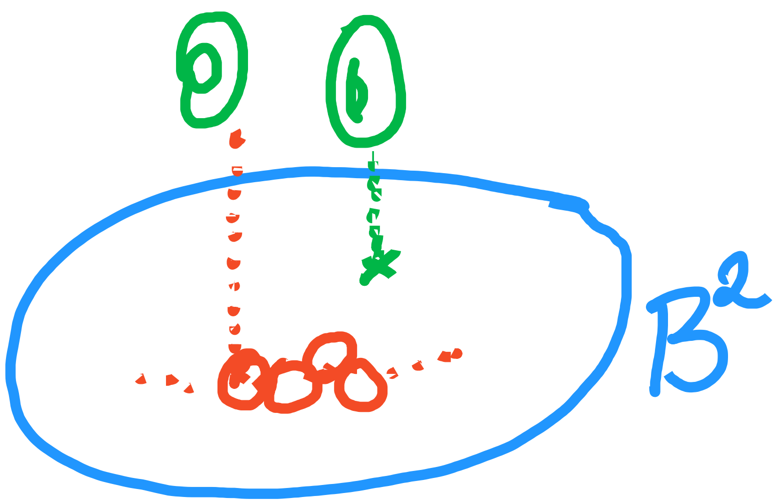
These theories have half as much SUSY.

Classifiable!

Idea: Based on realization of them in F-theory.

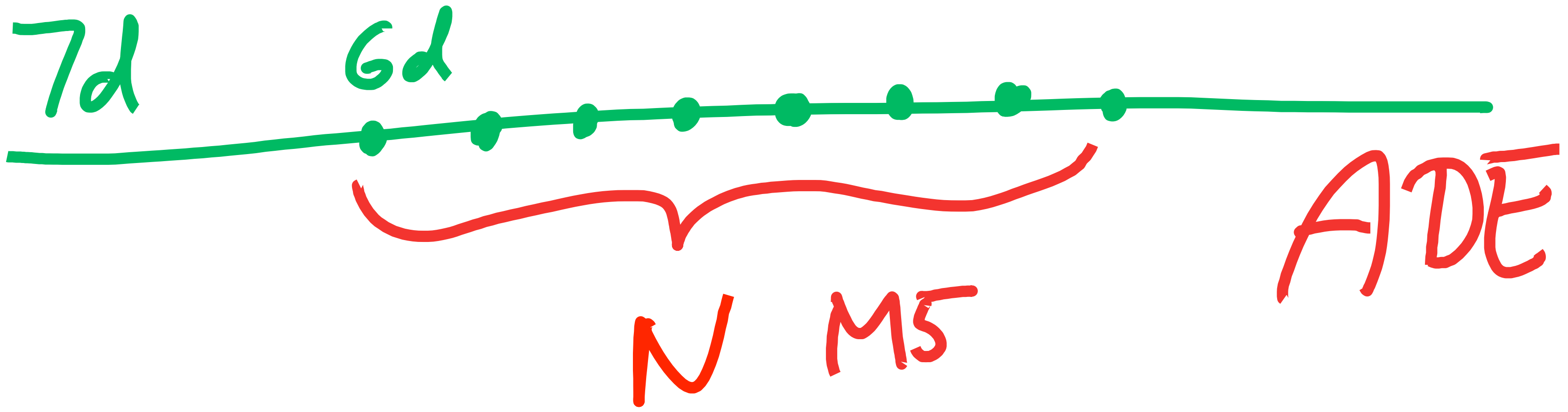
Dual to M-theory constructions in some examples.

Elliptic 3-folds:



To gain insight, let us consider a few examples first:

M-theory: N M5 branes probing ADE singularity



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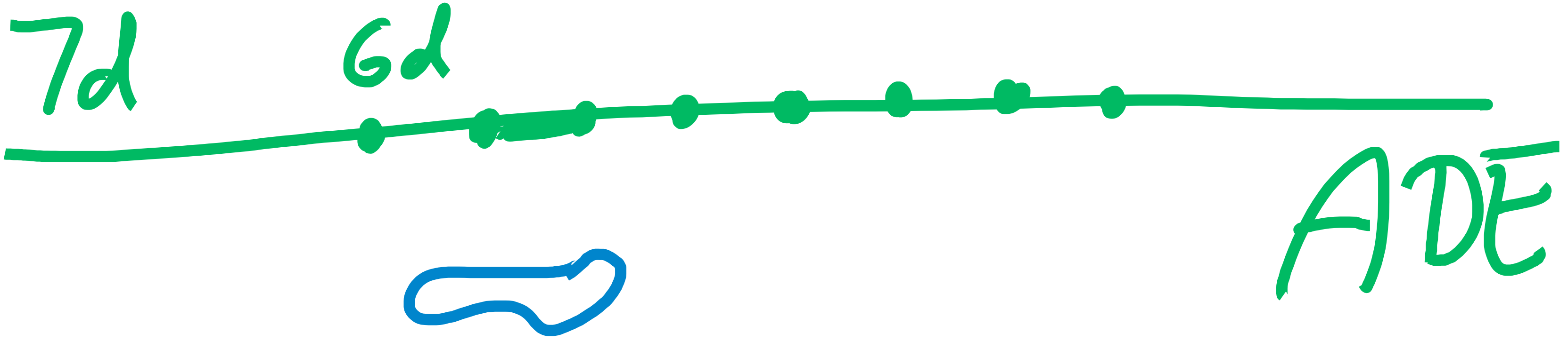

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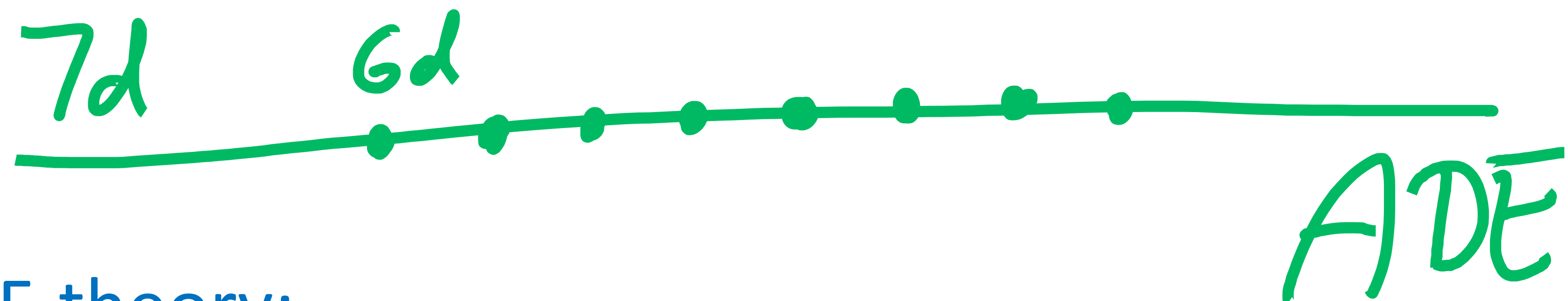
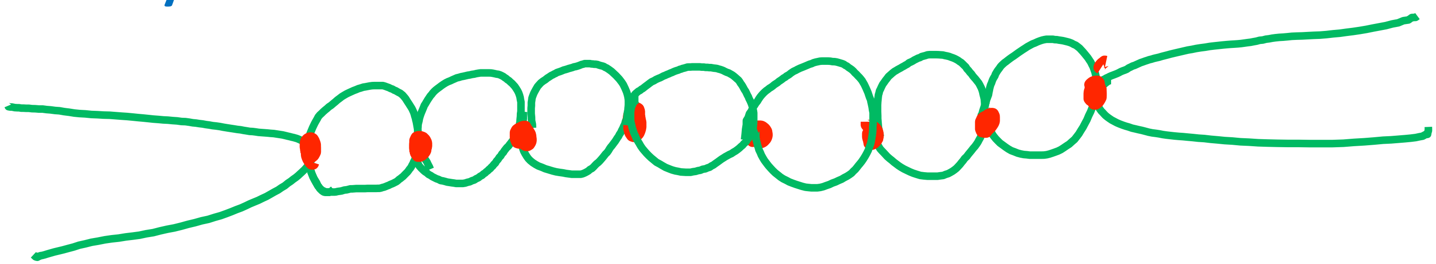


F-theory:



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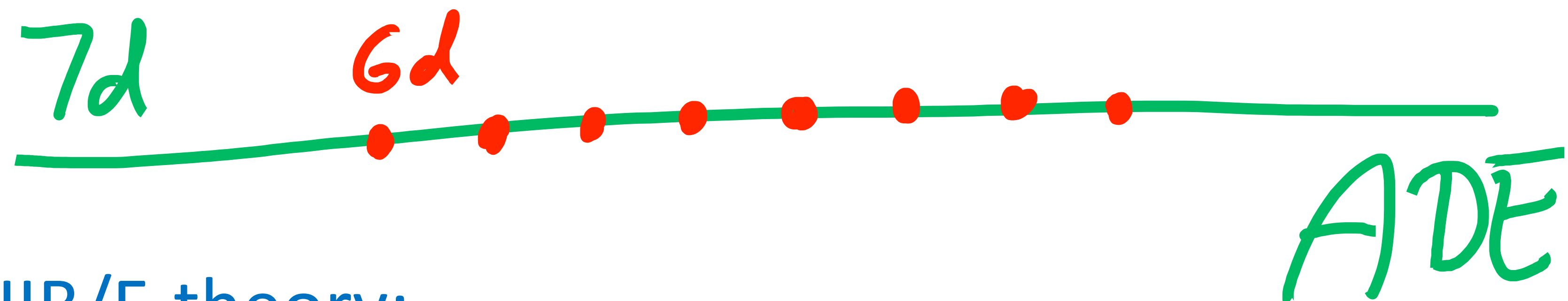
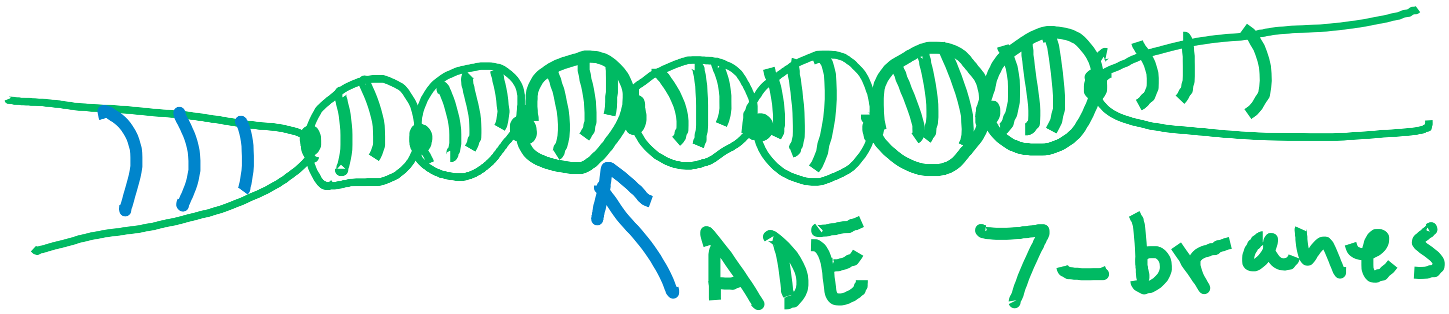


IIB/F-theory:



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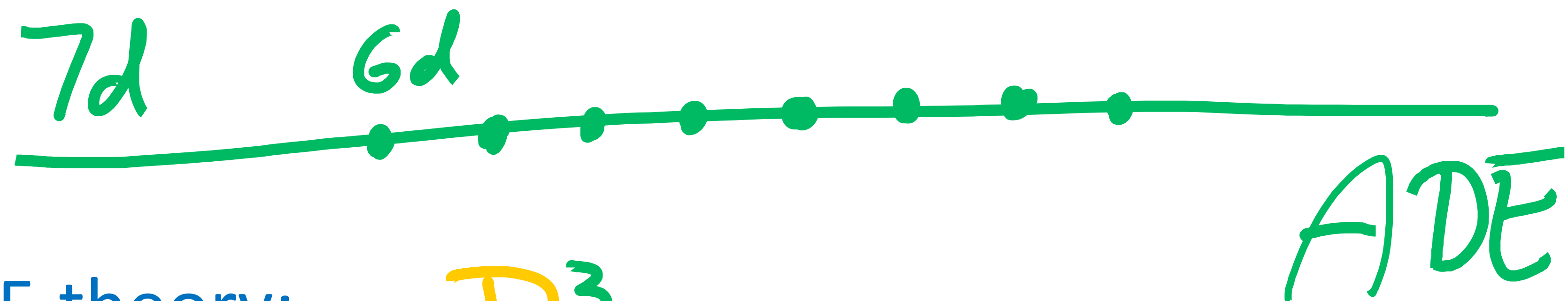
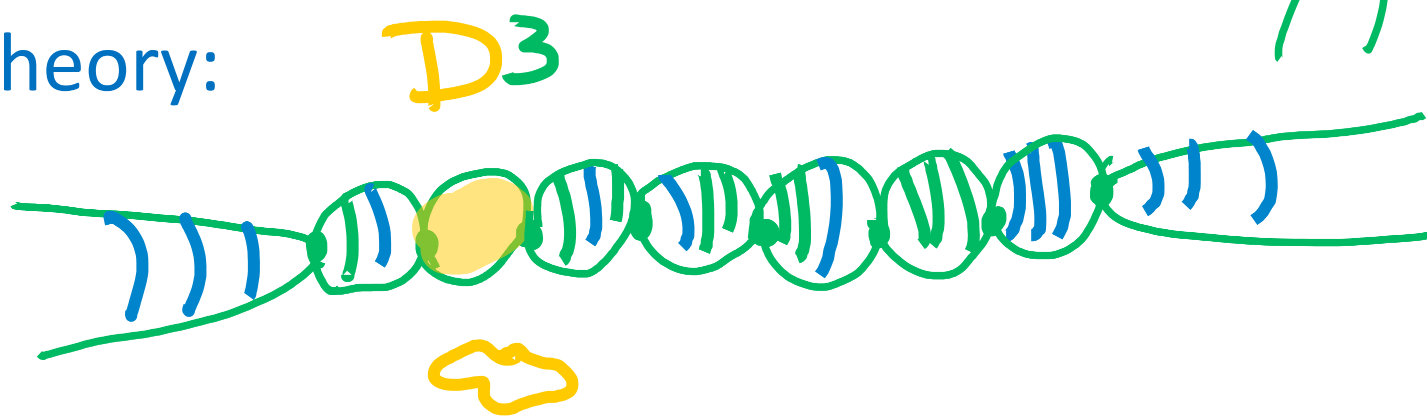
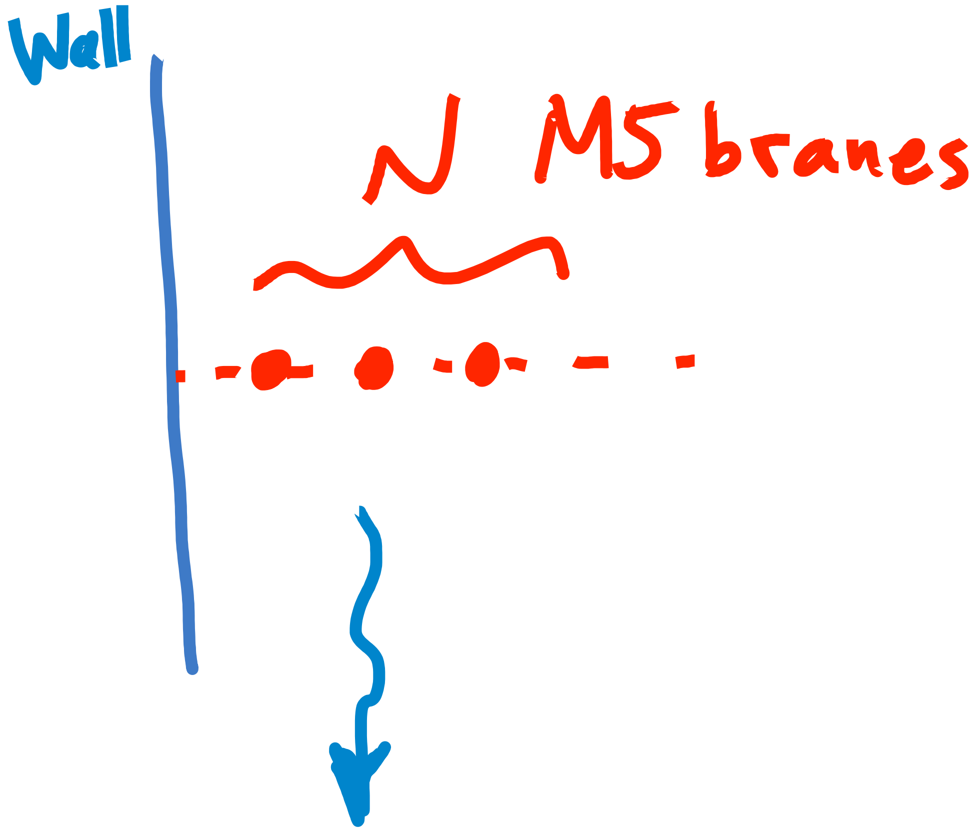


F-theory:



Another example: M5 branes at the Hořava-Witten wall

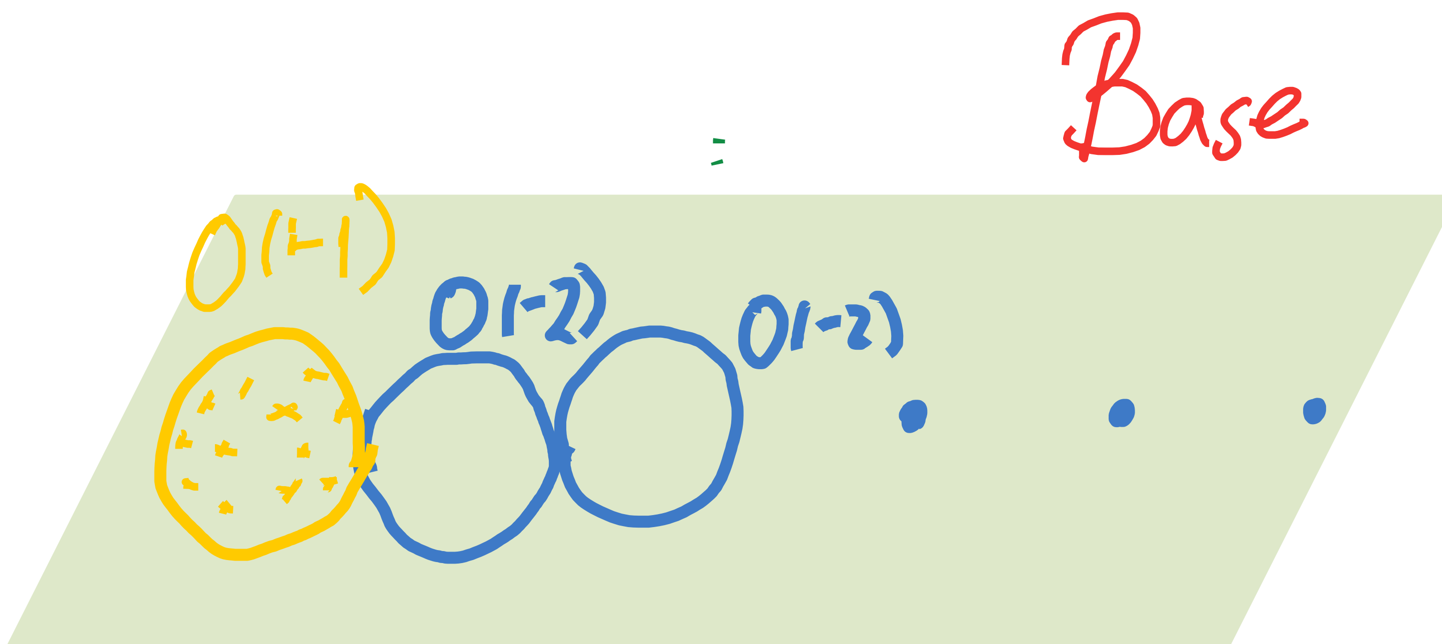


F-theory:

This (1,0) theory:

O(-1) curve, unlike the O(-2) curve, when shrunk leaves no imprint of singularity in the base.
122 corresponds to 3 blow ups of the base.

F-theory:



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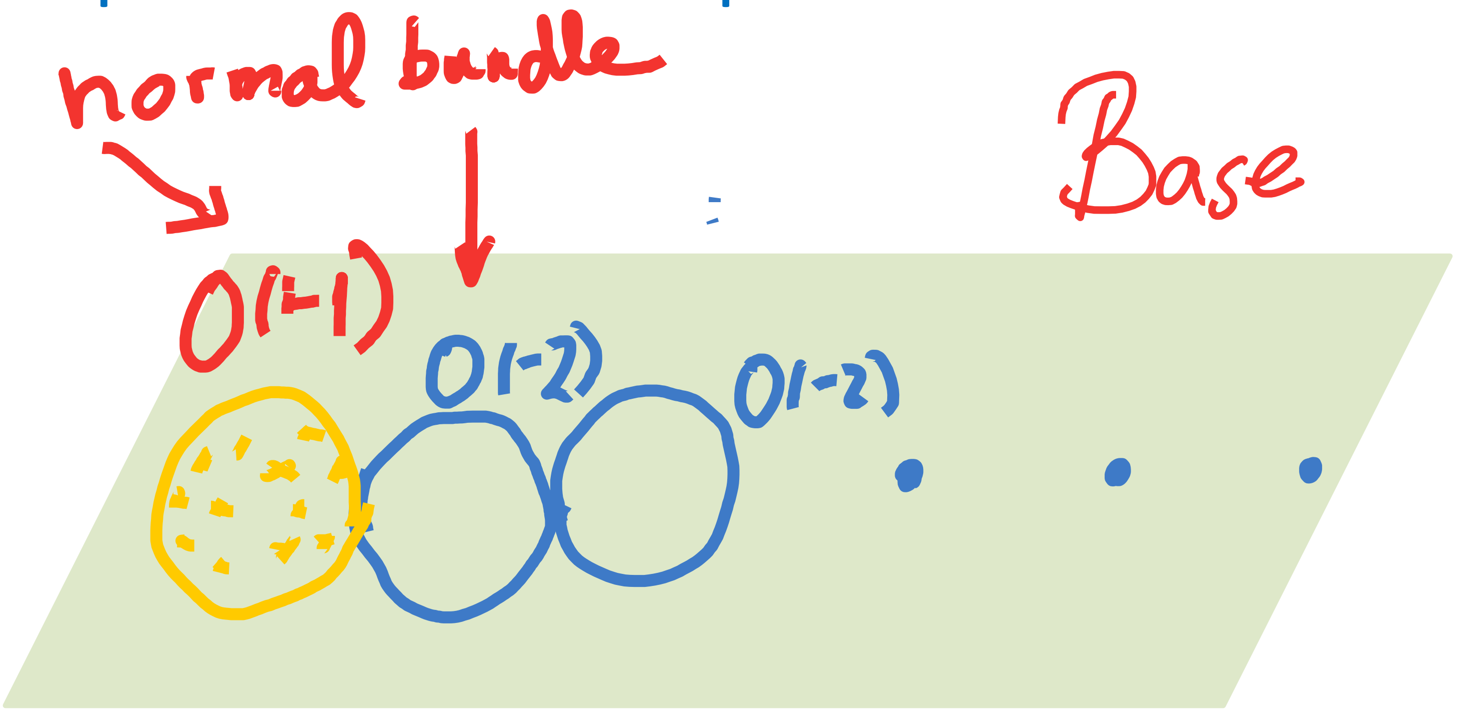

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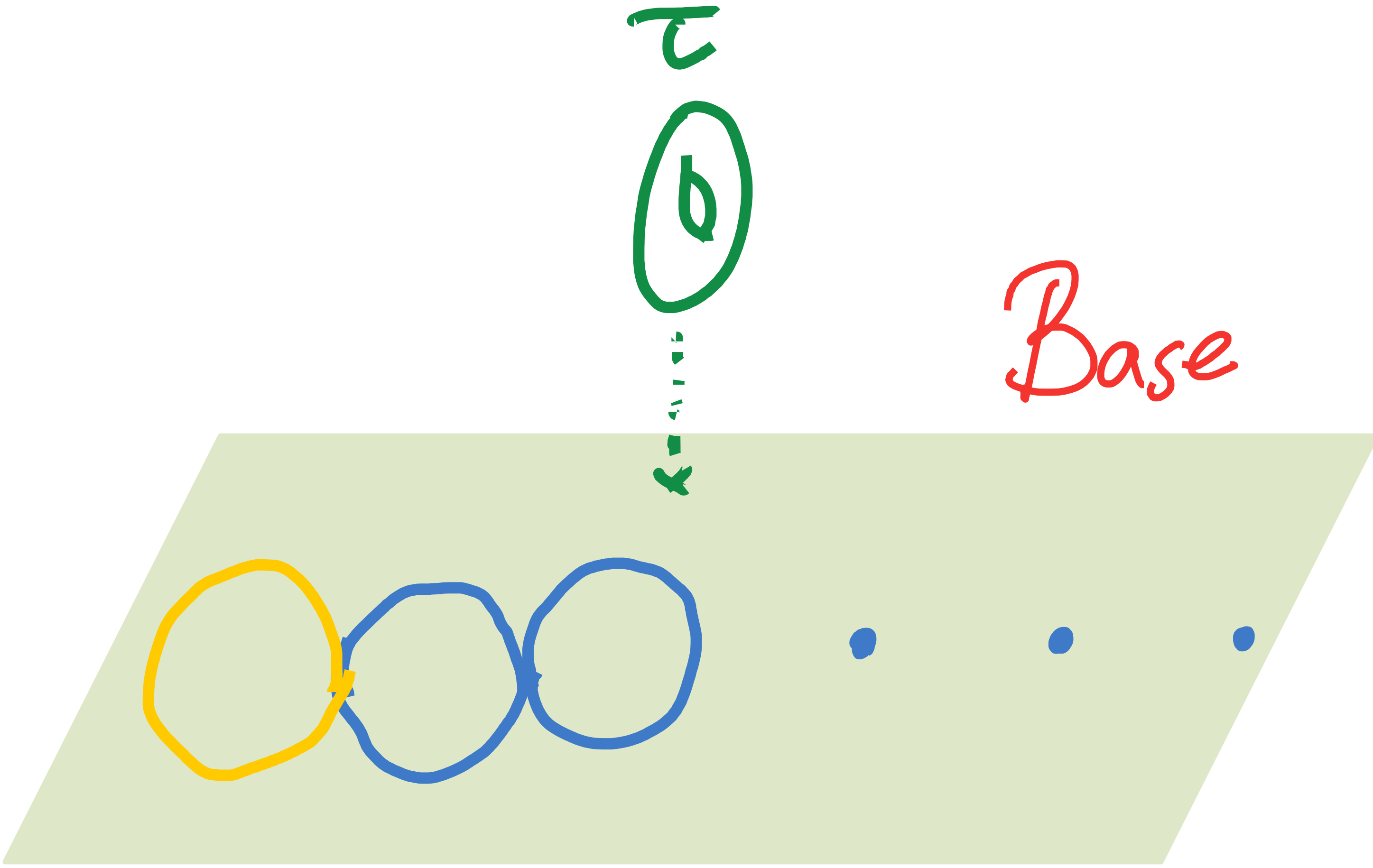

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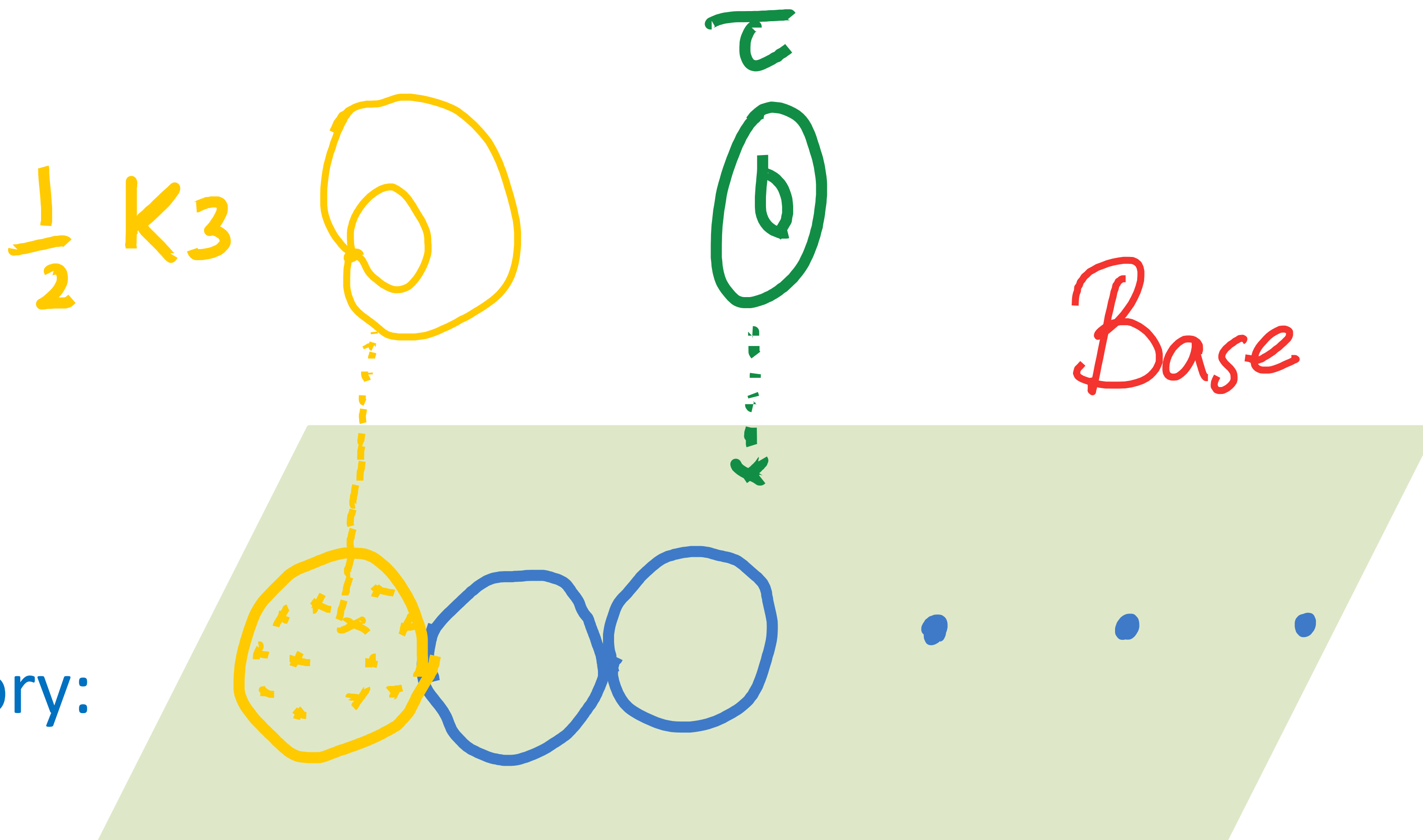

Another class of examples [W]:

F-theory on orbifolds

$$T^2 \times \overbrace{\mathbb{C} \times \mathbb{C}}^{\text{Base}} \Big/ Z_n$$

$$Z_n : \quad (\alpha^{-2}; \alpha, \alpha); \quad \alpha^n = 1$$

$$n = 2, 3, 4, 6, 8, 12$$

Another class of examples [W]:

F-theory on orbifolds $\mathcal{O}(-n) \to \mathbb{P}^1$: (n)

$$\left(T^2 \times \overbrace{\mathbb{C} \times \mathbb{C}}^{\mathcal{O}(-n)\to\mathbb{P}^1}\right) \Big/ \mathbb{Z}_n$$

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$$\mathbb{Z}_n : \quad (\alpha^{-2}; \underbrace{\alpha, \alpha})$$

Not $SU(2)$ but $U(2)$

$n = 2, 3, 4, 6, 8, 12$

$-n$ curve: $\mathcal{O}(-n) \to \mathbb{P}^1$	gauge symmetry on the $\mathbb{P}^1$
3	$\mathfrak{su}_3$
4	$\mathfrak{so}_8$
5	$\mathfrak{f}_4$
6	$\mathfrak{e}_6$
7	$\mathfrak{e}_7 + (1/2)$ hyper
8	$\mathfrak{e}_7$
12	$\mathfrak{e}_8$

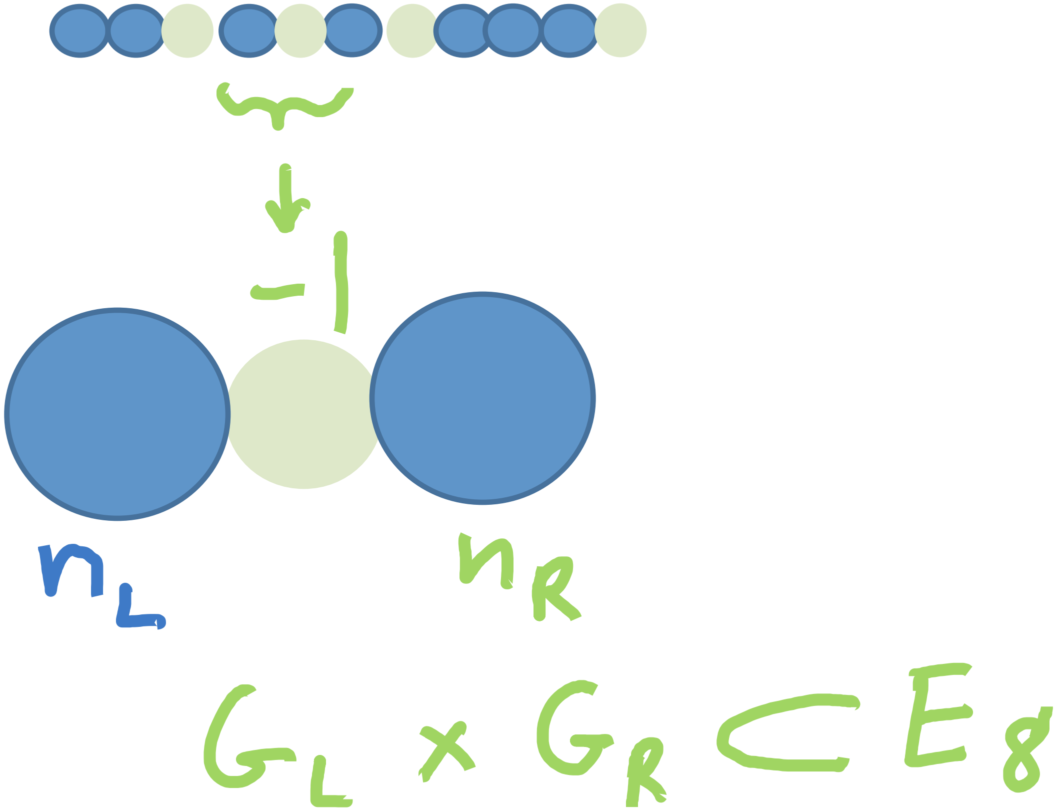
All the known examples of (2,0) and (1,0) CFT's can be realized in F-theory, which motivated:

Basic classification strategy [HMV,HMRV,B]-
Classify all the possible bases that can appear in F-theory, up to blow ups
and adding 7-branes wrapping the cycles
(i.e. classify the maximally higgsed phase).
Surprising result: All allowed endpoints correspond to orbifold singularities:

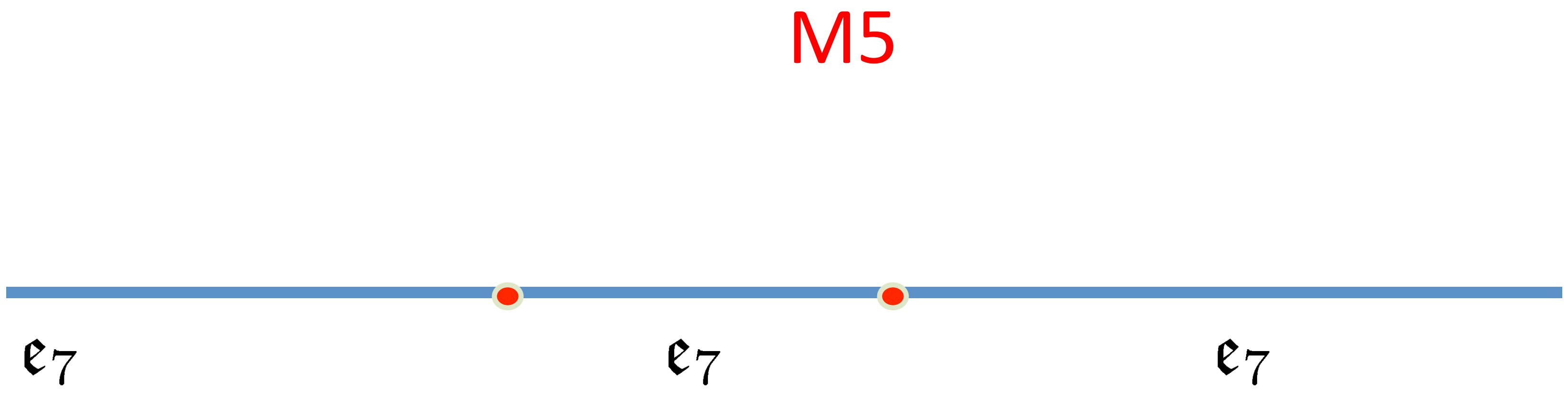
$$\mathbb{C}^2/\Gamma \qquad \Gamma \subset U(2)$$

for special subgroups.

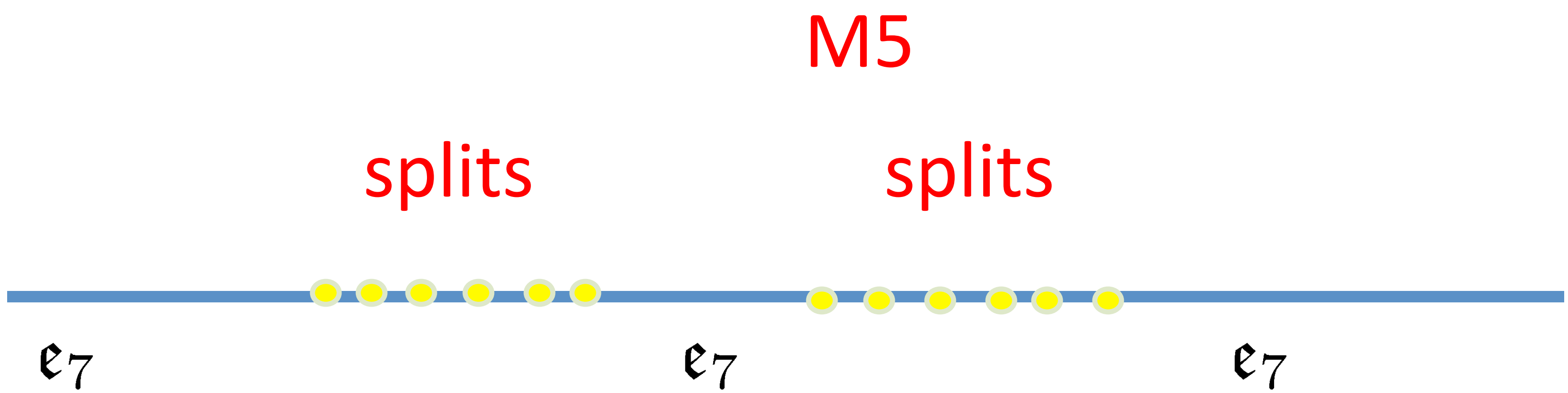
Each of these end points leads to a canonical tensor branch (which require additional blow ups for the elliptic 3-fold singularities) leading to a final geometry of blow ups



M-theory interpretation: M5 branes probing D,E singularities. For example:



M-theory interpretation: M5 branes probing D,E singularities. For example [DHTV]:



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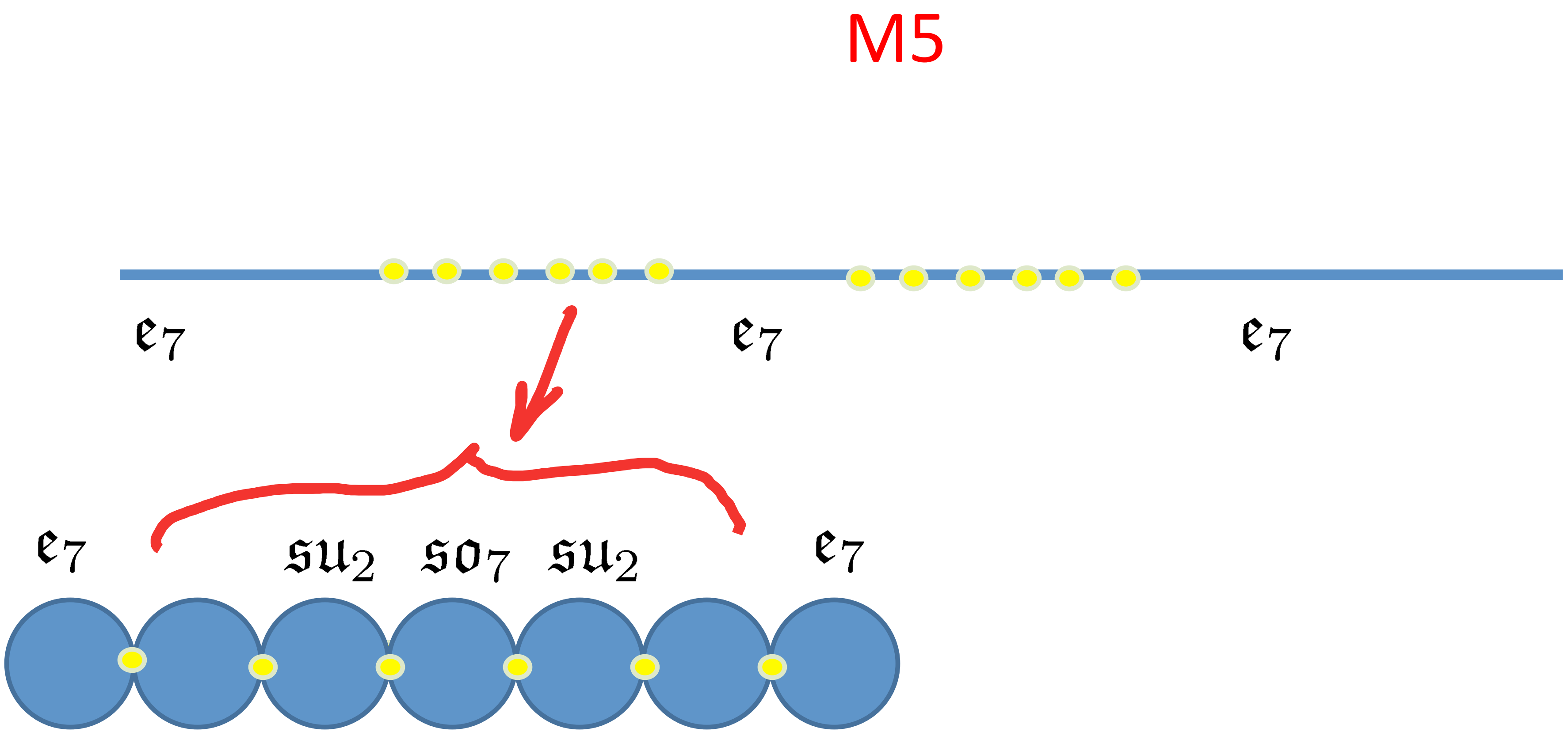
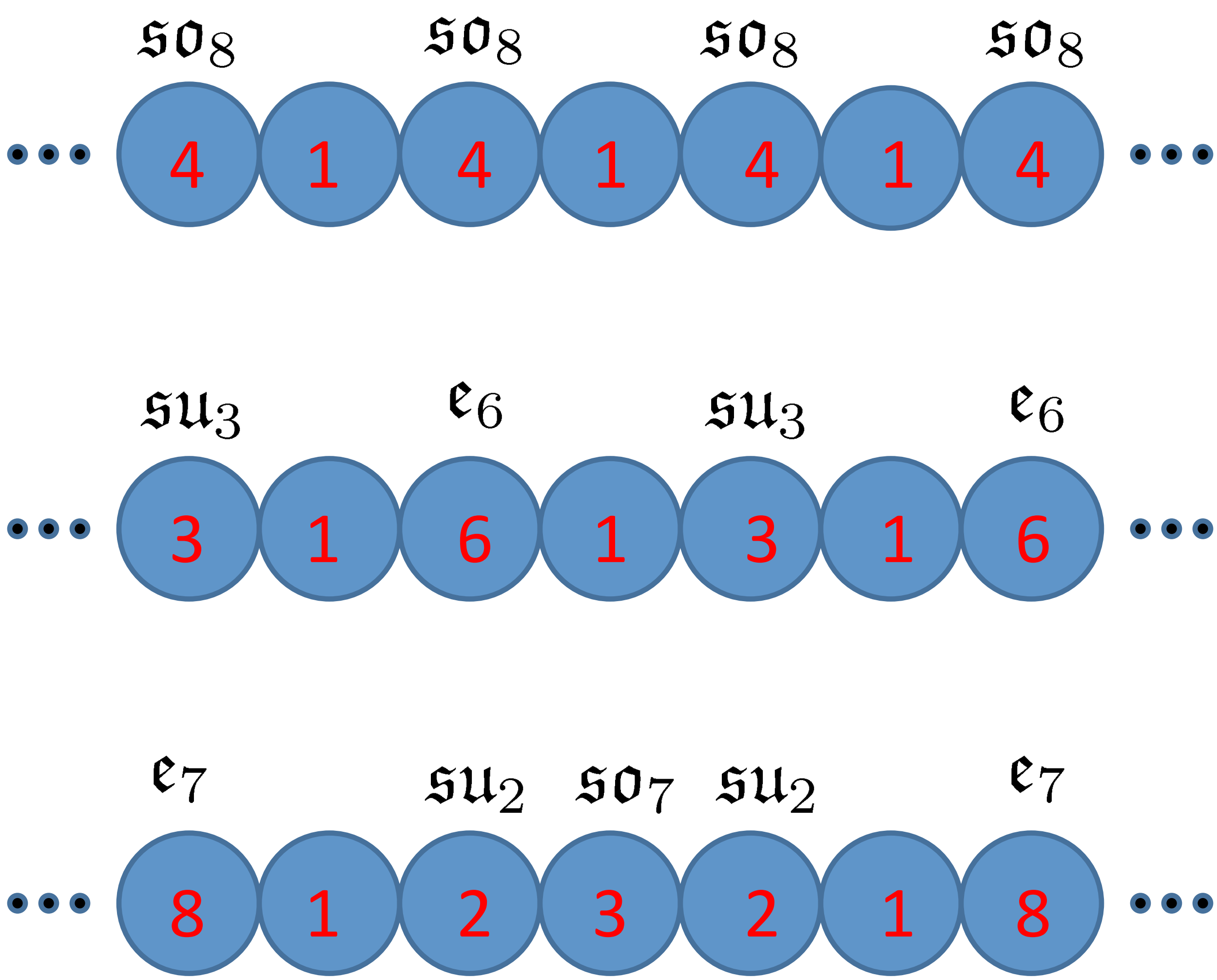
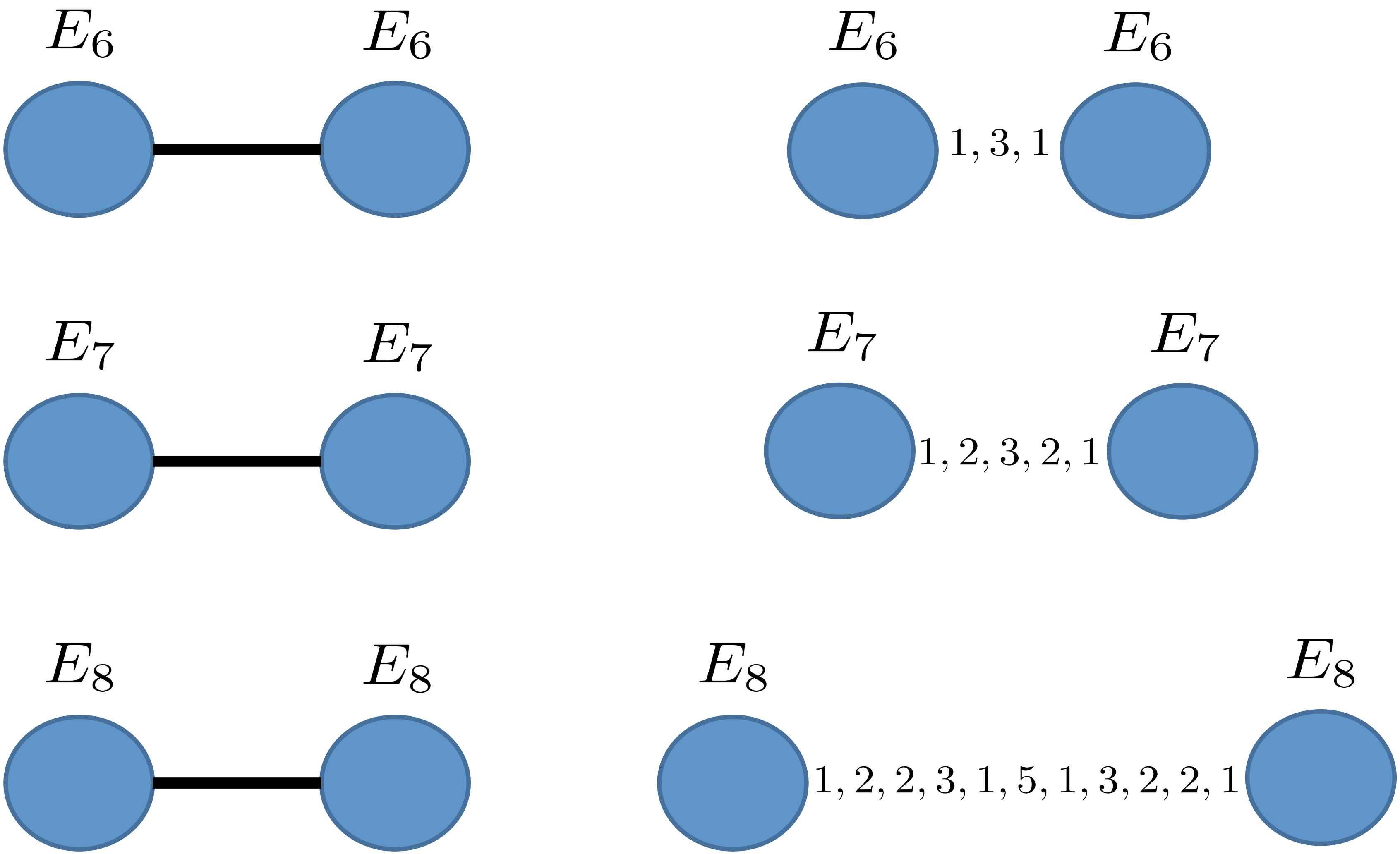


bifundamental `E7 matter' =SCFT

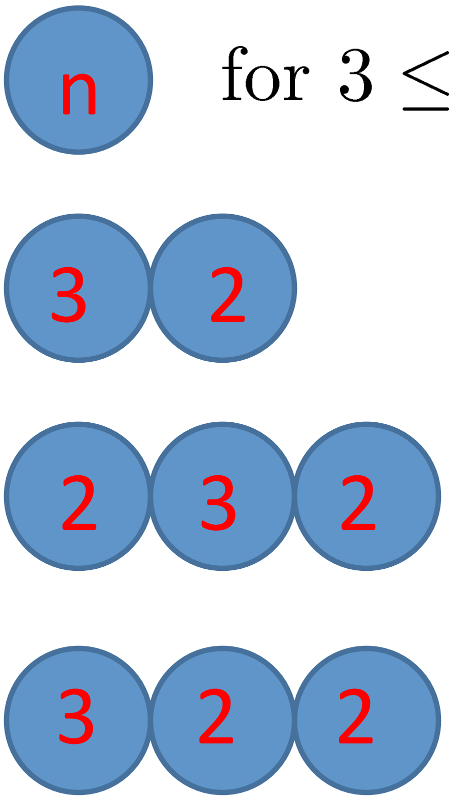
Emergence of repeating patterns



E-Type Quivers

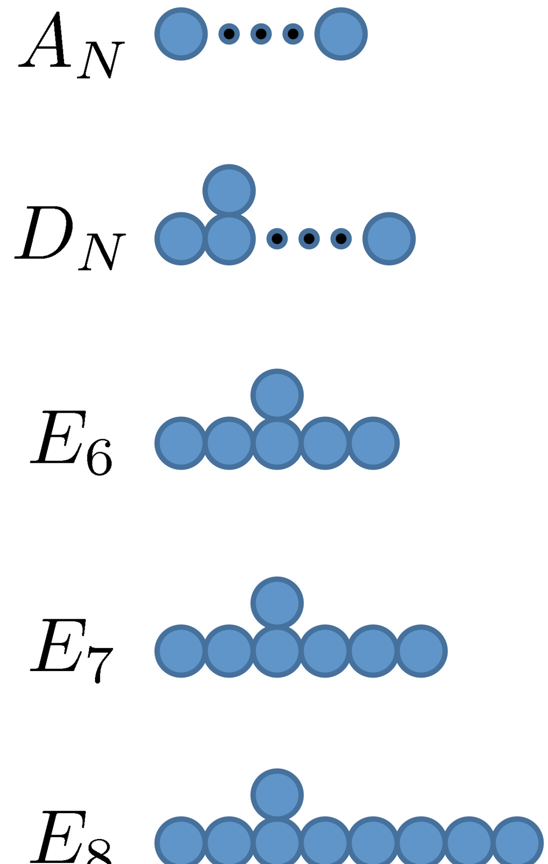


Building Blocks



for $3 \leq n \leq 12$

Non-Higgsable Clusters [MT]



The idea behind classification of 6d theories from F-theory perspective:

The intersection structure of the curves (string charge lattice) forms a positive definite quadratic form: Kinetic term of the scalars is positive definite.

The diagonal structure is special and intersection of nearby curves are also very special (dictated by non-Higgsable clusters).

Useful Terminology: I / II

Split Up NHCs into two groups:

$\mathbf{I}^l = 1, 2, ..., 2$

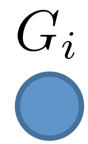
"instantons"

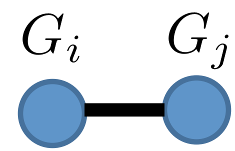
			$\frac{1}{2}\mathbf{56}$		$\mathbf{I}^3$	$\mathbf{I}^2$	$\mathbf{I}^1$	
	$\mathfrak{so}_8$	$\mathfrak{e}_6$	$\mathfrak{e}_7$	$\mathfrak{e}_7$	$\mathfrak{e}_8$	$\mathfrak{e}_8$	$\mathfrak{e}_8$	$\mathfrak{e}_8$
DE-type:	4,	6,	7,	8,	9,	10,	11,	12

non-DE-type: 1, 2, 3, 23, 232, 223, 5

Useful Terminology: II / II

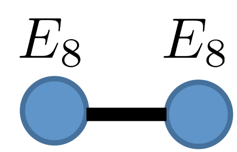
Define a Base Quiver by minimal fiber types:

Nodes: DE-type curves 

Links: Connecting DE-type curves 

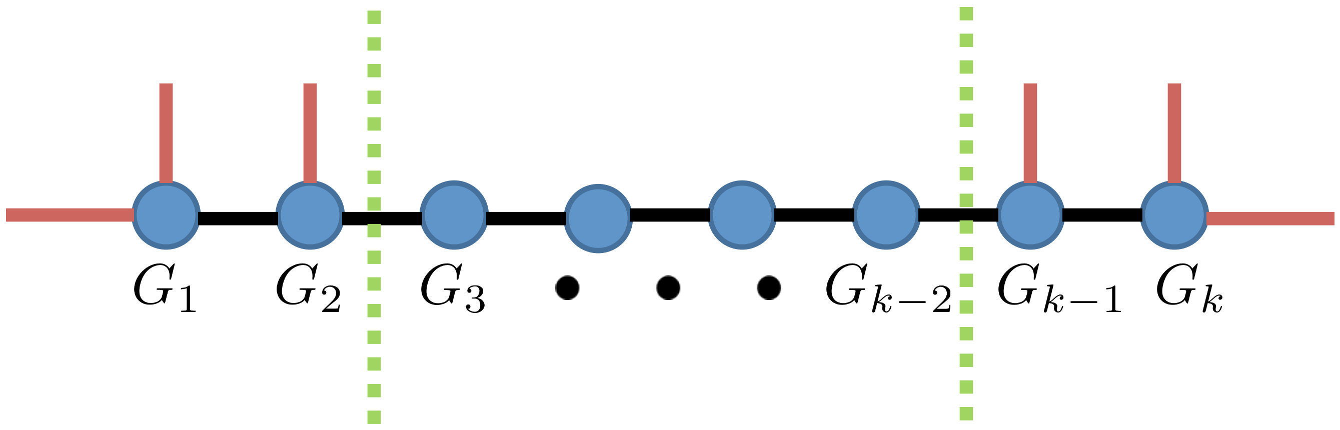
Example:

$$(12), 1, 223, 1, 5, 1, 322, 1, (12)$$



The Big Surprise

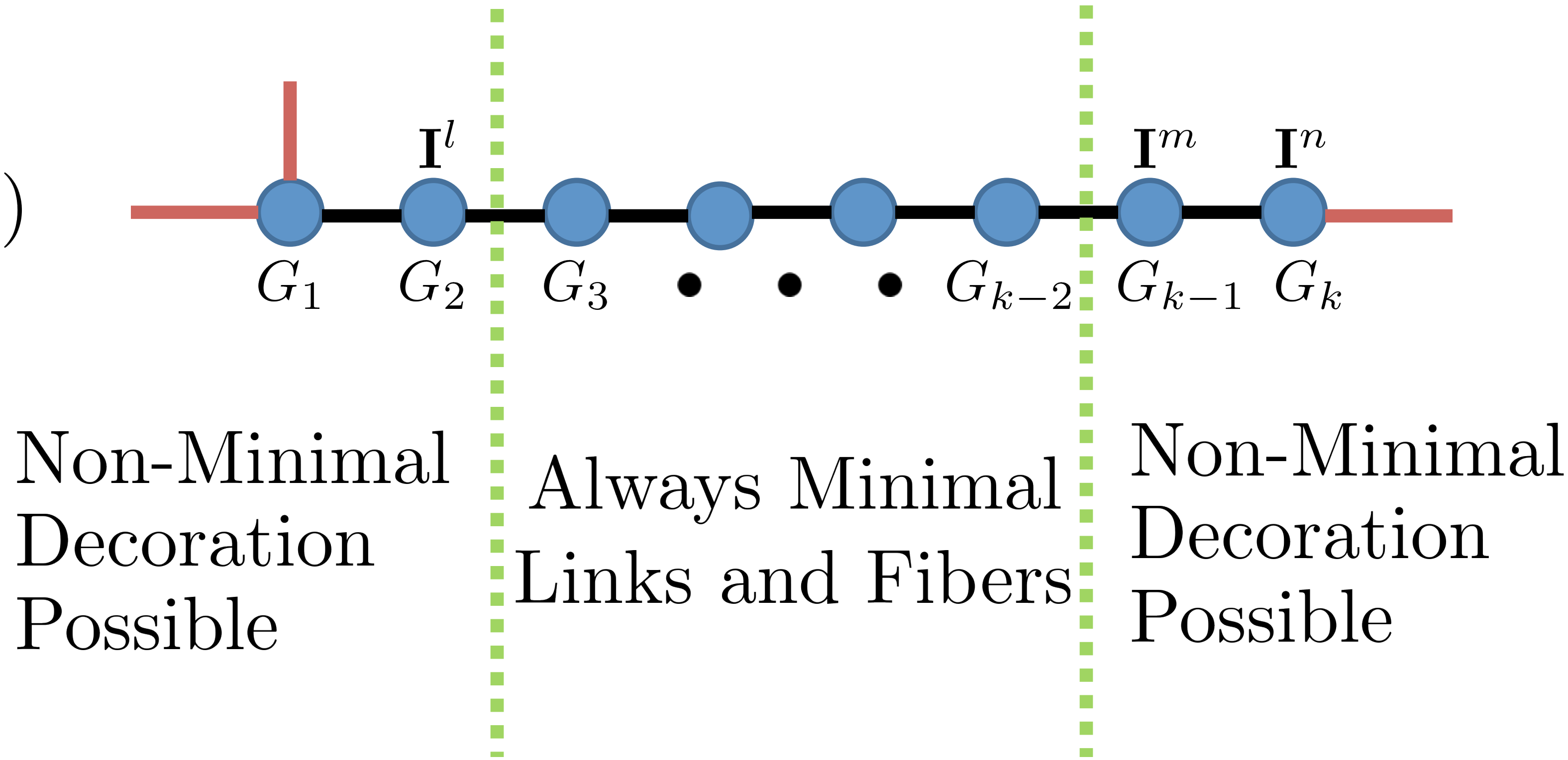
The Base Quivers have a *very* simple structure!



$$G_1 \subseteq G_2 \subseteq \cdots \subseteq G_m \supseteq \cdots \supseteq G_{k-1} \supseteq G_k$$

More Results...

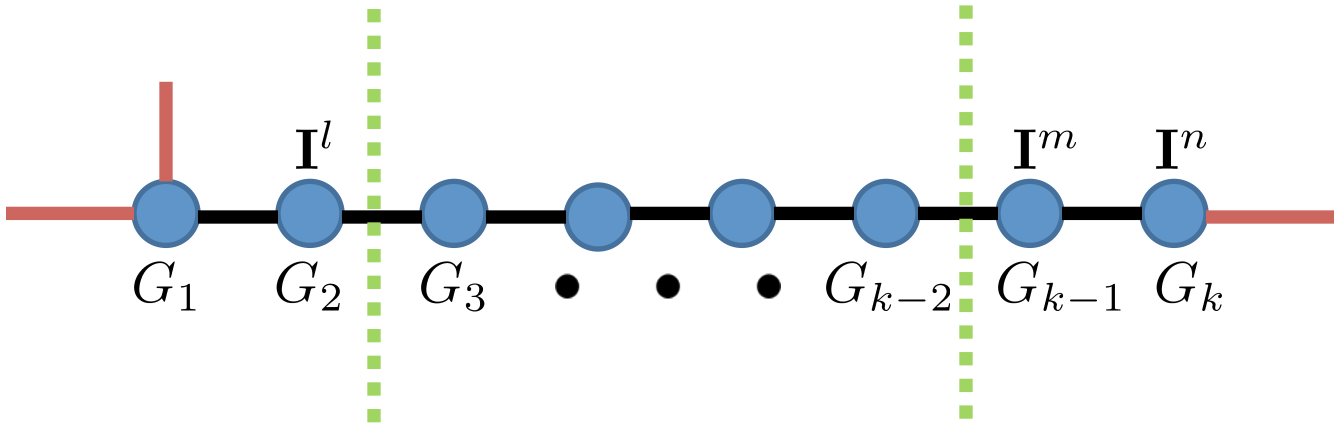
1)



$$\mathbf{I}^l = 1, 2, ..., 2$$

More Results...

1)

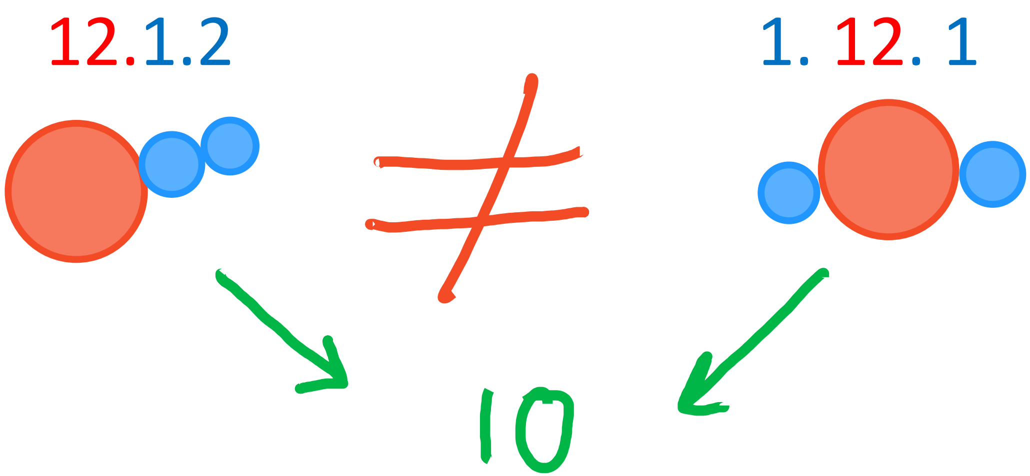


2) Classification of all possible links

3) Classification of all ways to wrap 7-branes

Dualities?

One may wonder whether any of the above theories are dual to one another. Evidence was found [HMRV] that there are no dualities among these theories extending the known fact for the (2,0) class to (1,0) case. An example:



Anomaly Polynomials

The 6d SCFT's are chiral and thus anomaly is an interesting aspect of them. Their coupling to gravity background and SU(2) R-symmetry will involve the 8-form anomaly polynomial:

$$I_8 = \frac{1}{4!}\left(\alpha c_2^2 + \beta c_2 p_1 + \gamma p_1^2 + \delta p_2\right)$$

The coefficients of the anomaly polynomial has been determined systematically [OSTY,I,CDI] for the quiver theories classified.

RG Flow and a-theorem in 6d

A generalization for an a-theorem has been proposed [C] which for even dimensions involves

$$\left\langle T^{\mu}_{\mu} \right\rangle = a E_d + \ldots$$

It is believed that a>0 and decreases along RG-flow.
There is no proof of this in 6d, unlike in 2d, 4d.
For supersymmetric theories one can expect `a' to be related to anomaly polynomials (as in 2d, 4d).

Based on various consistency arguments it was proposed [CDI] that for SCFT's in 6d `a' can be computed from anomaly polynomials

$$a = \frac{16}{7}(\alpha - \beta + \gamma) + \frac{6}{7}\delta$$

Many checks were made that this indeed decreases along RG-flows [HR].

Tensionless Strings

Contrast with fundamental strings:

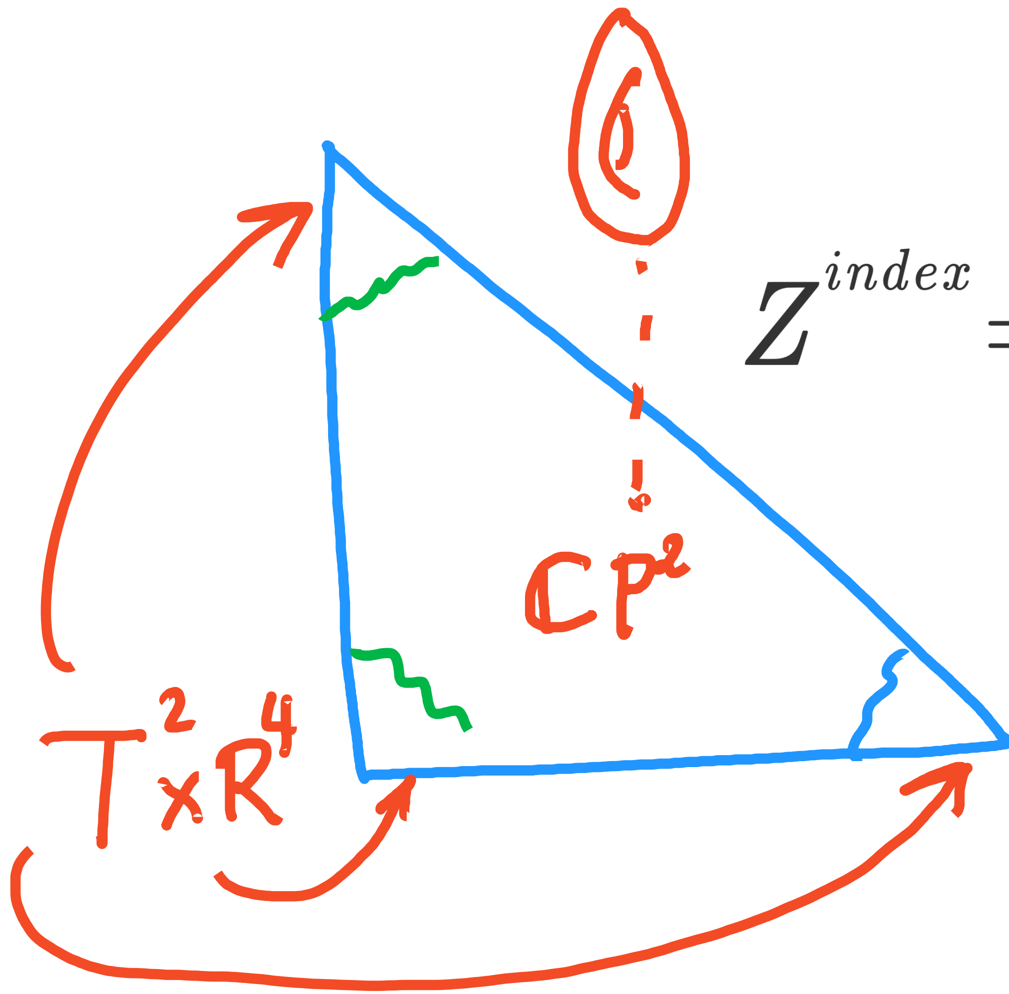
	Fundamental Strings	6d strings
Tension:	finite	finite or zero
Coupling const.	Yes	No
Quantization	Lorentz invariant	?

6d strings give rise to BPS states, and they enjoy (4,4) (in (2,0) case) or (4,0) SUSY (in (1,0) case) on their worldsheet.

One can ask what this 2d SUSY system is? It turns out that string duality can be used to find what are the IR degrees of freedom for such a string in many cases, which can be used to compute SUSY partition functions of it (and in particular on torus): $T^2 \times R^4$

Moreover the elliptic genus of these strings can be used to compute the superconformal index at the CFT point [LV,I,KKK]:

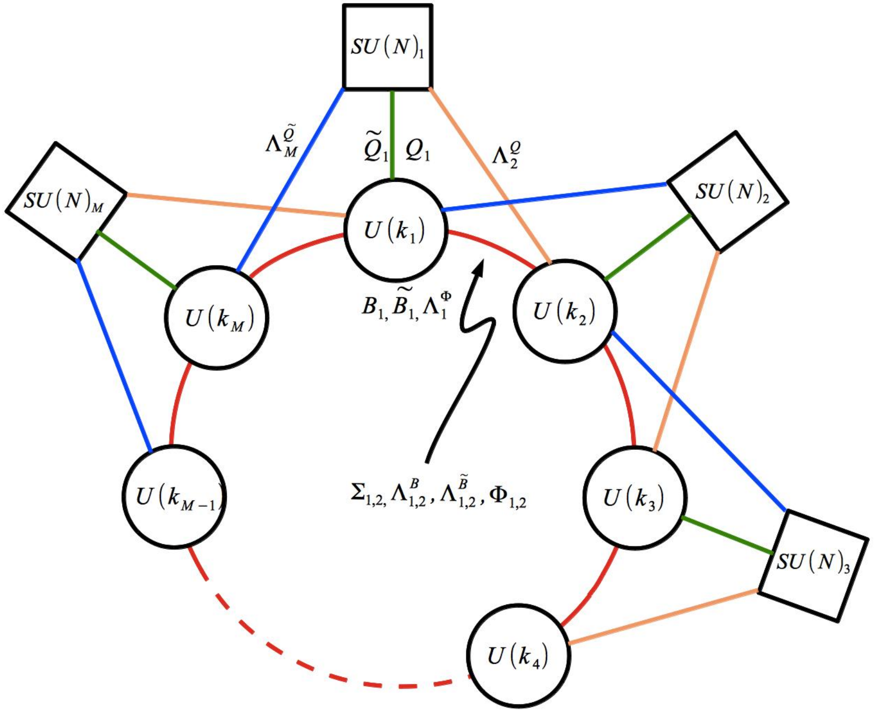
$$S^1\times S^5:\ \underbrace{S^1\times\left(L_{S^1}}_{T^2}\to CP^2\right)$$



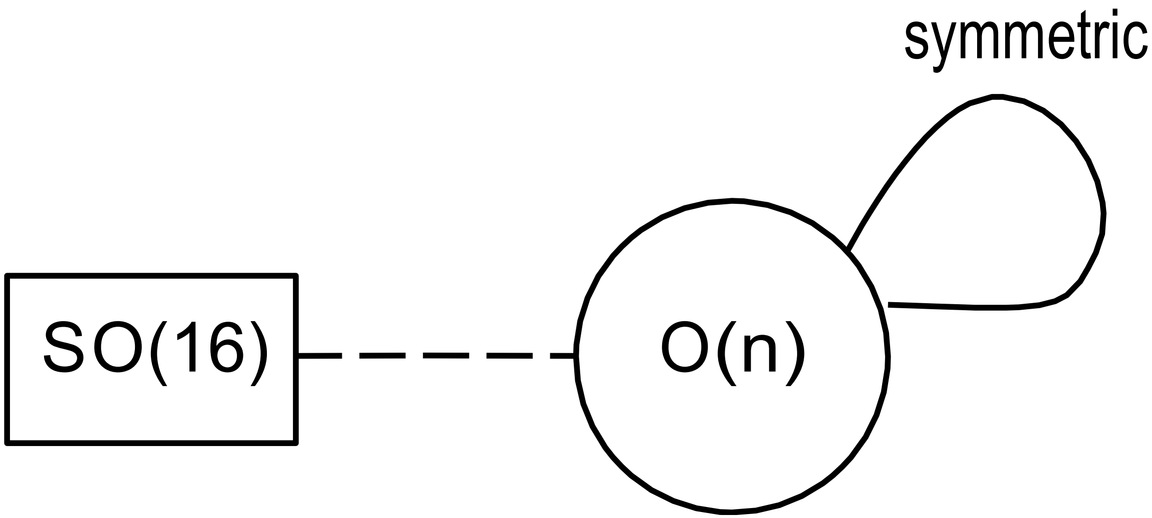
$$Z^{index}=Z_T\cdot Z_T\cdot Z_T$$

Examples:

The 2d (4,0) SUSY: quiver theory describing the associated strings for the M5 branes probing A-type Singularity [HKLV,HI].



For E-strings (the strings for the O(-1) SCFT) the quiver for n E-strings is given by [KKKPV]



Note that the n=1 is the familiar way the E8 current algebra arises in heterotic string

One learns unlike fundamental strings, tensionless strings do form bound states which is reflected in the partition function of 2 strings on torus;

$$Z_2(\tau) \neq \frac{1}{2}\left[Z_1^2(\tau) + Z_1(2\tau) + Z_1\left(\frac{\tau}{2}\right) + Z_1\left(\frac{\tau+1}{2}\right)\right]$$

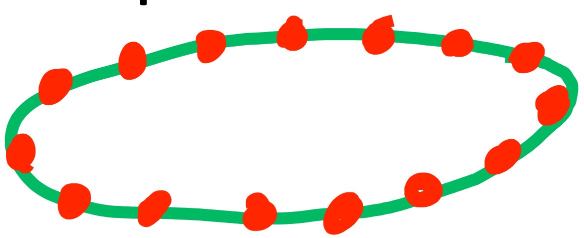
There are interesting probe strings for these theories which are classified by [DHPR]:

$$G = \Lambda^{*} / \Lambda$$

Where Λ is the lattice of string charges. They correspond to D3 branes wrapping non-compact two cycles in the base of F-theory, whose charge cannot be screened by the dynamical strings. They can be viewed as surface operators of the CFT.

Extension to classification of ``Little String Theories'' (see talks by Mina Aganagic and Tom Rudelius)

Little strings can be viewed as certain limits of string theory where gravity decouples but fundamental string does not [S]. So in particular they have a scale and thus not conformal supersymmetric theories in 6d. The basic example in M-theory:



M5-branes whose transverse direction contains a circle which we make arbitrarily small.

A More General Definition

LSTs are UV complete, non-local 6D theories with gravity decoupled and an intrinsic string scale

They are characterized by the presence of a non-dynamical tensor

6D SCFTs and LSTs

SCFTs	LSTs
Ω neg. def.	Ω neg. semi-def.
$(1,0)$ or $(2,0)$ SUSY	$(1,0)$, $(2,0)$, or $(1,1)$ SUSY
Scale-invariant	String scale M_s
	Flows to CFT in the IR

Strings in 6D

String Charge Lattice Γ

Dirac Pairing $\Omega : \Gamma \times \Gamma \to \mathbb{Z}$

Decouple Gravity $\Rightarrow \mathrm{Eig}(\Omega) \leq 0$

Strings in 6D

$$\text{Linear Alg.} \Rightarrow \text{Nullity}(\Omega) = 0, 1$$

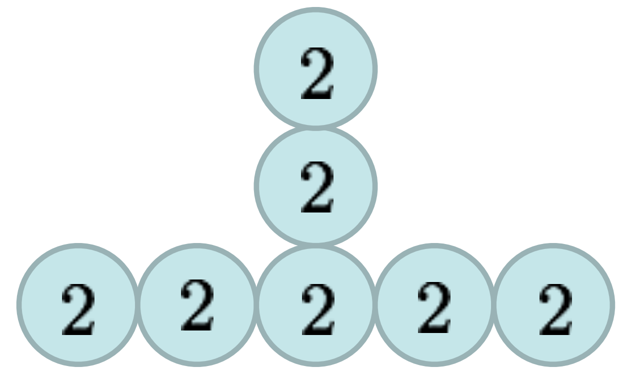
Vinberg '71

$$\mathcal{L}_{kin} \supset \Omega_{IJ} \partial_\mu t^I \partial^\mu t^J$$

$$\det(\Omega) = 0 \Rightarrow \text{Non-dyn. tensor}$$

LST Limit

Start: a smooth base B_2



End: To get an LST, simultaneously contract curves of B_2: one remains non-contractible (genus 0 or 1)

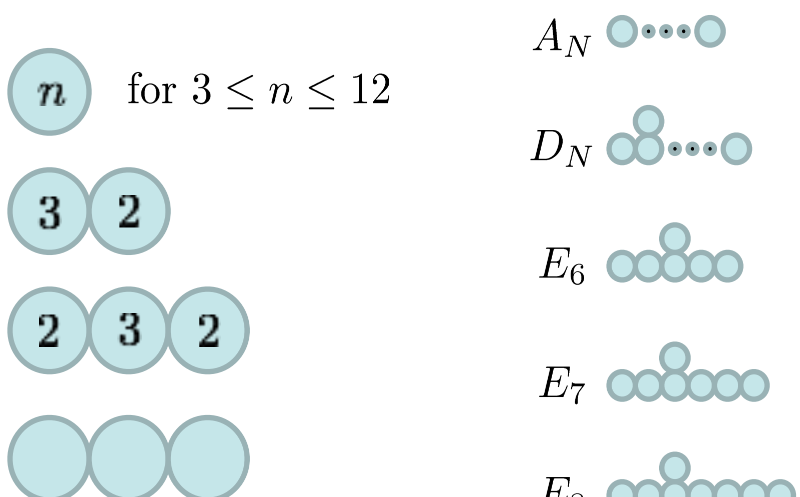
Necessary condition: Ω_{IJ} negative semi-definite with one null eigenvalue

Classifying LST Bases

[BDMRV]

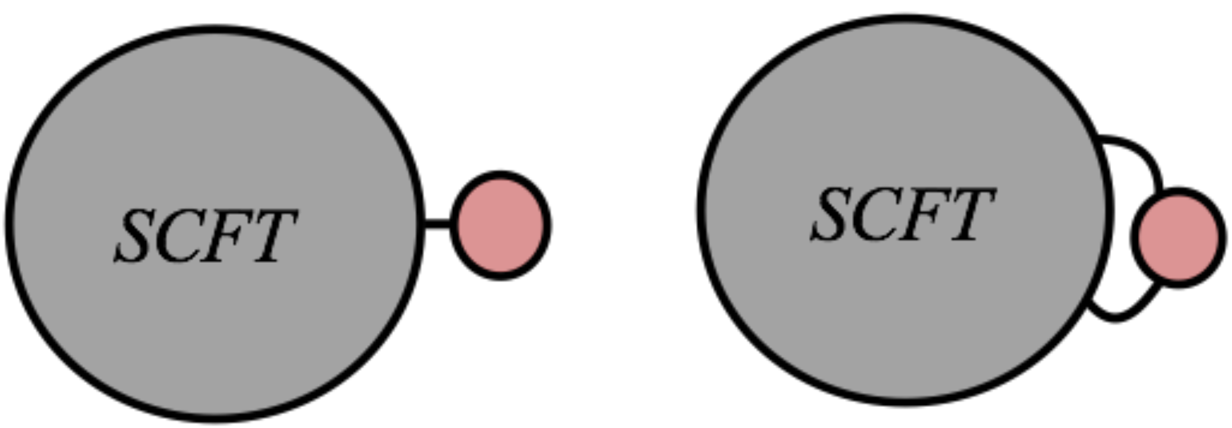
Find all bases B_2 with:

1. Ω_{IJ} neg. semi-def., one null eigenvalue
2. NHC "atoms" joined together by -1 curves

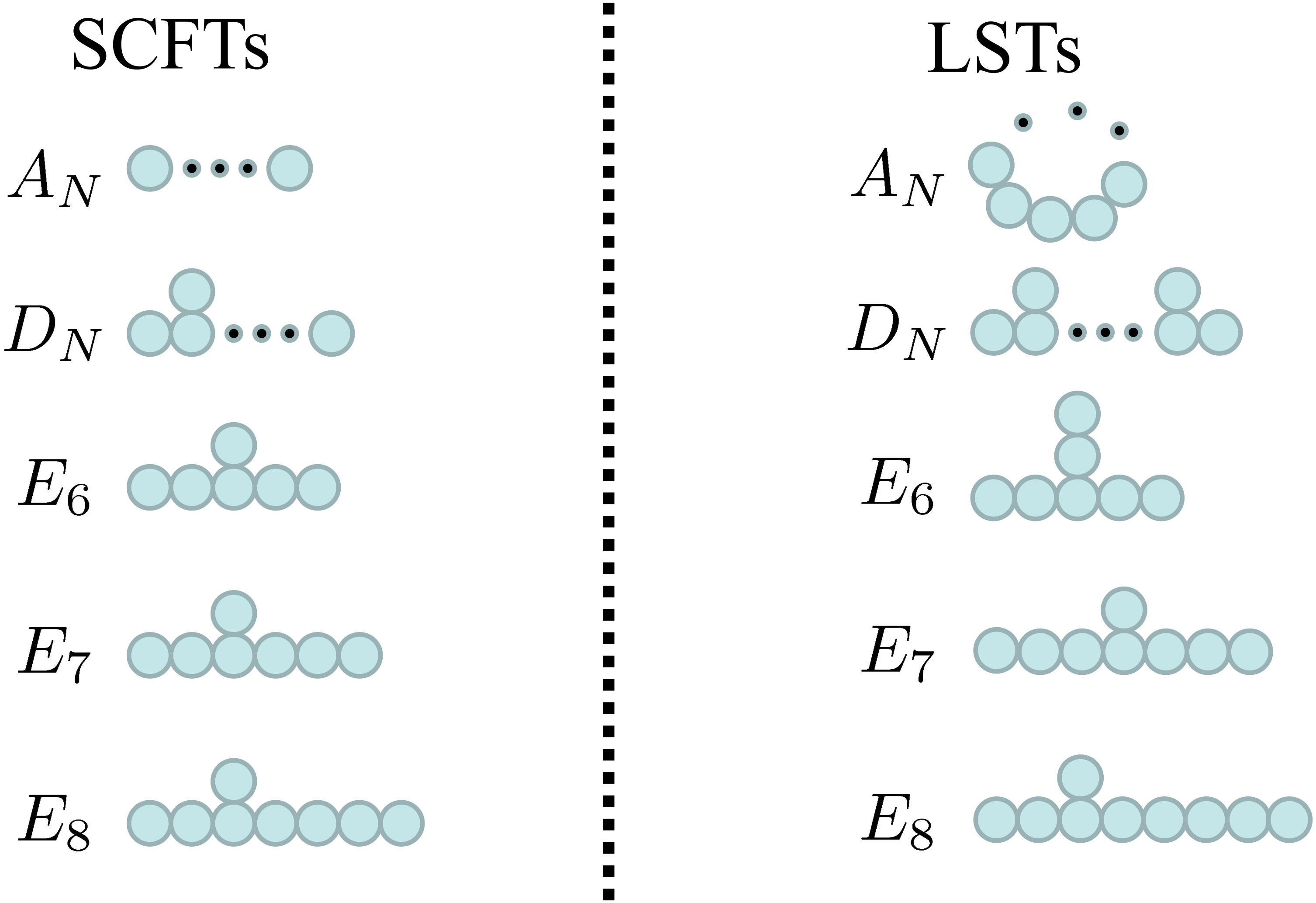


“Affinizing” SCFTs

All LSTs can be thought of as “affinizations” of SCFTs,
in which a single node is added to the quiver
(similar to affinizations of Lie algebras)

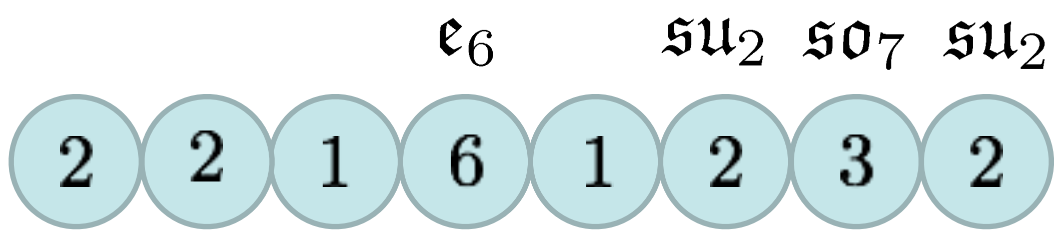


Example: (2,0) LSTs

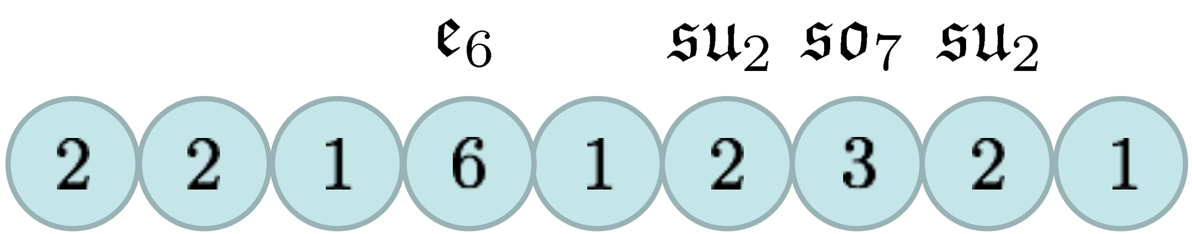


A (1,0) Example

SCFT

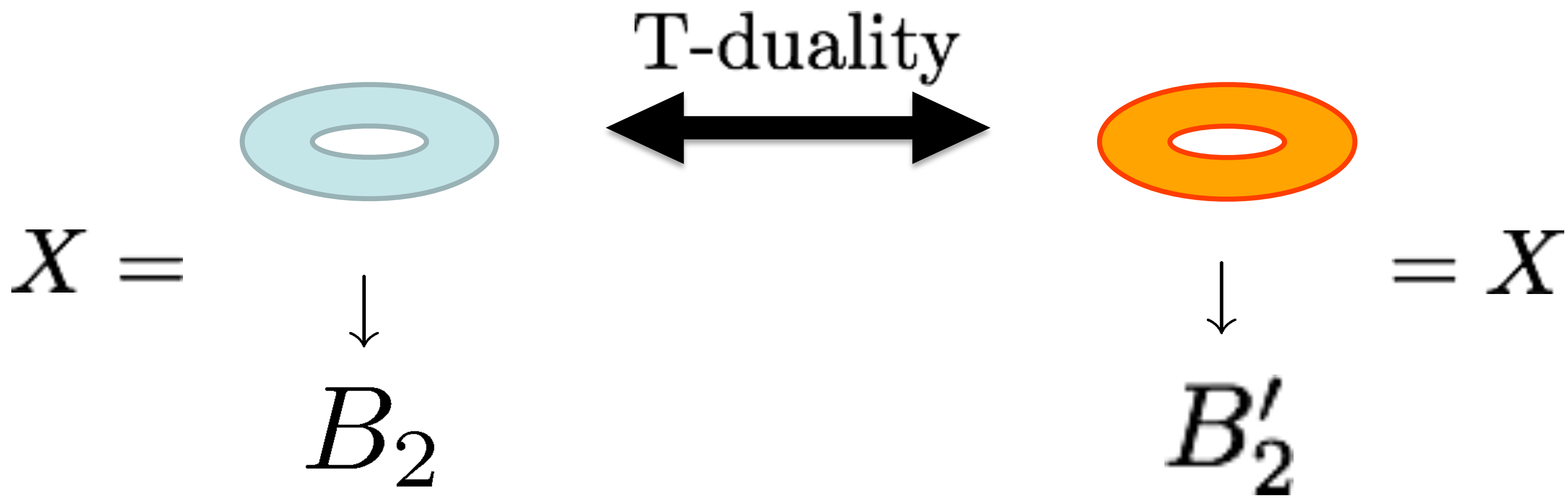


LST



T-duality and LST's

F-theory models of many LSTs feature a total space with **two** elliptic fibrations



Double Elliptic Fibrations

F-theory on $T^2 \to X \downarrow B_2$

F-theory on $T^{2\prime} \to X \downarrow B_2'$

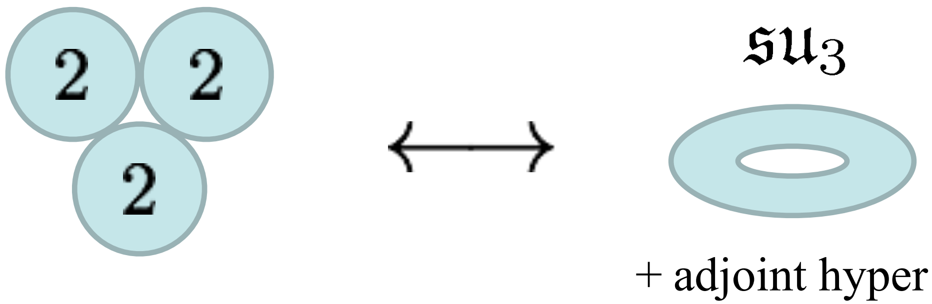
$S^1 \searrow$ $\swarrow S^1$

M-theory on X

In other words, the theories become equivalent upon circle compactification—a hallmark of T-duality.

Double Elliptic Fibrations

T-duality of (2,0) $\longleftrightarrow$ (1,1)



Other examples:

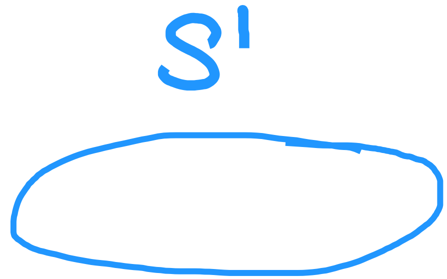
Affine A_k base with elliptic A_n singularity fiber (base fiber exchange) is T-dual to k $\leftrightarrow$ n

Note k=n is a self-T-dual LST.

Lower dimensional CFT's

We can compactify (1,0) and (2,0) SCFT's to lower dimensions on various manifolds and obtain new CFT's in IR

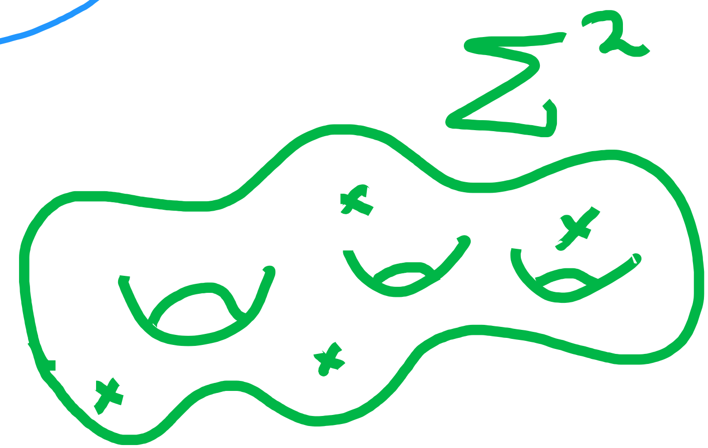
To 5d:



To 4d:

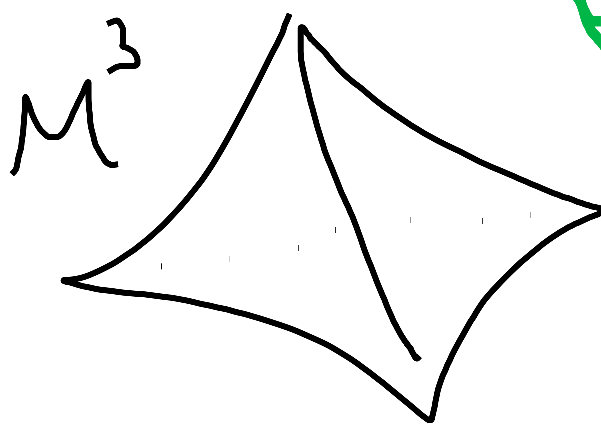
[G,GMN,AGT,BBBW; (2,0)]

[GR,OSTY,DVX; (1,0)]



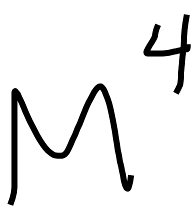
To 3d:

[DGG,CCV,D]



To 2d:

[GGP]



Common Themes:

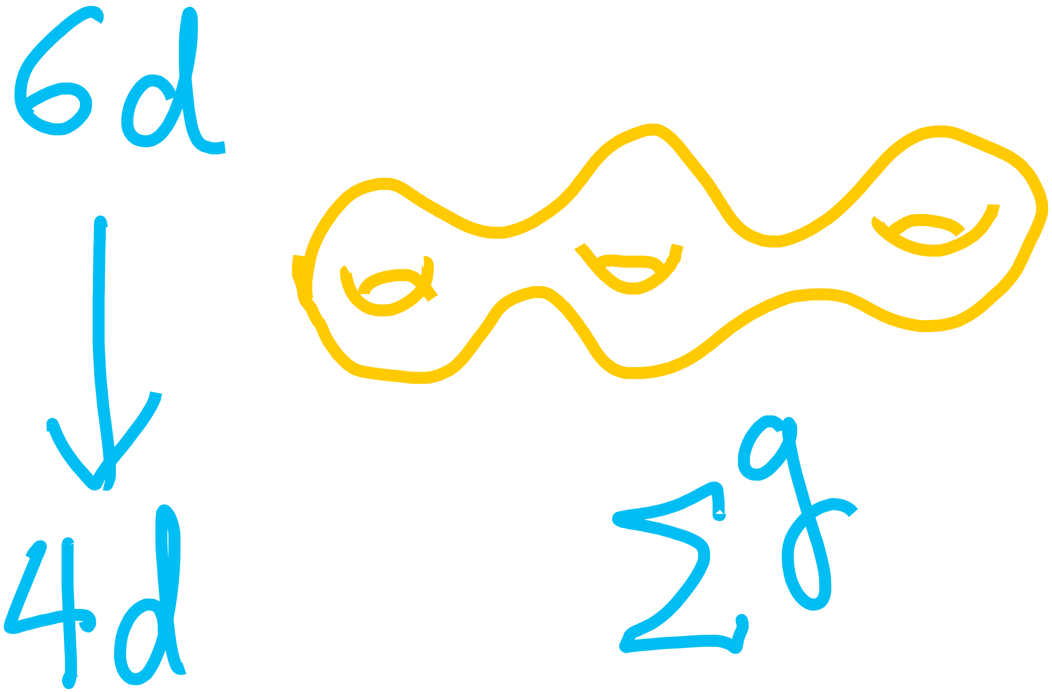
1) Dualities in lower dimensions are translated to geometric statement about cutting and pasting the internal manifold in different ways.

2) The amount of supersymmetry preserved can be adjusted by suitable choice of compactification geometry.

3) Sometimes one can get the same lower dimensional theory from two different higher dimensional theories.

A particular case: 6d→4d

Given the powerful classification of 6d SCFT’s it is natural to ask what it teaches us about 4d SCFT's:



To preserve supersymmetry, we need to twist (related to normal geometry of the 6d theory). Equivalent to picking U(1) inside the R-symmetry group and turning it on to lead to covariantly constant spinors.

(2,0) theories: g= 1 → N=4 SCFT

General g →

U(1) sitting in SO(2) of SO(5) R-symmetry group

→ N=2 theories of class S [G]

U(1) sitting in SO(2)xSO(2) of SO(5)

→ N=1 theories [BBBW]

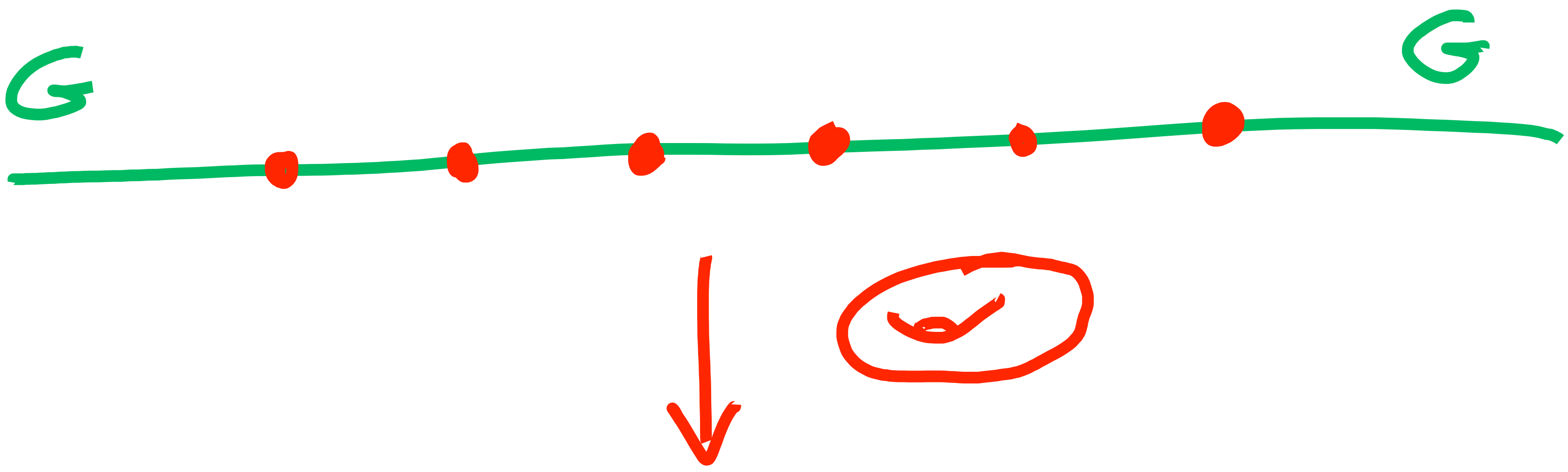
The IR theory leads to SCFT whose moduli is at least that of the moduli space of Riemann surface.

For (1,0) theories:
The R-symmetry is SU(2) and thus the choice of U(1) is fixed.

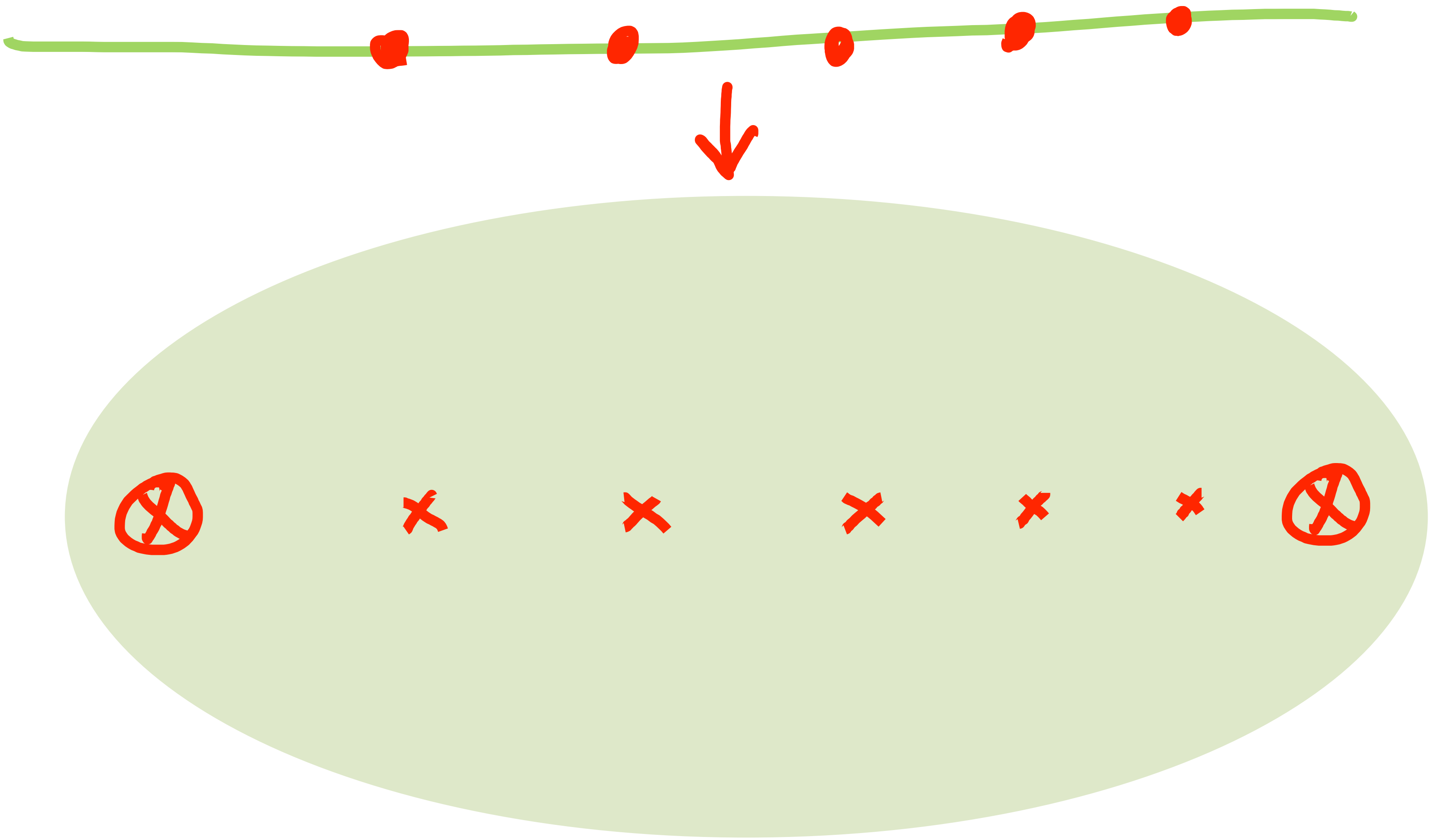
g=1 → N=2 theories in d=4
General g → N=1 theories in d=4

One may think that this always leads to theories in the IR which are SCFT's whose moduli at least includes the moduli of Riemann surfaces.
Not so: e.g.
For example: O(-3) theory has no such limit.

Consider as a simple example the theory of N M5 branes probing G=ADE singularity. The resulting 6d (1,0) theory has GxG symmetry. It has been argued [OSTY] that upon compactification on T^2, if one turns off the GxG:

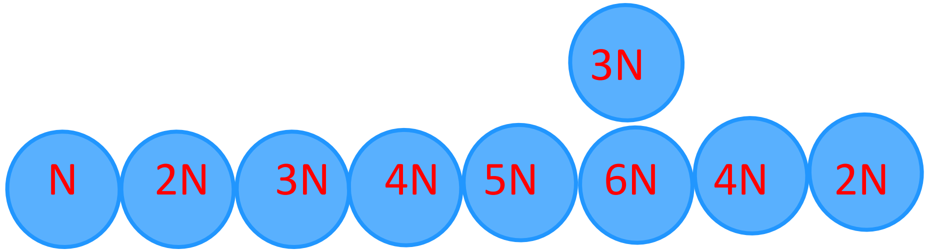


τ does not show up in the most obvious IR limit



A genus 0 Class S theory of G-type with 2 full punctures +simple punctures

However, one can also consider compactifications of (1,0) theories where the moduli of Riemann surface is part of the 4d IR moduli space [DVX]. In fact for this class since N M5 branes on T^2 is the same as N D3 branes in the IIB setup we get 4d affine ADE quivers whose moduli is that of ADE flat connection on T^2. For example for N M5 Branes probing E8 we get the N=2 SU quiver theory:



More generally one can ask: Which 6d (1,0) theories lead to 4d theories where the moduli of Riemann Surface is part of the IR moduli of the CFT?

Answer [MV]: Those whose description involves in 6d ADE oribfolds can lead to 4d theories where the moduli space of Riemann surface will show up as moduli of 4d CFT.

For the (1,0) theory of N M5 branes probing ADE compactified on a more general Riemann surface:

Genus 0 leads to new N=1 theories which generically have no known Lagrangian formulation [GR].

Moduli space of genus g can involve the moduli space of a pair of flat ADE bundles on the Riemann surface (or an abelian subspace thereof) [RV].

Conclusions

I hope I have conveyed the rich structure of SCFT's in 6d. A lot has been done, but many significant gaps remain.

- Complete the field theoretic reasoning for classification (address some of the remaining issues).
- The algebraic meaning of the Dynkin-like diagrams that emerge remains mysterious.
- Better understand the role of strings as excitations.
- Bootstrap methods need to be developed further for a more effective study of these theories.
- Compactification to lower dimensions needs to be studied further, especially for the (1,0) case.