Holographic reconstruction of quartic vertices in higher-spin gravity

Xavier BEKAERT

Laboratoire de Mathématiques et Physique Théorique (Tours, France)
&
Institute for Basic Science (Daejeon, South Korea)

4 August 2016 @ Beijing

From joint works with J. Erdmenger, D. Ponomarev and C. Sleight

- JHEP **1503** (2015) 170 [arXiv:1412.0016 [hep-th]]
- JHEP **1511** (2015) 149 [arXiv:1508.04292 [hep-th]]

Higher-spin gravity & holography

Higher-spin gravity

- $\equiv$ interacting theory with at least one gauge field of spin
  - greater than two ("higher-spin")
  - equal to two ("gravity")

  in the spectrum.
- typically have vertices involving infinitely many derivatives dressed by the only available dimensionfull scale (the AdS radius)
  $\Rightarrow$ their locality properties remain elusive.

Higher-spin holography

- $\equiv$ any holographic duality between a free (or integrable) CFT and a higher-spin gravity theory in the bulk.
- provides a (somewhat implicit) definition of the latter via holographic reconstruction.
- raises the question of bulk locality in the strongly curved regime of the AdS/CFT correspondence.

Outline

Quick review of higher-spin holography

Higher-spin holography

Basic idea behind the conjectured duality:
(Ferrara-Fronsdal, Konstein-Vasiliev-Zaikin, Sezgin-Sundell, Sundborg, Witten, Mikhailov, Klebanov-Polyakov, ...)

Free (or integrable) CFTs have an infinite number of higher-order conformal symmetries.

Higher-spin holography

Basic idea behind the conjectured duality:
(Ferrara-Fronsdal, Konstein-Vasiliev-Zaikin, Sezgin-Sundell, Sundborg, Witten, Mikhailov, Klebanov-Polyakov, ...)

Free (or integrable) CFTs have an infinite number of higher-order conformal symmetries.

Noether theorem $\Longrightarrow$

Their spectrum contains an infinite tower of traceless conserved currents with unbounded spin (including spin two).

Higher-spin holography

Basic idea behind the conjectured duality:
(Ferrara-Fronsdal, Konstein-Vasiliev-Zaikin, Sezgin-Sundell, Sundborg, Witten, Mikhailov, Klebanov-Polyakov, ...)

Free (or integrable) CFTs have an infinite number of higher-order conformal symmetries.

$\overset{\text{Noether theorem}}{\Longrightarrow}$

Their spectrum contains an infinite tower of traceless conserved currents with unbounded spin (including spin two).

$\overset{\text{AdS/CFT dictionary}}{\Longrightarrow}$

Free (or integrable) CFTs should be dual to "higher-spin gravity" theories whose spectrum contains an infinite tower of gauge fields with unbounded spin (including spin two).

Higher-spin holography

Remark:
The bulk dual of a *free* CFT should nevertheless be an *interacting* theory.

Higher-spin holography

Remark:
The bulk dual of a *free* CFT should nevertheless be an *interacting* theory.

All conserved currents are bilinears in the boundary free fields.

Higher-spin holography

Remark:
The bulk dual of a *free* CFT should nevertheless be an *interacting* theory.

All conserved currents are bilinears in the boundary free fields.

$\Longrightarrow$

Their n-point correlators, computed via Wick contractions, are all non-vanishing.

Higher-spin holography

Remark:
The bulk dual of a *free* CFT should nevertheless be an *interacting* theory.

All conserved currents are bilinears in the boundary free fields.

$\Longrightarrow$

Their n-point correlators, computed via Wick contractions, are all non-vanishing.

$\Longrightarrow$

The corresponding bulk n-point vertices should be non-vanishing.

Higher-spin holography: large-N vector model

"Simplest" example: The bulk dual of the *singlet sector* of the $O(N)$ *vector model* should be the *minimal higher-spin gravity* (all even spins).

Higher-spin holography: large-N vector model

"Simplest" example: The bulk dual of the *singlet sector* of the $O(N)$ *vector model* should be the *minimal higher-spin gravity* (all even spins).

Boundary spectrum

- $O(N)$-vector
  - **Conformal scalar fields** $\Delta = \frac{d-2}{2}$: $\phi^a \qquad (a = 1, 2, \ldots, N)$
- $O(N)$-singlets
  - **Bilinear ("Single-trace") operators**
    - *Scalar* $\Delta_0 = d - 2$: $\mathcal{O} = \phi^a \phi^a$
    - *Conformal currents* $\Delta_s = s + d - 2$:

$$\mathcal{J}_{i_1 \ldots i_s} = \phi^a \partial_{(i_1} \ldots \partial_{i_s)} \phi^a + \cdots \qquad (s = 2, 4, 6, \ldots)$$

Bulk spectrum

- Infinite tower of gauge fields of all even spins (s = 0, 2, 4, 6, . . .)

Higher-spin gravity

Higher-spin interactions appear to be generically

- quasi local in the sense that they possess a perturbative expansion (in powers of fields and their derivatives) where each individual term in the total Lagrangian is local.
- non local in the sense that the total number of derivatives is unbounded. This is a corollary of:
  - **Metsaev bounds:** The number of derivatives appearing in an on-shell non-trivial cubic vertex is bounded from
    - above by the sum of the spins involved
    - below by the highest spin involved

    (Metsaev, 1991-2008)
  - **Higher-spin algebra:** The Jacobi identity requires a spectrum with an infinite tower of fields with unbounded spin. (Fradkin-Vasiliev, 1987; Boulanger-Ponomarev-Skvortsov-Taronna, 2013)

Bulk locality of higher-spin gravity

An important question is whether higher-spin interactions nevertheless obey some refined notion of locality.

This issue is under current investigations, c.f. the proposals

- (Vasiliev, 2015) based on functional classes of star-product elements,
- (Skvortsov & Taronna, 2015) & (Taronna, 2016) based on classes of field redefinitions leaving Witten diagrams invariant.

Bulk locality of higher-spin gravity

Cubic interactions:
Individual cubic higher-spin interactions are local in the sense that the relevant cubic vertices for computing any 3-pt contact Witten diagram with fixed external legs involve a finite number of derivatives.

This follows as a corollary from

Metsaev upper bound: For any triplet of spins, the number of derivatives in any nontrivial cubic vertex on the free mass-shell is bounded from above by the sum of the spins.

Bulk locality of higher-spin gravity

Some arguments (based on AdS/CFT common lore and on standard properties of Mellin amplitudes) suggest that the quartic interactions of the bulk theory holographically reconstructed from the free $O(N)$ model might be weakly local. (XB-Erdmenger-Sleight-Ponomarev, 2016)

However, this picture is somewhat qualitative and relies on properties of Mellin amplitudes whose applicability remains questionable for free CFTs.

Bulk locality of higher-spin gravity

Some arguments (based on AdS/CFT common lore and on standard properties of Mellin amplitudes) suggest that the quartic interactions of the bulk theory holographically reconstructed from the free $O(N)$ model might be weakly local. (XB-Erdmenger-Sleight-Ponomarev, 2016)

However, this picture is somewhat qualitative and relies on properties of Mellin amplitudes whose applicability remains questionable for free CFTs.

Goal: Perform a purely holographic reconstruction of quartic AdS interations from free CFT in order to provide a more concrete playground to test bulk locality of higher-spin gravity.

Quartic AdS interactions from CFT

Cubic AdS interactions from CFT

Holographic reconstruction of cubic AdS interations from free CFT:

- Compute the 3-pt conformal correlators of bilinear singlets via Wick contraction
- Write the most general ansatz for relevant cubic vertices
- Compute the corresponding contact Witten diagram

Remark: 1-to-1 correspondence between individual

- 3-pt conformally-invariant correlators (Costa-Penedones-Poland-Rychkov, 2011)
- cubic gauge-invariant vertices (Joung-Taronna, 2011)

for any triplet of spin.

Cubic AdS interactions from CFT

Holographic reconstruction of cubic AdS interations from free CFT:

- Compute the 3-pt conformal correlators of bilinear singlets via Wick contraction
- Write the most general ansatz for relevant cubic vertices
- Compute the corresponding contact Witten diagram
- Fix the coefficients of vertices by matching with each correlator

Done ✓

- $0-0-0$ vertex (Petkou, 2003)
- $s-0-0$ vertices (XB-Erdmenger-Sleight-Ponomarev, 2015)
- $s-0-0$ vertices & parity-odd scalar (Skvortsov, 2015)
- all vertices (Sleight-Taronna, 2016)

Quartic AdS interactions from CFT

Holographic reconstruction of quartic AdS interations from free CFT:

- Compute the 3-pt and 4-pt conformal correlators via Wick contraction
- Write the most general ansatz for relevant cubic and quartic vertices
- Compute the corresponding exchange and contact amplitudes
- Fix the coefficients of vertices by matching the correlator with the total amplitude

Quartic AdS interactions from CFT

Holographic reconstruction of quartic AdS interations from free CFT:

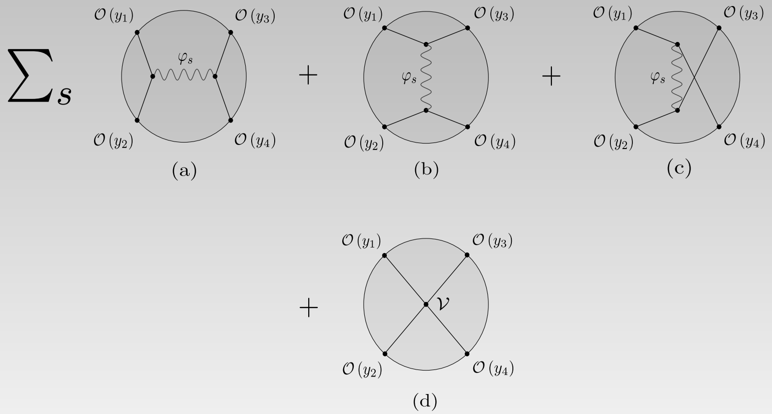
- Compute the 3-pt and 4-pt conformal correlators via Wick contraction
- Write the most general ansatz for relevant cubic and quartic vertices
- Compute the corresponding exchange and contact amplitudes
- Fix the coefficients of vertices by matching the correlator with the total amplitude

Remark: In principle, it is not guaranteed that a purely holographic reconstruction gives a result compatible with the Noether procedure. However, it is natural to expect that these two perturbative procedures are compatible since the Ward identities of the boundary CFT should be dual to the Noether identities of the AdS theory.

Quartic AdS interactions from CFT

Simplest non-trivial example

Holographic reconstruction of the quartic self-interaction of the AdS_4 scalar field in the higher-spin multiplet dual to the $d = 3$ free $O(N)$ model.



Quartic AdS interactions from CFT

Important technical simplifications for this example:

Scalar field:

- The bulk cubic vertex $s-0-0$ is of Noether type $\varphi_s J_s$:

  Gauge field φ_s $\times$ Conserved current $J_s = \varphi_0 (\nabla)^s \varphi_0 + \ldots$

  (Minkowski: Berends, Burgers, van Dam, 1986;

  Anti de Sitter: XB, Meunier, 2010)

Quartic AdS interactions from CFT

Important technical simplifications for this example:

Scalar field:

- The bulk cubic vertex $s-0-0$ is of Noether type $\varphi_s J_s$
- All bulk quartic vertices $0-0-0-0$ are of current exchange type $J_s \, \Box^m J_s$

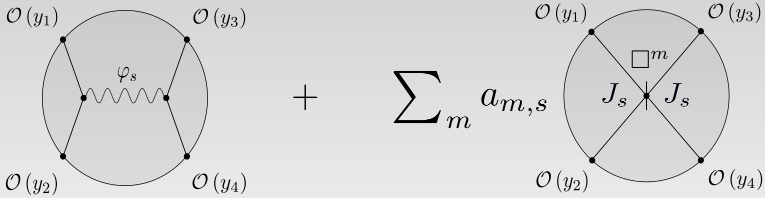
Quartic AdS interactions from CFT

Important technical simplifications for this example:

Scalar field:

- The bulk cubic vertex $s-0-0$ is of Noether type $\varphi_s J_s$:
- All bulk quartic vertices $0-0-0-0$ are of current exchange type $J_s \,\Box^m J_s$

$\Longrightarrow$ The exchange and contact Witten diagrams are of the same type and can be easily compared for each spin s and in each channel.



Quartic AdS interactions from CFT

Important technical simplifications for this example:

Dimension $d = 3$:

A celebrated simplification of higher-spin holography in this case is that the AdS_{d+1} scalar in the higher-spin multiplet is conformal.

(because $d - 2 = \frac{d+1}{2} - 1 \Leftrightarrow d = 3$)

Quartic AdS interactions from CFT

Important technical simplifications for this example:

Dimension $d = 3$:

A celebrated simplification of higher-spin holography in this case is that the AdS_{d+1} scalar in the higher-spin multiplet is conformal.

(because $d - 2 = \frac{d+1}{2} - 1 \Leftrightarrow d = 3$)

$\Longrightarrow$ The conserved currents J_s bilinear in the AdS_4 scalar field can be assumed to be on-shell traceless. (Anselmi, 2000)

Quartic AdS interactions from CFT

Important technical simplifications for this example:

Dimension $d = 3$:

A celebrated simplification of higher-spin holography in this case is that the AdS_{d+1} scalar in the higher-spin multiplet is conformal.

(because $d - 2 = \frac{d+1}{2} - 1 \Leftrightarrow d = 3$)

$\Longrightarrow$ The conserved currents J_s bilinear in the AdS_4 scalar field can be assumed to be on-shell traceless. (Anselmi, 2000)

$\Longrightarrow$ The traces of the exchanged gauge fields in AdS_4 do not contribute to the amplitudes.

Quartic AdS interactions from CFT

Important technical simplifications for this example:

Identical bosonic operators/fields: The sum over the three distinct channels is equivalent to a mere Bose symmetrisation.

Quartic AdS interactions from CFT

Important technical simplifications for this example:

Identical bosonic operators/fields: The sum over the three distinct channels is equivalent to a mere Bose symmetrisation.

$\Longrightarrow$ A formal holographic reconstruction in a single channel may lead to the correct result after suitable symmetrisation.

Quartic AdS interactions from CFT

Strategy:

In order to perform the holographic matching, it is convenient to write both sides in terms of the same building blocks.

Quartic AdS interactions from CFT

Strategy:

In order to perform the holographic matching, it is convenient to write both sides in terms of the same building blocks.

- Conformal block decomposition
  $\Longleftrightarrow$ decomposition into irreps of conformal group

In the direct channel (12)(34)

$$\langle \mathcal{O}(y_1)\,\mathcal{O}(y_2)\,\mathcal{O}(y_3)\,\mathcal{O}(y_4)\rangle = \frac{1}{\left(y_{12}^2 y_{34}^2\right)^{d-2}}\left\{1+\sum_{\Delta,s} c_{\Delta,s}^2\, G_{\Delta,s}(u,v)\right\}$$

where $c_{\Delta,s}$ is the OPE coefficient in

$$\mathcal{O}\mathcal{O} \sim \mathbb{I} + \sum_{\text{primary}} c_{\Delta,s}\,\mathcal{O}_{\Delta,s} + \text{descendants}$$

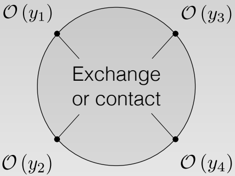
and $G_{\Delta,s}(u,v)$ is the conformal block for the operator $\mathcal{O}_{\Delta,s}$.

Quartic AdS interactions from CFT

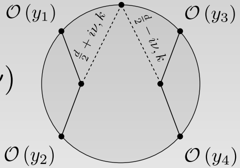
Strategy:

In order to perform the holographic matching, it is convenient to write both sides in terms of the same building blocks.

- Conformal block decomposition
  $\Longleftrightarrow$ decomposition into irreps of conformal group
- Contour integral representation
  The split representation allows to express Witten diagrams only in terms of boundary variables, as a contour integral.



$$= \sum_k \int_{-\infty}^{\infty} d\nu \; g_k(\nu)$$



(split representation of propagator: Fronsdal, 1974; Dobrev, 1999; Leonhardt-Manvelyan-Ruhl, 2003)

Quartic AdS interactions from CFT

Strategy:

In order to perform the holographic matching, it is convenient to write both sides in terms of the same building blocks.

- Conformal block decomposition
  $\Longleftrightarrow$ decomposition into irreps of conformal group
- Contour integral representation

  $\Longrightarrow$ One should obtain the contour integral representation of the conformal block decomposition (also called conformal partial wave expansion) of the 4-point correlator.

$$\langle \mathcal{O}(y_1)\,\mathcal{O}(y_2)\,\mathcal{O}(y_3)\,\mathcal{O}(y_4)\rangle$$
$$= \frac{1}{\left(y_{12}^2 y_{34}^2\right)^{\Delta}} \left\{ 1 + \sum_s \int_{-\infty}^{\infty} d\nu \; f_s(\nu)\, G_{\frac{d}{2}+i\nu,s}(u,v) \right\}$$

(Dobrev-Petkova-Petrova-Todorov, 1976)

Quartic AdS interactions from CFT

Summary: Achieving the holographic reconstruction required to

- build on scattered results in the litterature:
  - Various former results on OPE and conformal block decomposition (Dolan-Osborn, 2001; Diaz-Dorn, 2006)
  - Holographic match of the cubic vertices $s - 0 - 0$ with the corresponding 3-pt correlators (Costa-Gonçalves-Penedones, 2014)
  - Basis of quartic vertices $0 - 0 - 0 - 0$ in *flat* spacetime (Heemskerk-Penedones-Polchinski-Sully, 2009)
  - Harmonic analysis and split representation of (transverse) traceless part of AdS higher-spin (gauge) field propagators (Leonhardt-Manvelyan-Ruhl, 2003; Costa-Gonçalves-Penedones, 2014)

Quartic AdS interactions from CFT

Summary: Achieving the holographic reconstruction required to

- overcome various technical hurdles:
  - Split representation of AdS *massless* higher-spin field propagators
  - OPE coefficients for the scalar *double-trace* operators in any d
  - Contour integral form of the conformal block expansion of 4-point
    - Conformal correlator of scalar single-trace operators
    - Exchange Witten diagrams
    - Contact Witten diagram
  - Summation over the three channels ("$\frac{1}{3}$ trick")

Boundary side

Relevant boundary operators

The boundary operators relevant for the present computation are:

- $O(N)$-vector
  - **Fundamental conformal scalar fields $\Delta = \frac{d-2}{2}$**

$$\phi^a \qquad (a = 1, 2, \ldots, N)$$

- $O(N)$-singlets
  - **Single-trace operators**
    - Scalar $\Delta_0 = d - 2$: $\qquad \mathcal{O} = \frac{1}{\sqrt{2N}} \phi^a \phi^a$
    - Conformal currents $\Delta_s = s + d - 2$:

$$\mathcal{J}_{i_1 \ldots i_s} = \phi^a \partial_{(i_1} \ldots \partial_{i_s)} \phi^a + \cdots \qquad (s = 2, 4, 6, \ldots)$$

  - **Double-trace operators $\Delta_{n,s} = d - 2 + 2n + s$**

$$\mathcal{O}^{(2)}_{n, i_1 \ldots i_s} = \mathcal{O} \Box^n \partial_{(i_1} \ldots \partial_{i_s)} \mathcal{O} + \cdots \qquad (n = 0, 1, 2, \ldots)$$

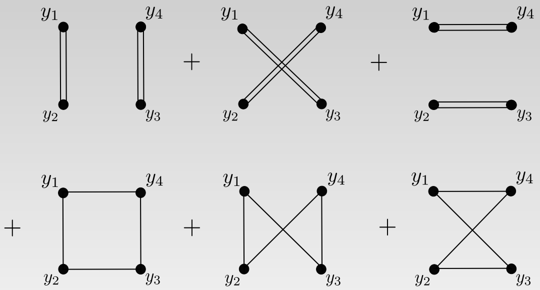
Four-point function of scalar single-trace operators

The full scalar single-trace operator 4-point function

$$\langle \mathcal{O}(y_1)\,\mathcal{O}(y_2)\,\mathcal{O}(y_3)\,\mathcal{O}(y_4)\rangle = \frac{1}{(y_{12}^2 y_{34}^2)^{d-2}} \times$$

$$\times \left\{ \left(1 + u^{d-2} + \left(\frac{u}{v}\right)^{d-2}\right) + \frac{4}{N}\left(u^{\frac{d}{2}-1} + \left(\frac{u}{v}\right)^{\frac{d}{2}-1} + u^{\frac{d}{2}-1}\left(\frac{u}{v}\right)^{\frac{d}{2}-1}\right)\right\}$$

is obtained via Wick contractions



Four-point function of scalar single-trace operators

The conformal block decomposition of the scalar single-trace operator 4-point function

$$\langle \mathcal{O}(y_1)\,\mathcal{O}(y_2)\,\mathcal{O}(y_3)\,\mathcal{O}(y_4)\rangle = \frac{1}{\left(y_{12}^2 y_{34}^2\right)^{d-2}} \times$$

$$\times \left\{ 1 + \sum_s c_s^2\, G_{s+d-2,s}(u,v) + \sum_{n,s} c_{n,s}^2\, G_{\Delta_{n,s},s}(u,v) \right\}$$

can be determined from the OPE of the scalar single-trace operator

$$\mathcal{O}\mathcal{O} \sim \mathbb{I} + \sum_s c_s\, \mathcal{J}_s + \sum_{n,s} c_{n,s}\, \mathcal{O}^{(2)}_{n,s} + \text{descendants},$$

Four-point function of scalar single-trace operators

The conformal block decomposition of the scalar single-trace operator 4-point function

$$\langle \mathcal{O}(y_1)\,\mathcal{O}(y_2)\,\mathcal{O}(y_3)\,\mathcal{O}(y_4)\rangle = \frac{1}{\left(y_{12}^2 y_{34}^2\right)^{d-2}} \times$$

$$\times \left\{ 1 + \sum_s c_s^2\, G_{s+d-2,s}(u,v) + \sum_{n,s} c_{n,s}^2\, G_{\Delta_{n,s},s}(u,v) \right\}$$

can be determined from the OPE of the scalar single-trace operator

$$\mathcal{O}\mathcal{O} \;\sim\; \mathbb{I} + \sum_s c_s\, \mathcal{J}_s + \sum_{n,s} c_{n,s}\, \mathcal{O}^{(2)}_{n,s} \;+\; \text{descendants},$$

The OPE coefficients

- c_s of the conformal current $\mathcal{J}_s$ were known (Dolan & Osborn, 2001; Diaz & Dorn, 2006)
- $c_{n,s}$ of the double-trace operator $\mathcal{O}^{(2)}_{n,s}$ were only known for $d = 4$ (Dolan & Osborn, 2001) so they had to be determined for $d = 3$.

Double-trace OPE coefficients

$$c^2_{n,s} = \frac{[(-1)^s+1]\,2^s\left(\frac{d}{2}-1\right)^2_n(d-2)^2_{s+n}}{s!n!\left(s+\frac{d}{2}\right)_n(d-3+n)_n(2d+2n+s-5)_s\left(\frac{3d}{2}-4+n+s\right)_n} \times\left(1+(-1)^n\frac{4}{N}\frac{\Gamma(s)}{2^s\Gamma\left(\frac{s}{2}\right)}\frac{\left(\frac{d}{2}-1\right)_{n+\frac{s}{2}}}{\left(\frac{d-1}{2}\right)_{\frac{s}{2}}(d-2)_{n+\frac{s}{2}}}\right)$$

As a preliminary result, the explicit form of the double-trace operator $\mathcal{O}^{(2)}_{n,s}$ had to be determined and is extremely complicated. The above generic form of the OPE coefficient $c_{n,s}$ remains a conjecture but it

- reproduces known results for
  - $d=4$ and $\forall n$, $\forall s$ (Dolan & Osborn, 2001)
  - $N=\infty$ and $\forall d$, $\forall n$, $\forall s$ (Fitzpatrick & Kaplan, 2011)
- was explicitly computed for
  - $s=0$ and $\forall d$, $\forall n$
  - $n=0,1$ and $\forall d$, $\forall s$

Contour integral representation

For each spin s, find a function $f_s(\nu)$ such that

$$\sum_{\Delta} c^2_{\Delta,s}\, G_{\Delta,s}(u,v) = \int_{-\infty}^{\infty} d\nu\, f_s(\nu)\, G_{\frac{d}{2}+i\nu,s}(u,v)$$

where one closes the contour in the lower-half plane.

It will turn out to be convenient to set

$$f_s(\nu) = p_s(\nu)\, \kappa_s(\nu)$$

where $p_s(\nu)$ is an even function of ν and

$$\kappa_s(\nu) = \frac{2^{-2i\nu+2s-3}\,\Gamma\left(i\nu+\frac{1}{2}\right)\Gamma\left(\frac{2s-2i\nu+1}{4}\right)^2\Gamma\left(\frac{2s+2i\nu+3}{4}\right)^2}{\pi^{5/2}\Gamma(i\nu)(2i\nu+2s+1)}$$

Contour integral representation

For the spin-s conformal current:

$$p_{\mathcal{J}_s}(\nu) = \frac{\pi\, 2^{8-s}}{N} \frac{1}{\nu^2 + (s-\frac{1}{2})^2} \frac{1}{\Gamma\left(\frac{2s-2i\nu+1}{4}\right)^2 \Gamma\left(\frac{2s+2i\nu+1}{4}\right)^2}.$$

For the spin-s double-trace operator contribution:

$$p_{\mathcal{O}_s^{(2)}}(\nu) = \frac{\pi^{\frac{3}{2}}\, 2^{s+4}\Gamma\left(s+\frac{3}{2}\right)}{\Gamma(s+1)\,\Gamma\left(s+\frac{1}{2}+i\nu\right)\Gamma\left(s+\frac{1}{2}-i\nu\right)} +$$

$$\frac{1}{N} \frac{(-1)^{\frac{s}{2}}\,\pi^{\frac{3}{2}} 2^{s+4}\Gamma\left(s+\frac{3}{2}\right)\Gamma\left(\frac{s}{2}+\frac{1}{2}\right)}{\sqrt{2}\,\Gamma\left(\frac{s}{2}+1\right)\Gamma(s+1)\,\Gamma\left(\frac{3}{4}-\frac{i\nu}{2}\right)\Gamma\left(\frac{3}{4}+\frac{i\nu}{2}\right)\Gamma\left(s+\frac{1}{2}+i\nu\right)\Gamma\left(s+\frac{1}{2}-i\nu\right)}$$

Bulk side

Cubic vertices

Relevant cubic vertex

$$\mathcal{V}^{(3)} = \sum_s g_s \, \mathcal{V}_s^{(3)}$$

expanded in a basis of on-shell non-trivial cubic vertices

$$\mathcal{V}_s^{(3)} = \varphi_{\mu_1 \cdots \mu_s} \, J^{\mu_1 \cdots \mu_s} \qquad (s \in 2\mathbb{N})$$

where

$$J^{\mu_1 \cdots \mu_s}(x) = \varphi_0 \nabla_{\mu_1} \cdots \nabla_{\mu_s} \varphi_0 + \cdots$$

is a basis of on-shell conserved & traceless bilinears in the scalar field φ_0.

Cubic vertices

Compute the amplitude

$$\mathcal{A}^{\text{contact}}_{s+d-2,s}\left(y_1,y_2;y,z\right) \propto \frac{1}{\left(y_{12}^2\right)^{\frac{d}{2}-1}\left(y_{13}^2\right)^{\frac{d}{2}-1}\left(y_{23}^2\right)^{\frac{d}{2}-1}}\left(\frac{y_{13}\cdot z}{y_{13}^2}-\frac{y_{23}\cdot z}{y_{23}^2}\right)^s$$

by means of the bulk-to-boundary propagators, as in (Costa, Gonçalves, Penedones, 2014).

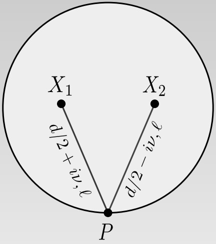
Fix the values of the coefficients g_s by imposing

$$\langle\mathcal{O}\left(y_1\right)\mathcal{O}\left(y_2\right)\mathcal{J}_s\left(y_3,z\right)\rangle = \mathcal{A}^{\text{contact}}_{s+d-2,s}\left(y_3,y_4;y,z\right)$$

Split representation of propagators

$$\Pi_s\left(X_1,u_1;X_2,u_2\right) \propto \sum_{k=0}^{\left[\frac{s}{2}\right]} \left(u_1^2\right)^k \left(u_2^2\right)^k \int_{-\infty}^{\infty} d\nu\, g_{s,k}\left(\nu\right) \times$$

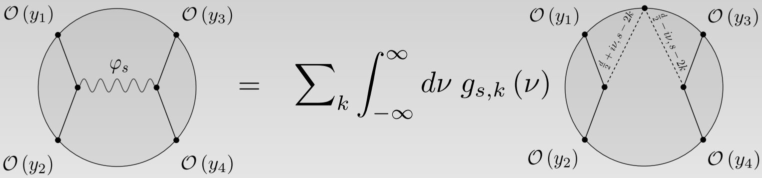
$$\int_{\partial \mathsf{AdS}} d^d P\, \Pi_{\frac{d}{2}+i\nu,s-2k}(X_1,u_1;P,\hat{\partial}_z)\, \Pi_{\frac{d}{2}-i\nu,s-2k}\left(X_2,u_2;P,z\right)$$



Split representation of exchange Witten diagrams

$$\mathcal{A}_s^{\text{exchange}}(y_1, y_2; y_3, y_4)$$

$$= \sum_{k=0}^{\left[\frac{s}{2}\right]} \int_{-\infty}^{\infty} d\nu \; g_{s,k}(\nu) \int_{\partial \text{AdS}} d^d y \; \mathcal{A}^{\text{contact}}_{\frac{d}{2}+i\nu,s}(y_1, y_2; y, \partial_z) \, \mathcal{A}^{\text{contact}}_{\frac{d}{2}-i\nu,s}(y_3, y_4; y, z)$$



Split representation of exchange Witten diagrams

d=3: only the term $k = 0$ in the sum

$$\mathcal{A}^{\text{exchange}}_{s}(y_1, y_2; y_3, y_4)$$
$$= \int_{-\infty}^{\infty} d\nu \; g_s(\nu) \int_{\partial\text{AdS}} d^3y \; \mathcal{A}^{\text{contact}}_{\frac{3}{2}+i\nu,s}(y_1, y_2; y) \, \mathcal{A}^{\text{contact}}_{\frac{3}{2}-i\nu,s}(y_3, y_4; y)$$
$$= \frac{1}{y_{12}^2 y_{34}^2} \int_{-\infty}^{\infty} d\nu \; \frac{1}{\nu^2 + (s - \frac{1}{2})^2} \, \kappa_s(\nu) \, G_{\frac{3}{2}+i\nu,s}(u, v)$$

Quartic vertices

Relevant quartic vertex

$$\mathcal{V} = \sum_{m,s} a_{m,s}\, \mathcal{V}_{m,s}$$

expanded in a basis of on-shell non-trivial quartic vertices

$$\mathcal{V}_{m,s} \;=\; J_{\mu_1\cdots\mu_s}\, \Box^m\, J^{\mu_1\cdots\mu_s} \qquad (s=2k, \quad k\geq m\geq 0, \quad k,m\in\mathbb{N})$$

Quartic vertices

Relevant quartic vertex

$$\mathcal{V} = \sum_{m,s} a_{m,s}\, \mathcal{V}_{m,s}$$

expanded in a basis of on-shell non-trivial quartic vertices

$$\mathcal{V}_{m,s} \;=\; J_{\mu_1\cdots\mu_s}\,\Box^m\, J^{\mu_1\cdots\mu_s} \qquad (s=2k, \quad k\geq m\geq 0, \quad k,m\in\mathbb{N})$$

Remark: If we relax the bound $m \leqslant k$ (as will turn out to be convenient technically), then the set of vertices $\mathcal{V}_{m,s}$ remains a generating set of quartic vertices but they are no more independent on-shell.

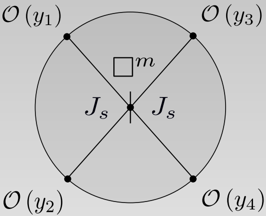
In principle, one should rewrite the final expression in terms of the genuine basis in order to compute the corresponding coefficients (similarly to the recent analysis of cubic vertices arising from Vasiliev equations by Boulanger, Kessel, Skvortsov and Taronna).

Split representation of contact Witten diagrams

$$\mathcal{A}_s^{\text{contact}}(y_1, y_2; y_3, y_4) = \sum_m a_{m,s} \mathcal{A}_{m,s}^{\text{contact}}(y_1, y_2; y_3, y_4)$$

where

$$\mathcal{A}_{m,s}^{\text{cont.}}(y_1, y_2; y_3, y_4) =$$



$$= \frac{1}{y_{12}^2 y_{34}^2} \int_{-\infty}^{\infty} d\nu \, (\nu^2 + s + \tfrac{9}{4})^m \, \kappa_s(\nu) \, G_{\frac{3}{2}+i\nu, s}(u, v)$$

Quartic AdS interactions from CFT

Both the 4-point

1. correlator of scalar single-trace operators, and the
2. amplitudes of the previous s-channel Witten diagrams

have been expressed in the contour integral representation in terms of given spin conformal blocks in the direct channel (12)(34).

Quartic AdS interactions from CFT

Both the 4-point

1. correlator of scalar single-trace operators, and the
2. amplitudes of the previous s-channel Witten diagrams

have been expressed in the contour integral representation in terms of given spin conformal blocks in the direct channel (12)(34).

Remaining obstacle:
The total amplitude is the sum over all channels (s, t and u).

It is very hard to reexpress this total amplitude in terms of a single channel. (Being able to rewrite a conformal block into another channel is essentially equivalent to solving the conformal bootstrap.)

Quartic AdS interactions from CFT

Both the 4-point

1. correlator of scalar single-trace operators, and the
2. amplitudes of the previous s-channel Witten diagrams

have been expressed in the contour integral representation in terms of given spin conformal blocks in the direct channel (12)(34).

Remaining obstacle:
The total amplitude is the sum over all channels (s, t and u).

It is very hard to reexpress this total amplitude in terms of a single channel. (Being able to rewrite a conformal block into another channel is essentially equivalent to solving the conformal bootstrap.)

Trick: Perform a formal holographic matching in a single channel and make sure that the total amplitude after symmetrisation gives the correct result.

Quartic AdS interactions from CFT

Final solution: $\mathcal{V} = \sum_{s\in\mathbb{N}} \mathcal{V}_s$ with

$$\mathcal{V}_s \;=\; J_{\mu_1\cdots\mu_s}\, a_s\left(\Box\right)\, J^{\mu_1\cdots\mu_s}$$

where the generating functions

$$a_s\left(\nu^2+s+\tfrac{9}{4}\right) = \sum_{m=0}^{\infty} a_{m,s}\,\left(\nu^2+s+\tfrac{9}{4}\right)^m$$

$$\propto \frac{2^{8-s}}{\nu^2+(s-\frac{1}{2})^2}\left[\frac{\pi}{\Gamma\left(\frac{2s-2i\nu+1}{4}\right)^2\Gamma\left(\frac{2s+2i\nu+1}{4}\right)^2}-\frac{1}{\Gamma\left(s\right)^2}\right]$$

$$-\frac{(-1)^{\frac{s}{2}}\,\pi^{\frac{3}{2}}2^{s+5}\Gamma\left(s+\frac{3}{2}\right)\Gamma\left(\frac{s}{2}+\frac{1}{2}\right)}{\sqrt{2}\,\Gamma\left(\frac{s}{2}+1\right)\Gamma\left(s+1\right)\Gamma\left(\frac{3}{4}-\frac{i\nu}{2}\right)\Gamma\left(\frac{3}{4}+\frac{i\nu}{2}\right)\Gamma\left(s+\frac{1}{2}+i\nu\right)\Gamma\left(s+\frac{1}{2}-i\nu\right)}$$

are entire functions (though it may not be manifest).

Summary of results and perspectives

Main result

Explicit holographic reconstruction of the latter quartic vertex

Based on various technical intermediate results

- Split representation of AdS gauge fields propagators
- Holographic reconstruction of cubic vertices $s-0-0$
- Generating set of quartic vertices $0-0-0-0$
- OPE coefficients for the scalar double-trace operators
- Contour integral form of the conformal block expansion of four-point
    - Conformal correlator of scalar single-trace operators
    - Exchange Witten diagrams
    - Contact Witten diagram
- Summation over the three channels

Perspectives

1. Extend the holographic reconstruction to
   - spin $s \neq 0$ (use twistors)
   - boundary dimension $d \neq 3$
2. Compare explicitly with
   - Vasiliev higher-spin gravity
   - Mellin amplitude programme