Tesytlin string, $O(D, D)$ and Seiberg-Witten map

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Strings 2016, Tsinghua University

Outlines

1. Tseytlin string
2. Open string configuration and Seiberg-Witten map
3. Seiberg-Witten map from $O(D, D)$
4. The general descriptions of Seiberg-Witten map
5. Open-Closed configuration (Controversial), time permitting

Joint work with Peng Wang and Houwen Wu. Based on arXiv:1501.01550,1505.02643 and works in progress.

Q: Can we manifest T-duality in the world sheet string action?

Tseytlin string: $O(D, D)$ symmetry and T-duality

To achieve this purpose, we bear in mind:

1. The continuous $O(D, D)$ symmetry is defined as $\Omega\eta\Omega^T = \eta$,

$$\eta_{MN} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

2. Compactification of $d = D - n$ dimensions breaks the continuous $O(D, D)$ into an $O(n, n) \times O(d, d; \mathbb{Z})$ group.
3. $O(n, n)$ relates flat background, and $O(d, d; \mathbb{Z})$ represents T-duality in the compactified background.

How about an intermediate theory: Polyakov $+ \; O(D, D)$?

Tseytlin string: Action

Bosonic $O(D,D)$ invariant extension of Polyakov action is the Tseytlin's action (Tseytlin 1990PLB; 1991 NPB)

$$S = -\frac{1}{4\pi\alpha'}\int_{\Sigma}\left(-\partial_1 X^M \mathcal{H}_{MN}\partial_1 X^N + \partial_1 X^M \eta_{MN}\partial_0 X^N\right),$$

where $\partial_0 = \partial_\tau$, $\partial_1 = \partial_\sigma$ and

$$\mathcal{H}_{MN} = \begin{pmatrix} g & -gB^{-1} \\ B^{-1}g & g^{-1} - B^{-1}gB^{-1} \end{pmatrix}, \quad \eta_{MN} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad X^M = \begin{pmatrix} X^i \\ \tilde{X}_i \end{pmatrix},$$

where $M, N = 1, 2, \ldots, 2D$ are $O(D,D)$ indices,

g is D dimensional spacetime metric,

B is an anti-symmetric field.

Originally, the Tseytlin string was proposed for closed string only!

Tseytlin string: EOM

The EOM and boundary conditions can be obtained by varying the action,

$$
\begin{aligned}
\delta S \quad = \quad & -\frac{1}{2\pi\alpha'}\int_\Sigma \delta X^M \partial_1 \left(\mathcal{H}_{MN}\partial_1 X^N - \eta_{MN}\partial_0 X^N\right) \\
& -\frac{1}{2\pi\alpha'}\int_\Sigma \partial_1 \left[\delta X^M \left(\mathcal{H}_{MN}\partial_1 X^N - \frac{1}{2}\eta_{MN}\partial_0 X^N\right)\right] \\
& -\frac{1}{4\pi\alpha'}\int_\Sigma \partial_0 \left[\delta X^N \eta_{MN}\partial_1 X^M\right] \\
& +\frac{1}{4\pi\alpha'}\int_\Sigma \delta X^M \partial_1 X^A \partial_M \mathcal{H}_{AN}\partial_1 X^N,
\end{aligned}
$$

The EOM is

$$
\partial_1 \left(\mathcal{H}_{MN}\partial_1 X^N - \eta_{MN}\partial_0 X^N\right) = \frac{1}{2}\partial_1 X^A \partial_M \mathcal{H}_{AN}\partial_1 X^N.
$$

The annoying term on the r.h.s. turns out to be immaterial for our discussions. So the EOM can be integrated to first order PDE.

Tseytlin string: Closed-Closed configuration

For simplicity, we consider vanishing B field at first, so the EOM in components

EOM

$$g_{ij}\partial_1 X^j - \partial_0 \tilde{X}_i = f_1(\tau)\,, \qquad g^{ij}\partial_1 \tilde{X}_j - \partial_0 X^i = f_2(\tau)\,.$$

B.C.

$$\delta X^i \left(g_{ij}\partial_1 X^j - \frac{1}{2}\partial_0 \tilde{X}_i \right) + \delta \tilde{X}_i \left(g^{ij}\partial_1 \tilde{X}_j - \frac{1}{2}\partial_0 X^i \right) \Bigg|_{\partial\Sigma} = 0.$$

1. Closed-Closed boundary condition

$$\tilde{X}(\sigma,\tau) = \tilde{X}(\sigma+2\pi,\tau), \qquad \text{and} \qquad X(\sigma,\tau) = X(\sigma+2\pi,\tau).$$

EOM is simplified by shifting X and $\tilde{X}$

$$g_{ij}\partial_1 X^j - \partial_0 \tilde{X}_i = 0, \qquad g^{ij}\partial_1 \tilde{X}_j - \partial_0 X^i = 0.$$

- After integrating out $\tilde{X}$ (or X), we recover the Polyakov string.
- The low energy limit is Double Field Theory.
- Open question: non-commutative gravity?

A natural question is that:

Can the Tseytlin string also describes open strings $O(D, D)$ covariantly?

Open-open configuration

EOM

$$g_{ij}\partial_1 X^j - \partial_0 \tilde{X}_i = f_1(\tau)\,, \qquad g^{ij}\partial_1 \tilde{X}_j - \partial_0 X^i = f_2(\tau)\,.$$

B.C.

$$\delta X^i \left(g_{ij}\partial_1 X^j - \frac{1}{2}\partial_0 \tilde{X}_i \right) + \delta \tilde{X}_i \left(g^{ij}\partial_1 \tilde{X}_j - \frac{1}{2}\partial_0 X^i \right) \Bigg|_{\partial\Sigma} = 0.$$

2. Open-Open boundary condition (Polyakov, Wang, Wu and Yang arXiv:1501.01550)
One $O(D,D)$ covariant B.C. is

$$\begin{aligned} \delta \tilde{X}\Big|_{\partial\Sigma} &= \partial_0 \tilde{X}\Big|_{\partial\Sigma} = 0, \\ g\partial_1 X - \frac{1}{2}\partial_0 \tilde{X}\Big|_{\partial\Sigma} &= 0 \Rightarrow \partial_1 X|_{\partial\Sigma} = 0. \end{aligned}$$

precisely represents an open string configuration. (Another equivalent scenario is achieved by exchanging X and $\tilde{X}$). Applying the EOM on B.C to find $f_1(\tau) = 0$. $f_2(\tau)$ can be removed by shifting $X \to X - \int d\tau f_2$. Again, the Polyakov string is recovered after integrating out $\tilde{X}$ (or X).

EOM and BC

The $O(D, D)$ covariant open-open configuration is

EOM

$$g_{ij}\partial_1 X^j - \partial_0 \tilde{X}_i = 0, \qquad g^{ij}\partial_1 \tilde{X}_j - \partial_0 X^i = 0.$$

The second order EOM and B.C. are

$$\begin{aligned} ({\partial_1}^2 - {\partial_0}^2)X &= 0, \\ \partial_1 X|_{\partial\Sigma} &= 0, \end{aligned}$$

and

$$\begin{aligned} ({\partial_1}^2 - {\partial_0}^2)\tilde{X} &= 0, \\ \partial_0 \tilde{X}\Big|_{\partial\Sigma} &= 0. \end{aligned}$$

Propagators and Non-commutativity

It is easy to figure out the propagators

$$\left\langle X^{i}\left(z,\bar{z}\right)X^{j}\left(z',\bar{z}'\right)\right\rangle = -\alpha'\left(g^{ij}\log\left|z-z'\right| + g^{ij}\log\left|z-\bar{z}'\right|\right). \qquad (1)$$

$$\left\langle \tilde{X}_{i}\left(z,\bar{z}\right)\tilde{X}_{j}\left(z',\bar{z}'\right)\right\rangle = -\alpha'\left(g_{ij}\log\left|z-z'\right| - g_{ij}\log\left|z-\bar{z}'\right|\right). \qquad (2)$$

From these two propagators, X and $\tilde{X}$ are both commutative. From the first order EOM, the mixed propagators are

$$\begin{aligned} \left\langle X^{i}\left(z,\bar{z}\right)\tilde{X}_{j}\left(z',\bar{z}'\right)\right\rangle &= -\frac{\alpha'}{2}g^{ik}g_{kj}\left(\log\frac{z-z'}{\bar{z}-\bar{z}'} - \log\frac{z-\bar{z}'}{\bar{z}-z'}\right), & (3)\\ \left\langle \tilde{X}_{i}\left(z,\bar{z}\right)X^{j}\left(z',\bar{z}'\right)\right\rangle &= -\frac{\alpha'}{2}g_{ik}g^{kj}\left(\log\frac{z-z'}{\bar{z}-\bar{z}'} + \log\frac{z-\bar{z}'}{\bar{z}-z'}\right). & (4)\end{aligned}$$

where non-commutativity arises on the boundary

$$\begin{aligned} [X^{i}(\tau), X^{j}(\tau')] &= [\tilde{X}^{i}(\tau), \tilde{X}^{j}(\tau')] = 0,\\ [\tilde{X}_{i}(\tau), X^{j}(\tau)] &= i2\pi\alpha'\delta_{i}{}^{j}. \qquad (5)\end{aligned}$$

Implication: T-dual fields are non-commutative!

We now go to a general phase frame by a pure coordinate transformation

$$\Omega = \begin{pmatrix} 1 & -B^{ij} \\ 0 & 1 \end{pmatrix}, \tag{6}$$

where B^{ij} is an antisymmetric tensor. The generalized metric h_{MN} is then rotated to

$$H_{MN} = \Omega^T h_{MN} \Omega = \begin{pmatrix} g & -gB^{-1} \\ B^{-1}g & g^{-1} - B^{-1}gB^{-1} \end{pmatrix}, \tag{7}$$

accompanied by the coordinate transformation

$$\begin{aligned} X^{i\prime} &= X^i + B^{ij}\tilde{X}_j, \\ \tilde{X}'_j &= \tilde{X}_j. \end{aligned} \tag{8}$$

It is easy to see that $\tilde{X}'$ is still commutative but X' is non-commutative on the boundary from the propagator

Propagators:

$$\begin{aligned}
\left\langle X^{i\prime}(z,\bar z)\, X^{j\prime}(z',\bar z')\right\rangle &= -\alpha'\Big[\left(g^{ij} - B^{ik} g_{k\ell} B^{\ell j}\right) \log\left|z - z'\right| \\
&\quad + \left(g^{ij} + B^{ik} g_{k\ell} B^{\ell j}\right) \log\left|z - \bar z'\right| + B^{ij}\left(\log\frac{z-\bar z'}{\bar z - z'}\right)\Big], \\
\left\langle \tilde X'_i(z,\bar z)\, \tilde X'_j(z',\bar z')\right\rangle &= -\alpha'\left(g_{ij}\log\left|z-z'\right| - g_{ij}\log\left|z-\bar z'\right|\right), \\
\left\langle \tilde X'_i(z,\bar z)\, X^{j\prime}(z',\bar z')\right\rangle &= -\frac{\alpha'}{2}\delta_i^{\ j}\left(\log\frac{z-z'}{\bar z-\bar z'} + \log\frac{z-\bar z'}{\bar z - z'}\right) \\
&\quad -\alpha' B_i^{\ j}\left(\log\left|z-z'\right| - \log\left|z-\bar z'\right|\right). \qquad (9)
\end{aligned}$$

with commutators

$$\begin{aligned}
\left[X^{i\prime}(\tau), X^{j\prime}(\tau)\right] &= i\pi\alpha' B^{ij} \\
[\tilde X'_i(\tau), \tilde X'_j(\tau)] &= 0 \\
[\tilde X'_i(\tau), X^{j\prime}(\tau)] &= i2\pi\alpha'\delta_i^{\ j}.
\end{aligned}$$

Expressed with g, B, we thus expect that the DBI of X is non-commutative but that of $\tilde{X}$ is commutative (We remove the prime for convenience). Applying the corresponding EOM and B.C.to remove half of the D.O.F, we find the DBI

$$S_{DBI}(X) = \frac{1}{g_s} \int d^D x \sqrt{\det\left(\frac{1}{g^{-1} + B^{-1}} + F(x)\right)}, \tag{10}$$

which is non-commutative by the Seiberg-Witten map

$$F^* = \frac{1}{1 + FB^{-1}} F. \tag{11}$$

The DBI of $\tilde{X}$ is

$$S_{DBI}(\tilde{X}) = \frac{1}{g_s} \int d^D \tilde{x} \sqrt{\det(g^{-1} + B^{-1} + F(\tilde{x}))}, \tag{12}$$

which is commutative.

Open-Closed relation as an $O(D,D)$ element

X and $\tilde{X}$ are $O(D,D)$ related by

$$X \leftrightarrow \tilde{X}, \qquad g \leftrightarrow \hat{g}^{-1}, \qquad B \leftrightarrow \hat{B}^{-1}$$

with the identification

$$\eta \begin{pmatrix} \hat{g}^{-1} & -\hat{g}^{-1}\hat{B} \\ \hat{B}\hat{g}^{-1} & \hat{g} - \hat{B}\hat{g}^{-1}\hat{B} \end{pmatrix} \eta = \begin{pmatrix} g & -gB^{-1} \\ B^{-1}g & g^{-1} - B^{-1}gB^{-1} \end{pmatrix}.$$

solved precisely by the open-closed relations:

$$\begin{aligned} g_{ij} &= \left(\hat{g} - \hat{B}\hat{g}^{-1}\hat{B}\right)_{ij}, \\ B^{ij} &= -\left(\frac{1}{\hat{g}+\hat{B}}\hat{B}\frac{1}{\hat{g}-\hat{B}}\right)^{ij}, \\ \hat{g}^{ij} &= \left(g^{-1} - B^{-1}gB^{-1}\right)^{ij} \\ \hat{B}^{ij} &= \left(B^{-1} - g^{-1}Bg^{-1}\right)^{ij} \end{aligned}$$

Thus, if we rotate X instead of $\tilde{X}$, we will get commutative X theory and non-commutative $\tilde{X}$ theory expressed by $\hat{g}$ and $\hat{B}$.

The Seiberg-Witten map can be interpreted within $O(D, D)$!

The general descriptions of Seiberg-Witten map

In the Seiberg-Witten map

$$F^* = \frac{1}{1 + FB^{-1}} F, \tag{13}$$

where

$$B^{ij} = -\left(\frac{1}{\hat{g} + \hat{B}} \hat{B} \frac{1}{\hat{g} - \hat{B}} \right)^{ij}, \tag{14}$$

is fixed. It is proposed to generalize the map to

$$F^* = \frac{1}{1 + F\theta} F, \tag{15}$$

for varying θ. This can be naturally realized in $O(D, D)$ formalism by an extra rotation

$$\Omega' = \begin{pmatrix} 1 & -B^{-1} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\Phi & 1 \end{pmatrix} = \begin{pmatrix} 1 + B^{-1}\Phi & -B^{-1} \\ -\Phi & 1 \end{pmatrix}, \tag{16}$$

where B and Φ are two-forms.

Then after a careful identification of the rotation and tedious calculation, we have the DBI

$$\begin{aligned} S_{DBI} &= \frac{1}{g_s}\int d^D x\sqrt{\det\left(\frac{1}{g^{-1}+B^{-1}}+F+\Phi\right)} \\ &= \frac{1}{G_s}\int d^D x\sqrt{\det\left(g+F^*\right)}, \end{aligned}$$

with the non-commutative gauge field defined

$$F^* = \frac{1}{1+F\theta}F,$$

and the constraint for θ

$$g^{-1}+\theta = \frac{1}{\Phi+\frac{1}{g^{-1}+B^{-1}}}.$$

θ is free to vary provided Φ varying accordingly for fixed g and B. So, $O(D,D)$ group parameter Φ plays the role of the general description parameter.

Open-Closed configuration (Controversial), time permitting

It is curious to ask:

Q: Is an Open(X)-Closed($\tilde{X}$) configuration allowed?

The third boundary condition

Look at the B.C. again

B.C.

$$\begin{aligned} \delta X^i \left(g_{ij}\partial_1 X^j - \frac{1}{2}\partial_0 \tilde{X}_i \right) + \delta \tilde{X}_i \left(g^{ij}\partial_1 \tilde{X}_j - \frac{1}{2}\partial_0 X^i \right) \Big|_\sigma &= 0, \\ \delta X^i \partial_1 \tilde{X}_i + \delta \tilde{X}_i \partial_1 X^i |_\tau &= 0, \end{aligned}$$

We missed a third $O(D, D)$ covariant boundary condition!

$$\left(g_{ij}\partial_1 X^j - \frac{1}{2}\partial_0 \tilde{X}_i \right) \Big|_\sigma = \left(g^{ij}\partial_1 \tilde{X}_j - \frac{1}{2}\partial_0 X^i \right) \Big|_\sigma = 0.$$

Note the Polyakov action cannot be reproduced with this B.C.

The third boundary conditions

To consider this boundary condition, we can again absorb $f_i(\tau)$ by shifting X and $\tilde{X}$

$$\tilde{X} \to \tilde{X} - \int d\tau f_1(\tau), \qquad X \to X - \int d\tau f_2(\tau).$$

Then the decoupled second order EOM is

$$({\partial_1}^2 - {\partial_0}^2)X = 0, \qquad ({\partial_1}^2 - {\partial_0}^2)\tilde{X} = 0,$$

with the first order constraint,

$$g\partial_1 X - \partial_0 \tilde{X} = 0, \qquad g^{-1}\partial_1 \tilde{X} - \partial_0 X = 0,$$

and the boundary conditions (good news and bad news: B.C. is the same as EOM),

$$\begin{aligned} \delta X^i \left(g_{ij}\partial_1 X^j - \partial_0 \tilde{X}_i\right) + \delta\tilde{X}_i \left(g^{ij}\partial_1 \tilde{X}_j - \partial_0 X^i\right)|_\sigma &= 0, \\ g_{ij}\delta X^i \partial_0 X^j + g^{ij}\delta\tilde{X}_i \partial_0 \tilde{X}_j|_\tau &= 0. \end{aligned}$$

How to get Open-Closed? Decoupling of X and $\tilde{X}$ near the boundary only!!

$$g_{ij}|_{\partial\Sigma} \gg 1 \qquad \text{or} \qquad g_{ij}|_{\partial\Sigma} \ll 1$$

Required metric

From general guidances:

- Generalize Tseytlins action to nonlinear double sigma model.
- Near the boundaries, $g_{ij} \gg 1$ or $g_{ij} \ll 1$.
- For D-branes, $g_{\mu\nu}$ is reciprocal of g_{ab} and $g_{\mu a} = 0$.
- Metric on D-branes is conformally flat.
- $D = 5$ from the symmetry group of M theory.
- Consistent with Einstein equation.

it turns out that the only consistent choice is AdS_5:

$$ds^2 = \frac{r^2}{c^2}\eta_{\mu\nu}dx^\mu dx^\nu + \frac{c^2}{r^2}dr^2$$

Open-Closed configuration

It is crucial to remember:

- X and $\tilde{X}$ are always $O(D,D)$ related.
- EOM

$$\begin{aligned} g\partial_1 X - \partial_0 \tilde{X} &= 0, \\ g^{-1}\partial_1 \tilde{X} - \partial_0 X &= 0, \end{aligned}$$

couple the dual fields in the bulk.

After realizing the decoupling of X and $\tilde{X}$ near the boundaries, it is easy to understand that open/closed strings are $O(D,D)$ equivalent in an asymptotic AdS background!

Thank you!

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