Exploring black hole space-times

with

boundary conformal blocks

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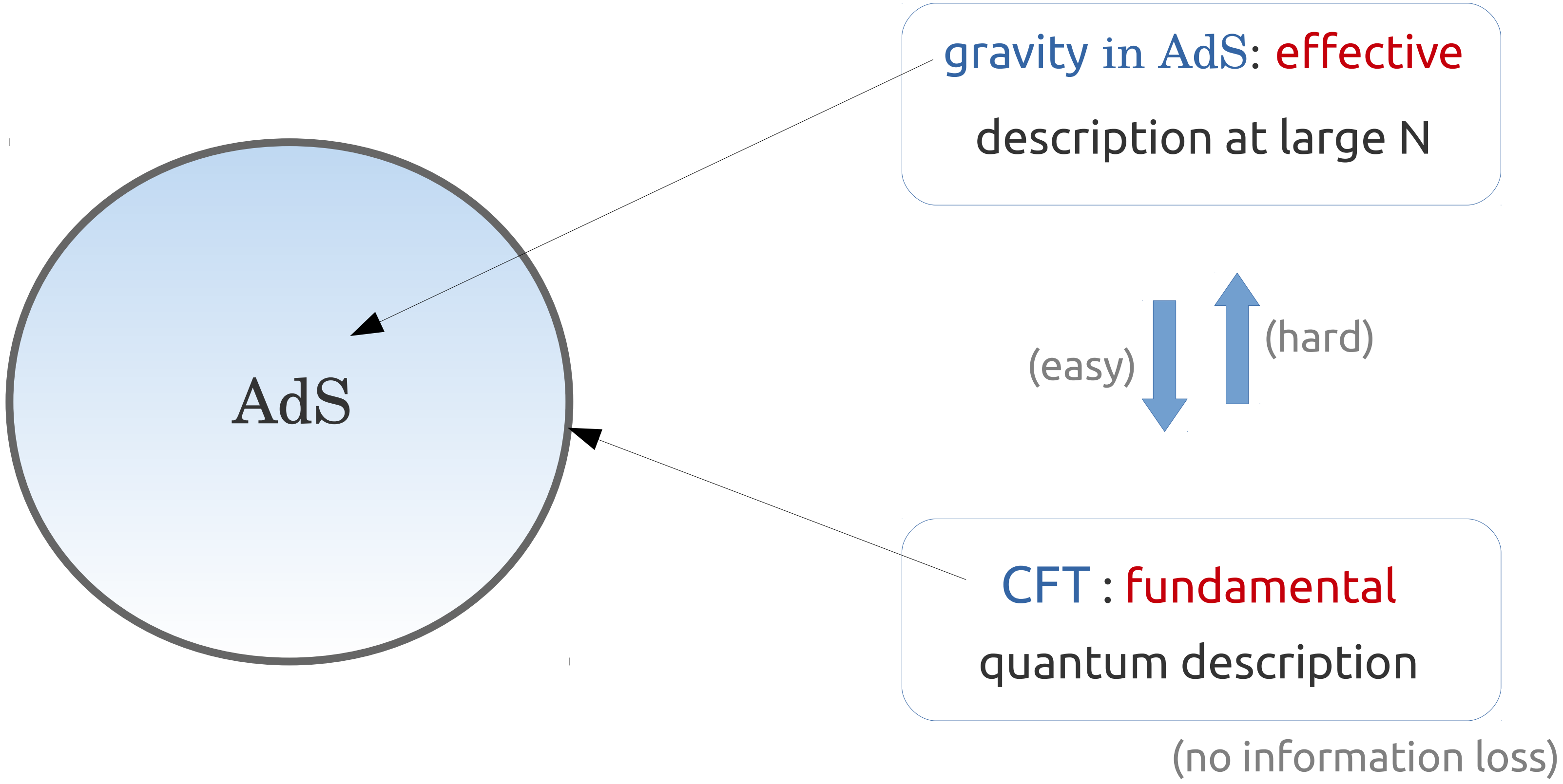
Motivation

- gravity & quantum mechanics (e.g., information loss) → AdS/CFT correspondence



gravity in AdS: effective description at large N

CFT : fundamental quantum description

(no information loss)

How can a CFT observer see/construct the dual space-time geometry?

- understand e.g. if horizon is smooth

The near horizon regions of black holes

- dynamics of bulk fields near a black hole's horizon → universal features

- wave equation $(\Box - m^2)\Phi = 0$ is separable

$$\Phi = e^{-i\omega t + i\kappa\phi} f(r)\, S(\theta)$$

- low energy, near horizon limit ($r << \lambda, \quad \lambda >> R_s$)

→ universal BTZ - like dynamics (only seen in momentum space)

(rotating black hole in AdS_3 ↔ thermal state in CFT_2)

Strominger, Maldacena; Cvetic, Larsen '97
Castro, Maloney, Strominger '10

→ CFT_2 description for general black holes?

- focus on asymptotically AdS black holes

How can a CFT observer see this universal near - horizon behaviour?

Monodromies of the bulk solution

- study analytic properties of $f(r)$ in the complex r plane ($\Phi = e^{-i\omega t + i\kappa\phi} f(r)\, S(\theta)$)
- $f(r)$ obeys a 2nd order differential equation with singular points at $r_\pm$, ∞ (and other)
- monodromies around $r_\pm$ → dual CFT temperatures Castro, Maloney, Lapan, Rodriguez '13

What is the interpretation of the singular points $r_\pm$ & monodromies

of the bulk field from the point of view of the dual CFT?

- ask for all higher-dimensional Kerr - AdS black holes
- focused on BTZ : radial wave equation has 3 singular points, $r_\pm$, ∞

  → solution exactly a hypergeometric
- lessons for higher-dimensional black holes

CFT interpretation of the bulk field

Why expect a dual spacetime?

- large N CFT → correlation functions of light operators factorize

$$\langle \mathcal{O}_1 \mathcal{O}_2 \ldots \mathcal{O}_{2n} \rangle = \langle \mathcal{O}_1 \mathcal{O}_2 \rangle \ldots \langle \mathcal{O}_{2n-1} \mathcal{O}_{2n} \rangle \; + \; perms.$$

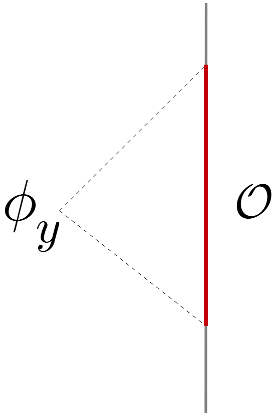
but $\mathcal{O}$ is not free, $\Box_d \mathcal{O} \neq 0$

El-Showk, Papadodimas '11

- linear structure realised by a free field in one dimension higher $(\Box_{d+1} - m^2)\,\phi(z,x) = 0$

$$\phi(y) = \int d^d x' \, K_g(y|x') \, \mathcal{O}(x')$$

background dependent!



- in the vacuum → K_{vac} is fixed by conformal symmetry

- in non-trivial states in which correlators factorize (e.g. thermal states) →

→ no natural way to predict K_g from the CFT point of view !

Extract meaning of the radial direction and the bulk field in pure CFT terms

Probing black holes with conformal blocks

- black hole in AdS → thermal state in dual CFT
- CFT heavy pure states that thermalize $\langle\Psi|\mathcal{O}_L\ldots\mathcal{O}_L|\Psi\rangle \approx \langle\mathcal{O}_L\ldots\mathcal{O}_L\rangle_T$

  → black hole (formation), single-sided

- very successful in AdS$_3$/CFT$_2$

$|\Psi\rangle = \mathcal{O}_H(\tau=-\infty)|0\rangle$ ⇨ $\langle\mathcal{O}_H\mathcal{O}_H\,\mathbb{P}_{\mathrm{I}}^{Vir}\mathcal{O}_L\mathcal{O}_L\rangle = \langle\mathcal{O}_L\mathcal{O}_L\rangle_T$ $\quad T=\frac{1}{2\pi}\sqrt{\frac{12\Delta_H}{c}-1}$

→ matches BTZ answer

Fitzpatrick, Kaplan, Walters '14

$|\Psi\rangle = \mathcal{O}_H^{smeared}(\tau=0)|0\rangle$ ⇨ identity Virasoro block $\quad G(t_1,t_2), S_E$

→ refined details of the black hole collapse (AdS$_3$ - Vaidya)

Anous, Hartman, Rovai, Sonner '16

- purely kinematic, but able to capture details of bulk dynamics
- non-local probes → need localized ones

Bulk fields from conformal blocks

- extract the bulk from the boundary conformal block without using any bulk information

vacuum AdS_{d+1}

free bulk field operator ⟷ boundary OPE

classical bulk field ⟷ boundary global conformal block

radial depth ⟷ CFT conformal ratios

purely kinematic!

Ferrara, Gatto, Grillo, Parisi ‘71

Hijano, Kraus, Perlmutter, Snively ‘15

BTZ

classical bulk field ⟷ boundary heavy-light Virasoro conformal block

radial depth ⟷ boundary conformal ratios

large **c** essential

singular points, monodromies

Fitzpatrick, Kaplan, Walters ‘14

- discuss how to extend lessons to higher dimensions

Bulk fields vs. boundary conformal blocks in

vacuum AdS

Bulk fields and the boundary OPE

- HKLL prescription for a bulk free field in vacuum AdS$_{d+1}$

$$\Phi^{(0)}(z,x) = \int K_{vac}\,\mathcal{O} = 2^\nu\Gamma(\nu+1)\int \frac{d^dp}{(2\pi)^d}\,\frac{e^{ip\cdot x}}{(-p^2)^{\nu/2}}\,z^{\frac{d}{2}}J_\nu(z\sqrt{-p^2})\,\mathcal{O}(p)$$

- FGG '71 contribution of descendants to the OPE $A(x)\,B(0) \quad\rightarrow\quad \mathcal{O}(0)+\ldots$

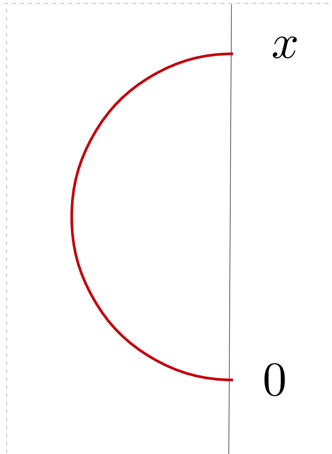
$$A(x)\,B(0) \sim \frac{B^{-1}_{AB}}{|x|^{\Delta_A+\Delta_B}}\int_0^1 \frac{du}{u(1-u)}\left(\frac{u}{1-u}\right)^{\frac{\Delta_{AB}}{2}} 2^\nu\Gamma(\nu+1)\,\times \qquad C^{\mathcal{O}}_{AB}=1$$

$$\times\boxed{\int \frac{d^dp}{(2\pi)^d}\,\frac{e^{iup\cdot x}}{(-p^2)^{\nu/2}}\,(u(1-u)x^2)^{\frac{d}{4}}J_\nu\left(\sqrt{-u(1-u)x^2p^2}\right)\,\mathcal{O}(p)}$$

- same for

spacelike

$$z(u) = \sqrt{u(1-u)\,\underline{\underline{x^2}}} \qquad\qquad x^\mu(u) = u\,x^\mu$$



- in AdS$_2$ /CFT$_1$, even simpler:

$$A(t_1)B(t_2) \sim \frac{2^\Delta}{|t_1-t_2|^{2\Delta_A}}\,\Phi^{(0)}\left(\frac{|t_1-t_2|}{2},\frac{t_1+t_2}{2}\right)$$

Remarks

$$\boxed{A(x)\, B(0) \sim \frac{B^{-1}_{AB}}{|x|^{\Delta_A+\Delta_B}} \int_{-\infty}^{\infty} d\lambda\, e^{-\lambda\Delta_{AB}}\, \Phi^{(0)}(y(\lambda))}$$

$\lambda \in \gamma_{AB}$

- free bulk field → convenient way to encode the conformal family of $\mathcal{O}$

- purely kinematic → $\Phi^{(0)}$ only becomes a local bulk field at large N

$$\Phi_{HKLL} = \underbrace{\int K\mathcal{O}}_{\Phi^{(0)}} + \frac{1}{N}\int K' : AB : + \dots$$

needed for bulk microcausality

- $t = e^{-\lambda}$ → OPE = Mellin transform of $\Phi^{(0)}$

  → use inverse Mellin transform to extract the bulk field?

  → formal procedure, independent of the bulk

  → dependence on $\Delta_{A,B}$ only through prefactor and Mellin parameter

- boundary Casimir operator ↔ bulk Laplacian

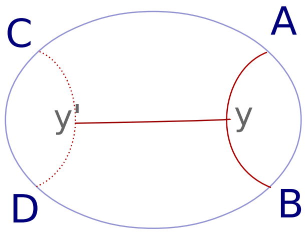
Geodesic Witten diagrams

- conformal block

$$W_{\mathcal{O}}(x_i) = \langle A(x_1)B(x_2)\,\mathbb{P}_{\mathcal{O}}\,C(x_3)D(x_4)\rangle$$

↕

geodesic Witten diagram

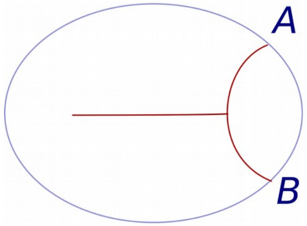


- correlator

$$\Phi_{\gamma_{AB}} \equiv \langle \Phi^{(0)}(y) A(x_1) B(x_2)\rangle = \int_{\gamma_{AB}} d\lambda\, e^{-\lambda \Delta_{AB}} \left\langle \Phi^{(0)}(y)\, \Phi^{(0)}(y'(\lambda))\right\rangle$$

↕

classical bulk field sourced on geodesic γ_{AB}



- related as

$$W_{\mathcal{O}} = \langle CD\,\mathbb{P}_{\mathcal{O}}\,AB\rangle = \int_{\gamma_{CD}} d\lambda\, e^{-\lambda\Delta_{CD}}\, \Phi_{\gamma_{AB}}(y(\lambda))$$

→ map between radial direction and CFT cross ratios: $\rho = \dfrac{x_{14}^2 x_{23}^2}{x_{12}^2 x_{34}^2}$ $\eta = \dfrac{x_{13}^2 x_{24}^2}{x_{12}^2 x_{34}^2}$

$$\cos^2 \rho(\lambda) = \frac{z^2(\lambda)}{z^2(\lambda) + x^2(\lambda)} = \left[\eta\,(1+e^{-2\lambda}) + \rho\,(1+e^{2\lambda})\right]^{-1}$$

Timelike separation

- $\Phi_{\gamma_{AB}} = \langle \Phi^{(0)}(y) A(x_1) B(x_2) \rangle$ = classical bulk field sourced on γ_{AB}

$$W_{\mathcal{O}} = \int_{\gamma_{CD}} e^{-\lambda \Delta_{CD}} \, \Phi_{\gamma_{AB}}$$

$\Big\}$ derived for spacelike separated operators

- timelike separation? → correlators are defined via analytic continuation

  → is the relation to the bulk field still valid?

  (should the bulk field be now sourced on a timelike geodesic?)

- Prescription: *the bulk field is the bulk field sourced on a geodesic in euclidean AdS;*

  analytically continuing the end result yields the correlator

  $\langle \Phi^{(0)}(y) A(x_1) B(x_2) \rangle$, *which in turn is related to the conformal block*

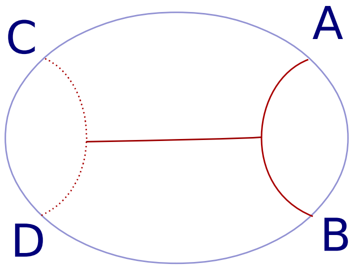
- insertions in the timelike euclidean direction Δ_{AB} ↔ frequency of the bulk field

Partial summary

- boundary OPE ↔ (normalizable) bulk field operator

- conformal block ↔ normalizable classical bulk field sourced on a geodesic in Euclidean AdS

$$\frac{z^2(\lambda)}{z^2(\lambda)+x^2(\lambda)} = \left[\eta\,(1+e^{-2\lambda})+\rho\,(1+e^{2\lambda})\right]^{-1}$$

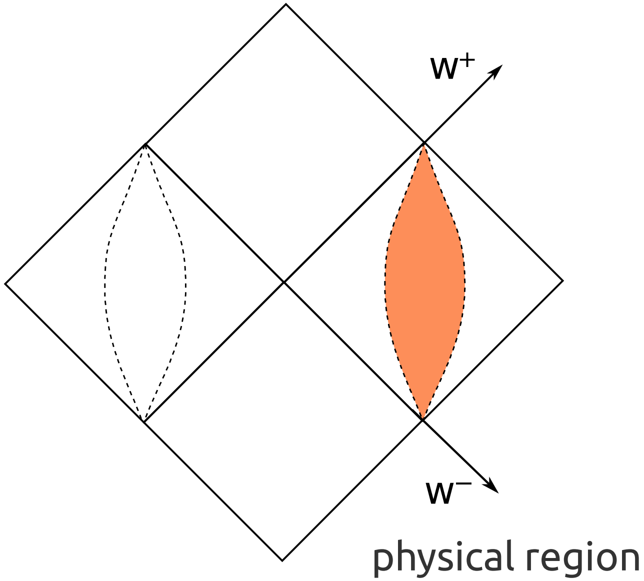


fixed frequency ↔ Δ_{AB}

- conformal block ↔ non-normalizable classical bulk field that is smooth in the interior of euclidean AdS
  + shadow

Bulk fields in BTZ

The BTZ black hole



- rotating black hole in AdS → coordinates r, t, ϕ

  → horizons at $r_\pm$

- locally pure AdS3 $$ds^2 = \frac{dw^+ dw^- + dz^2}{z^2}$$

$$w^\pm = \left(\frac{r^2 - r_+^2}{r^2 - r_-^2}\right)^{\frac{1}{2}} e^{2\pi T_\pm(\phi \mp t)}$$

$$T_\pm = \frac{1}{2\pi}(r_+ \pm r_-)$$

$$z = \left(\frac{r_+^2 - r_-^2}{r^2 - r_-^2}\right)^{\frac{1}{2}} e^{\pi T_+(\phi - t) + \pi T_-(\phi + t)}$$

- two quadrants → two sides of eternal BTZ

Using conformal blocks to study BTZ:

- model BTZ as a heavy pure state $|\Psi\rangle = \mathcal{O}_H|0\rangle \qquad \Delta_H \propto c$
- dual to single-sided black hole in AdS
- go to euclidean BTZ $t \to -it_E \;, \quad r_- \in i\mathbb{R}$

Thermalization in CFT$_2$

Fitzpatrick, Kaplan, Walters ‘14-’15

heavy-light **Virasoro block**

$$h_A, h_B \propto c$$

$$h_C, h_D, h_{AB}, h \propto \mathcal{O}(1)$$

⇔

global conformal block

in a conformally transformed

background w

$$\mathcal{V}_{\mathcal{O}}(h_i, z) = (2\pi i T_+)^{h_D - h}(w'(z))^{h_C}\, G_{\mathcal{O}}\left(\frac{h_{AB}}{2\pi i T_+}, h_{CD}, h, w\right)$$

$$\boxed{w = z^{2\pi i T_+}\ , \qquad \bar{w} = \bar{z}^{-2\pi i T_-}}$$

same coordinate transformation

as BTZ → Poincaré AdS

- **thermalization** $\langle O_H(\infty) O_H(0) O_L(z) O_L(1)\rangle \sim \langle O_H(\infty) O_H(0)\, \mathbb{P}_I^{Vir} O_L(z) O_L(1)\rangle =$

$$= \langle O_H(\infty) O_H(0)\, \mathbb{P}_I^{global} O_L(w) O_L(1)\rangle \propto \underbrace{\langle O_L(w) O_L(1)\rangle}_{\text{vacuum}} \sim \underbrace{\langle O_L(t) O_L(0)\rangle_T}_{\text{thermal}}$$

- **factorization**

- **HKLL formula** $\Phi^{(0)}(z, w^{\pm}) = \int d^2w'\, K_{Poincaré}(z, w^{\pm}|w'^{\pm})\, \mathcal{O}(w'^{\pm})$ $\Big\{$ agrees with BTZ expression; **state-dependent!**

Implications

$$\mathcal{V}_{\mathcal{O}}(z) \underset{\text{large } \mathbf{c}}{=} \underbrace{G_{\mathcal{O}}(w) = \int_{\gamma_{CD}} d\lambda\, e^{-\lambda\Delta_{CD}}\, \Phi^{(0)}_{\gamma_{AB}}(w;\lambda)}_{\text{kinematic}} \underset{\text{BTZ locally AdS}}{=} \int_{-\infty}^{\infty} d\lambda\, e^{-\lambda\Delta_{CD}}\, \Phi^{(0)}_{BTZ}(\lambda)$$

- check $h_{AB} \to \dfrac{h_{AB}}{2\pi i T_+}$ same
- state-independent way to extract bulk field from Virasoro conformal block
- obeys wave equation in BTZ backround because it's related to the effective global block

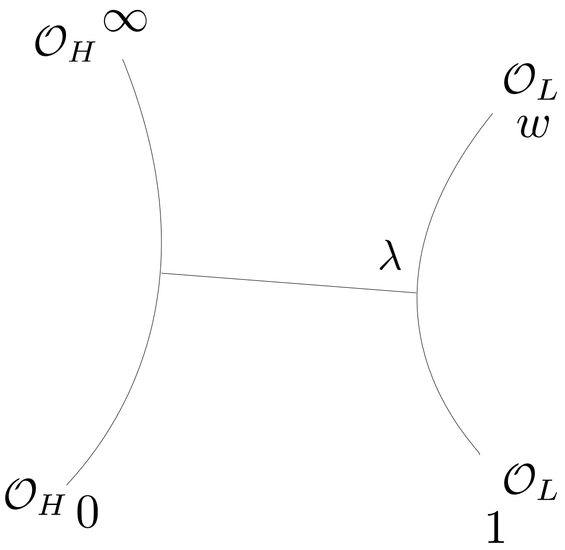
CFT z - plane ↔ effective block w -plane ↔ w - vacuum AdS ↔ BTZ radial plane

- BTZ radial plane ↔ conformal ratios in $\underline{w}$ - plane
- BTZ monodromies ↔ monodromies of the effective global conformal block

w - plane real!!!

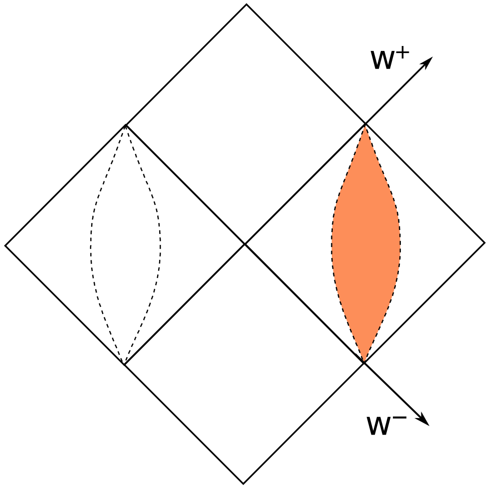
Mapping the singular points

- BTZ radial wave equation → singular points at $r_\pm, \infty$
- map to CFT conformal ratios on w plane $$\frac{r^2(\lambda)-r_-^2}{r_+^2-r_-^2} = \frac{1+e^{2\lambda}}{|1-w|^2}\left(1+|w|^2 e^{-2\lambda}\right)$$



$$\left.\begin{array}{l} \infty : \quad w, \bar{w} \to 1 \\ r_+ : \quad w, \bar{w} \to 0\,,\ \lambda \to -\infty \end{array}\right\} \text{Euclidean}$$

$$\left. r_- : \quad w \to \infty, \quad \bar{w} \to 0 \quad \right\} \text{Lorentzian!}$$



Going behind the horizon

→ need light insertions to belong to different quadrants!

→ reconstructs eternal BTZ

Lessons for higher dimensions

Lessons from 2d

1. Heavy operators act as a thermal background for the light ones

$$\langle \mathcal{O}_H | \mathcal{O}_L \ldots \mathcal{O}_L | \mathcal{O}_H \rangle \approx \langle \mathcal{O}_L \ldots \mathcal{O}_L \rangle_T$$

2. Virasoro heavy-light conformal block = effective global block (effective coordinate)

(resums interactions between the light operators and the background)

All bulk dynamics takes place in this effective coordinate.

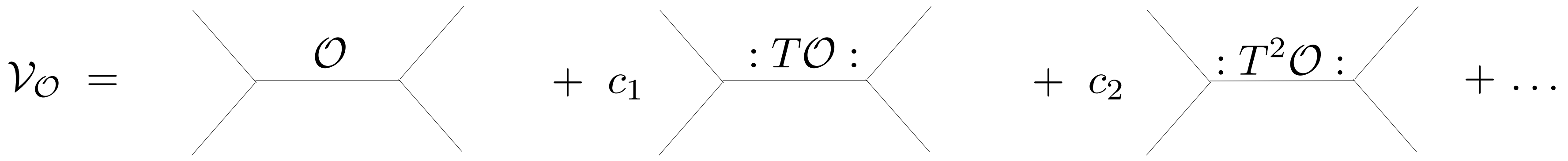
bulk Laplacian ↔ effective boundary Casimir (state-dependent)

3. The bulk field can be extracted from the effective block via a (state-independent) inverse

Mellin transform

- frequency of bulk field ↔ Δ_{AB}
- near-horizon region ↔ $z, \bar{z} \to 1$ kinematic limit

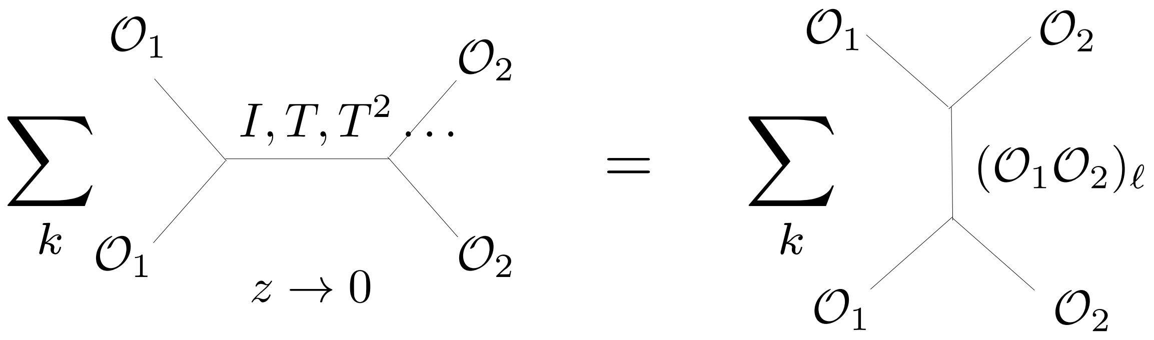
Effective conformal blocks in higher d



global $SL(2,\mathbb{R})$ blocks

- coefficents c_k fixed by Virasoro symmetry
- generalization to $d > 2$: define effective block as $G^{eff}_{\mathcal{O}}(z,\bar{z}) = \sum_k c_k \, G_{:T^k\mathcal{O}:}(z,\bar{z})$
- coefficents c_k fixed by analytic bootstrap in the kinematic limit $z \to 0 \; , \quad \bar{z} \to 1$

Fitzpatrick, Kaplan, Walters, Wang '15



$$\Delta = \Delta_1 + \Delta_2 + 2\ell + \gamma_\ell$$

$$\bar{z} \to 1$$

- evidence that $G^{eff}_{\mathcal{O}}(z) = G_{\mathcal{O}}(z_{eff})$ (with F. Alday)

Conclusions and future directions

- extract bulk field from boundary conformal block in a state-independent way
- argued bulk dynamics is captured by an effective global block depending on an effective CFT coordinate → state-dependent bulk equations of motion
- analysis can be generalized to higher dimensions
- can the CFT tell us where the spacetime stops being smooth (breakdown of analytic continuation)?
- can we understand the bulk field in a black hole background without using conformal blocks (just by including 1/N corrections to HKLL)?

Thank you !