Tachyon field theory description of (thermo)dynamics in dS space

Jian-Xin Lu

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(with Huiquan Li)

ICTS
University of Science & Technology of China

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Outline

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- Massive probe in static dS patch and unstable D0 (or homogeneous Dp) brane
- dS universe and S-brane
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Motivation

In string theory,

in addition to time-like D-branes,

there exist also the so-called space-like D-branes!

- describe the dynamics of unstable D-branes such as D-$\bar{\mathrm{D}}$-brane or non-BPS D-branes,
- exist only for a moment of time.

Motivation

In low energy,

from the unstable D(p + 1)-brane worldvolume,

SDp-branes

- arise as the time-dependent solutions of the tachyon field theory,
- describe the creation and subsequent decay of this unstable system.

Motivation

While from the bulk spacetime,

they are the time dependent solution (Chen et al (02); Kruczenski et al (02)):

$$ds^2 = F(\tau)^{\frac{p+1}{8}} g(\tau)^{\frac{1}{7-p} - \frac{p(p+1)}{8(7-p)}\delta_0} \left(-d\tau^2 + \tau^2 dH^2_{8-p}\right) + F(\tau)^{-\frac{7-p}{8}} g(\tau)^{\frac{p}{8}\delta_0} \sum_{i=1}^{p+1} (dx^i)^2, \quad (1.1)$$

where $(\tau \geq 0)$

$$F(\tau) = g(\tau)^{\alpha/2}\cos^2\theta + g(\tau)^{-\beta/2}\sin^2\theta, \quad g(\tau) = 1 + \frac{\tau_0^{7-p}}{\tau^{7-p}}, \quad (1.2)$$

with θ & τ_0 related to the charge, parameters δ_0, α & β satisfying

$$\alpha - \beta = 3\delta_0, \quad \frac{14+5p}{7-p}\delta_0^2 + \frac{1}{2}\alpha(\alpha - 3\delta_0) = \frac{8-p}{7-p}. \quad (1.3)$$

Motivation

A few remarks:

- SDp are time-dependent and non-SUSY branes, characterizing a dynamical process.
- they are located at $\tau = 0$, therefore existing only for a moment of time.
- from $\tau = 0$ to $\tau = \infty$, it represents the decay of the SDp.
- unlike the usual Dp, the so-called 'near-brane' limit of SDp has so far not been proved to give rise to $dS_{p+2} \times H_{8-p}$ space.

Motivation

- It gives at most something to take the form of (p + 1) + 1 dimensional dS space up to a conformal transformation, upon compactification on Hyperbolic space H_{8-p}. (see Nayek and Roy (2015)).
- This implies that only for very low energy modes, the 'near-brane' geometry of the bulk solution can be dS_{p+2}!

Motivation

So for these very low energy modes, one expects

$(p+2)$-dimensional tachyon field theory dynamics

$$\overset{?}{\Longleftrightarrow}$$

dynamics in dS_{p+2}.

Now focusing on the low energy dynamics:

dS spacetime

Let us first recall the dS spacetime structure (Spradlin et al (01)):

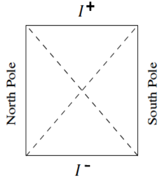


Figure 2: Penrose diagram for dS_d. The north and south poles are timelike lines, while every point in the interior represents an S^{d-2}. A horizontal slice is an S^{d-1}. The dashed lines are the past and future horizons of an observer at the south pole. The conformal time coordinate T runs from $-\pi/2$ at $\mathcal{I}^-$ to $+\pi/2$ at $\mathcal{I}^+$.

The static dS patch

As a first example, we would like to ask: what is the worldvolume correspondence of a massive probe in a static dS patch (the southern causal diamond)?

The metric is

$$ds^2 = -\left(1-\beta^2 r^2\right) dt^2 + \left(1-\beta^2 r^2\right)^{-1} dr^2 + r^2 d\Omega^2_{(d-2)}, \quad (2.1)$$

where $0 \le r \le r_H = 1/\beta$, with the pole at $r = 0$ and the horizon at $r_H = 1/\beta$, and the free constant β will attain a new meaning in string theory context.

The static dS patch

The Hawking temperature of this dS is

$$T_H = \frac{\beta}{2\pi}. \tag{2.2}$$

Re-define

$$T = r_H \tanh^{-1}\left(\frac{r}{r_H}\right), \tag{2.3}$$

then $r \in [0, r_H] \Longleftrightarrow T \in [0, \infty]$, and the above metric (2.1) becomes

A probe on static dS patch

$$ds^2 = \frac{1}{\cosh^2(\beta T)}\left[-dt^2 + dT^2 + r_H^2 \sinh^2(\beta T) d\Omega^2_{(d-2)}\right]. \quad (2.4)$$

- Near-origin geometry $\Longrightarrow$ a d-dimensional Minkowski spacetime,
- Near-horizon geometry $\Longrightarrow$ the Rindler spacetime times a (d - 2)-sphere.

The dS metric therefore interpolates between the two.

A probe on static dS patch

Consider now a massive probe with mass τ_0 moving along the radial direction in this background.

The geodesic can be described by the action

$$\begin{aligned} S_0 &= -\tau_0 \int dt \sqrt{-g_{\mu\nu}\dot{X}^\mu \dot{X}^\nu}, \\ &= -\int dt\, V(T)\sqrt{1-\dot{T}^2}, \qquad V(T) = \frac{\tau_0}{\cosh(\beta T)}, \end{aligned} \quad (2.5)$$

where the dot denotes the derivative with respect to t.

A probe on static dS patch and a tachyon on the worldvolume

- The second line of above turns out to be exactly the one for non-BPS D-particle (or homogeneous non-BPS brane) with the tachyon field T and the correct tachyon potential $V(T)$ from open string field theory.
- In other words, the radial geodesic motion of a massive particle in the causal diamond is nothing but the dynamics of the tachyon field of an unstable D-particle(or homogeneous unstable Dp-brane with $p = d - 1$).
- In order to bring this connection, we need to identify

$$\beta = \begin{cases} 1/(2l_s) & \text{for bosonic string} \\ 1/(\sqrt{2}\, l_s) & \text{for superstring} \end{cases} \tag{2.6}$$

A probe on static dS patch and a tachyon on the worldvolume

Let us take a close look of the connection between the two:

$T = 0$ (open string vacuum)

$\Updownarrow$

Near-origin geometry (the Minkowski spacetime),

&

$T = \infty$ (closed string vacuum)

$\Updownarrow$

Near-horizon geometry (the Rindler times a sphere).

A probe on static dS patch and a tachyon on the worldvolume

- So the tachyon rolling from the top of the potential to the closed string vacuum can be viewed as a geodesic motion from $r = 0$ (the south pole) to the horizon at $r = r_H$.
- therefore provide geometric pictures for the two vacua and the tachyon rolling process in the tachyon field theory of unstable brane, respectively.

A geometric understanding of tachyon condensation

The geometric picture may provide certain understanding of the tachyon condensation.

- The tachyon action (2.5) gives 00-component of E-M tensor

$$T_{00} = \frac{V(T)}{\sqrt{1 - \dot{T}^2}} = E \qquad (2.7)$$

- The energy is conserved due to the classical limit $g_s = 0$, i.e., ignoring the closed string radiation, giving rise to the end products as pressureless tachyon matter.
- The equation (2.7) implies mathematically that the tachyon field will accelerate to the critical value $|\dot{T}| = 1$ in the limit of $T \to \infty$, i.e. reaching the speed of light , consistent also with the geometric picture of geodesic at the horizon.

A probe on static dS patch and a tachyon on the worldvolume

Classically, so far so good!

We now go one step further considering their thermal properties (semi-classically).

- Note the derived Hawking temperature (2.3) $T_H = \beta/2\pi$ turns out to be true for any observer moving along a time-like geodesic in dS space (Spradlin et al (01)).

A probe on static dS patch and a tachyon on the worldvolume

- The temperature felt by an observer at $T \to \infty$ for thermal radiation in the $T = 0$ vacuum can be obtained using the tachyon field theory as $T_{\text{tachyon}} = \beta/2\pi$.

(2.7) $\Rightarrow \dot{T}^2 - 1 + V^2/E^2 = 0 \Leftrightarrow \ddot{T} = -VV'/E^2$

We can have $L(T, \dot{T}) = (\dot{T}^2 + 1 - V^2/E^2)/2$ and
$H = (\dot{T}^2 - 1 + V^2/E^2)/2 = (p_T^2 - 1 + V^2(T)/E^2)/2$

Classical $H = 0 \Rightarrow$ quantum mechanically $H\psi(T) = 0$.

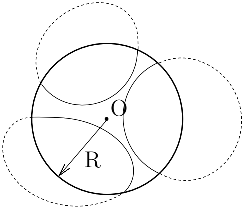
A probe on static dS patch and a tachyon on the worldvolume

- In string context, i.e., with the β given in (2.6), the thermal particles (closed string origin) in dS and the particles (open string origin) from tachyon field theory have the same thermal temperature.
- further this temperature turns out to be the Hagedorn one, signaling a phase transition of the decaying branes (open strings) into closed strings.

A probe on static dS patch and a tachyon on the worldvolume

Then how to understand this transition from the dS?

One possible way is ($R = \pi r_H/2 \sim l_s$)



Observer O sees only part of the closed strings

dS and S-brane

If consider anything more than that covered by the southern (or northern) causal diamond, we enter into a dS universe in other coordinates.

We try to seek a connection between the dynamics in d-dimensional dS universe and that in tachyon field theory of unstable Dp-brane with $d = p + 1$. For simplicity, here consider the expanding universe in planar coordinates and the corresponding tachyon field theory.

Consider first a homogeneous tachyon background relevant and the bosonic part of the DBI action of a non-BPS Dp-brane in flat Minkowski metric is

$$S_p = -\int d^{p+1}\sigma\, V(T)\sqrt{-\det(\eta_{\mu\nu} + \partial_\mu T \partial_\nu T + \cdots)}, \qquad (3.1)$$

where the tachyon potential $V(T) = \frac{\tau_p}{\cosh(\beta T)}$.

dS and S-brane

The tachyon background $T_0(\sigma^0)$ can be determined from the above action and satisfies the integrated equation:

$$\frac{V(T_0)}{\sqrt{1-\dot{T}_0^2}} = E, \tag{3.2}$$

where E is the conserved energy of the system in the limit $g_s = 0$.

The planar dS universe and the half S-brane

EOM for tachyon from its action (3.1) is

$$\left(\partial_\mu \partial^\mu T - V'/V\right)(1 + \partial T \cdot \partial T) = \frac{1}{2}\partial^\mu T \partial_\mu (1 + \partial T \cdot \partial T), \quad (3.3)$$

where $V' = dV/dT$.

Consider $T(\sigma^0, \vec{\sigma}) = T_0(\sigma^0) + \tau(\sigma^0, \vec{\sigma})$ with

$$\tau(\sigma^0, \vec{\sigma}) = \hat{\tau}(\sigma^0, \vec{\sigma}) / \cosh \beta T_0(\sigma^0). \quad (3.4)$$

To leading order in the perturbation,

$$\left[\partial^2_{\sigma^0} - \beta^2 - \frac{1}{l^2 \cosh^2(\beta T_0)} \vec{\nabla}^2\right] \hat{\tau}(\sigma^0, \vec{\sigma}) = 0. \quad (3.5)$$

where EOM for $T_0(\sigma^0)$ has been used.

The planar dS universe and the half S-brane

Now consider a probe scalar in dS universe in planar coordinates with metric

$$ds^2 = -dt^2 + e^{2\beta t} d\vec{x}^2_{(d-1)}, \tag{3.6}$$

where the spatial part of metric is a flat $(d-1)$-dimensional Euclidean space. It describes an expanding universe with time running from 0 to ∞.

With $\phi = e^{-(d-1)\beta t/2}\hat{\phi}(t,\vec{x})$, the probe scalar $\hat{\phi}(t,\vec{x})$ satisfies

$$\left[\partial_t^2 - \frac{1}{4}(d-1)^2\beta^2 + m_s^2 - e^{-2\beta t}\vec{\nabla}^2\right]\hat{\phi}(t,\vec{x}) = 0. \tag{3.7}$$

The planar dS universe and the half S-brane

We now try to seek under what conditions, (3.7) can be identified with (3.5), i.e., the scalar fluctuations in the dS with those of tachyon in a given tachyon background $T_0(\sigma^0)$.

- To be so, first the relevant tachyon background should be a future half S-brane solution of (3.2). In other words, we have $\sinh \beta T_0(\sigma^0) = \lambda e^{\beta \sigma^0}$.
- Secondly, consider $\sigma^0 = t$, running also from 0 to ∞, we need $\cosh \beta T_0(t) \approx \sinh \beta T_0(\sigma^0) = \lambda e^{\beta t}$, i.e. a large λ and a large T_0, so near by the closed vacuum.

The planar dS universe and the half S-brane

We then have from (3.5) for the tachyon fluctuation for the present case as

$$\left[\partial_t^2 - \beta^2 - \frac{1}{\lambda^2} e^{-2\beta t} \vec{\nabla}_{\vec{\sigma}}^2\right] \hat{\tau}(t, \vec{\sigma}) = 0, \tag{3.8}$$

where we have set $\sigma^0 = t$. Then both (3.7) and (3.8) can indeed be identified if we set $x^i = \lambda \sigma^i$ with $i = 1, \cdots, p$ and the scalar mass is given by $m_s^2 = (d-3)(d+1)\beta^2/4$ for any allowed d.

Summary

In this talk, we provide evidence supporting that the (thermo)dynamics in dS space can be described by the tachyon field theory of unstable D-branes. In particular,

- We show that the radial geodesic motion of a massive probe particle in static dS space turns out to be the same as that of the tachyon field theory derived from open string field theory.
- Further we show that certain low energy linear scalar dynamics in dS space can be identified with the tachyon fluctuations on a homogeneous tachyon background.

Summary

- In addition, we show that the thermal temperature of tachyon radiation agrees with that felt by any time-like observer in dS space. In string theory context, this temperature is actually the Hagedorn one, signaling a transition of open strings to closed strings.
- An understanding of this transition in dS space is also provided.

THANK YOU!