A One-Dimensional Theory for Higgs Branch Operators

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Based on work with Mykola Dedushenko and Ran Yacoby, to appear

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Motivation (Part I)

- Correlation functions of **local operators** are interesting observables in QFT.
- Hard to compute in interacting **non-perturbative** examples.
- **This talk:** computation of correlation functions of (certain) 1/2-BPS **local** operators in 3d $\mathcal{N} = 4$ QFTs.
  - Any number of operator insertions.
- Method: place theory on S^3 and use supersymmetric localization.

Supersymmetric localization

SUSic localizaiton works as follows:

- Choose supercharge $\mathcal{Q}$ such that $\mathcal{Q}^2 = \text{isometry} + R\text{-symmetry}$.
- Deform action by positive definite $\mathcal{Q}$-exact term:

$$S + t\{\mathcal{Q}, V\}\,.$$

- Can compute $\mathcal{Q}$-invariant observables by noticing that

$$\langle \mathcal{Q}\text{-inv. observable} \rangle = \int DX\, e^{-S - t\{\mathcal{Q}, V\}} (\mathcal{Q}\text{-inv. observable})$$

  is **independent of** t.
- Take $t \to \infty \implies$ path integral localizes to $\{\mathcal{Q}, V\} = 0$; saddle point approx becomes exact.
- **Path integral can simplify dramatically**, e.g. it can become a matrix model.

A trade-off

There is a trade-off between:

1. obtaining a simple theory after localization
   - Ideally, the isometry ($\subset \mathcal{Q}^2$) has no fixed points.
2. existence of $\mathcal{Q}$-invariant **local** operators.
   - $[\mathcal{Q}, \mathcal{O}(x)] = 0 \implies \quad [\mathcal{Q}^2, \mathcal{O}(x)] = 0 \quad \implies x \in$ fixed point of isometry ($\subset \mathcal{Q}^2$).

This talk: isometry ($\subset \mathcal{Q}^2$) fixes a great circle on $S^3 \implies$ 3d $\mathcal{N} = 4$ QFT localizes to a 1d theory living on a great circle of S^3.

- Certain local operators in the 3d $\mathcal{N} = 4$ QFT **are** observables in the 1d theory.

Motivation (Part II)

- $\mathcal{N} = 4$ **SCFTs** in flat space have two 1d topological sectors, one for **Higgs branch operators** and one for **Coulomb branch operators** [Beem, Lemos, Liendo, Peelaers, Rastelli '13; Chester, Lee, SSP, Yacoby '14; Beem, Peelaers, Rastelli '16] .
  - Argument based on properties of the superconformal algebra $\mathfrak{osp}(4|4)$.
- $\mathfrak{osp}(4|4) \supset \mathfrak{so}(3,2) \oplus \mathfrak{su}(2)_H \oplus \mathfrak{su}(2)_C$
  $\mathfrak{su}(2)_H$ fundamental indices: $a, b, c, \ldots$
  $\mathfrak{su}(2)_C$ fundamental indices: $\dot{a}, \dot{b}, \dot{c}, \ldots$
- Higgs branch operators = scalar ops w/ $\Delta = (\mathfrak{su}(2)_H \text{ spin})$
  - Example: scalars in hypermultiplet.
- Coulomb branch operators = scalar ops w/ $\Delta = (\mathfrak{su}(2)_C \text{ spin})$
  - Example: scalars in vector multiplet.

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Topological 1d theory

- Let me describe the topological theory for Higgs branch ops.

$$\mathcal{O}_{a_1 a_2 \ldots a_{2j}}(\vec{x}) \qquad\qquad \Delta = j$$

How it works:

- Define 1d "twisted Higgs branch operator"

$$\mathcal{O}(x) = u^{a_1}(x) u^{a_2}(x) \cdots u^{a_{2j}}(x) \mathcal{O}_{a_1 a_2 \ldots a_{2j}}(0,0,x)\,, \qquad u^a = (1, x)\,.$$

- The correlation functions

$$\langle \mathcal{O}_1(x_1) \cdots \mathcal{O}_m(x_m) \rangle$$

are independent of the distance between insertions, but depend on the ordering of the $x_i \Longrightarrow$ **topological 1d theory.**

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Topological 1d theory: Example

- Example: free hypermultiplet has complex scalars $q_a, \tilde{q}_a$. In 3d,

$$\langle q_a(\vec{x}_1)\tilde{q}_b(\vec{x}_2)\rangle = \frac{\epsilon_{ab}}{4\pi|\vec{x}_1 - \vec{x}_2|}\,, \qquad \epsilon_{21} = -\epsilon_{12} = 1\,.$$

- Twisted operators

$$Q(x) = q_1(0,0,x) + x q_2(0,0,x)\,, \qquad \tilde{Q}(x) = \tilde{q}_1(0,0,x) + x\tilde{q}_2(0,0,x)$$

- The 2-pt function of twisted operators is **topological**:

$$\langle Q(x_1)\tilde{Q}(x_2)\rangle = \frac{x_1 - x_2}{4\pi|x_1 - x_2|} = \frac{\mathrm{sgn}(x_1 - x_2)}{4\pi}\,.$$

Topological 1d theory

Why it works:

- One can find a supercharge $\mathcal{Q}^H$ (in fact, a 1-parameter family) such that
  - twisted operators $\mathcal{O}(x)$ are $\mathcal{Q}^H$-invariant.
  - $\partial_x \mathcal{O}$ is $\mathcal{Q}^H$-exact $\longrightarrow$ topological correlation functions.
- $\mathcal{Q}^H =$ Poincaré $+$ superconformal supercharge.

Rest of the talk: Obtain an action for this topological 1d theory.

- Map to S^3 and localize w.r.t. $\mathcal{Q}^H$.
- $(\mathcal{Q}^H)^2 =$ rotation around x_3 axis + $\mathfrak{su}(2)_C$ generator.
  - Stereographic projection ($\mathbb{R}^3 \to S^3$):
    the x_3 axis $\to$ great circle parameterized by $\varphi \in [-\pi, \pi]$.

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Key observation #1

- **Group theory:** after stereographic projection, $\mathcal{Q}^H$ can be embedded into a "massive $\mathcal{N} = 4$ algebra"

$$\mathfrak{su}(2|1) \oplus \mathfrak{su}(2|1) \quad \subset \mathfrak{osp}(4|4)$$

- Massive $\mathcal{N} = 4$ algebra contains: isometries of S^3, $U(1)^2$ R-symmetry, 8 supercharges [Assel, Gomis '15] .
- So: the 1d theory construction I reviewed **does not require an SCFT**! We can, more generally, study $\mathcal{N} = 4$ QFTs on S^3.
- Construct action from vector multiplets $\mathcal{V} = (A_\mu, \lambda_{\alpha a \dot{a}}, \Phi_{\dot{a}\dot{b}}, D_{ab})$ (w/ gauge group G) and hypermultiplets $\mathcal{H} = (q_a, \tilde{q}_a, \psi_{\alpha\dot{a}}, \tilde{\psi}_{\alpha\dot{a}})$ in irrep $\mathcal{R}$ of G.
    - For now, think of quiver gauge theories with action $S_{\text{YM}} + S_{\text{kin hyper}}$.

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Key observation #2

- **Hard calculation:** Yang-Mills action is $\mathcal{Q}^H$-exact!! $S_{\text{YM}} = \{\mathcal{Q}^H, V\}$.

Two consequences:

1. The S^3 partition function and the correlation functions of $\mathcal{Q}^H$-invariant observables are independent of g_{YM}.
2. Take $g_{\text{YM}} \to 0 \implies$ vector multiplet fields localize to zero, except for one scalar in the vector multiplet that localizes to a constant σ.
   - Same as previous SUSic localization computations [Kapustin, Willet, Yaakov '09; Jafferis '10; Hama, Hosomichi, Lee '10] that use a different supercharge $\mathcal{Q}^{\text{KWY}}$.
   - $\mathcal{Q}^{\text{KWY}}$ belongs to an $\mathcal{N} = 2$ subalgebra, i.e. to one of the $\mathfrak{su}(2|1)$ factors in $\mathfrak{su}(2|1) \oplus \mathfrak{su}(2|1)$).
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Key observation #3

- After vector multiplet localization, the partition function becomes

$$Z = \int_{\text{Cartan}} d\sigma \; \det{}_{adj}(2\sinh(\pi\sigma)) \int DqD\tilde{q}D\psi D\tilde{\psi} e^{-S[\sigma,q,\tilde{q},\psi,\tilde{\psi}]} \quad (*)$$

  - $\det{}_{adj}(2\sinh(\pi\sigma))$ is the 1-loop det of the vector multiplet.

- For fixed σ, $S[\sigma, q, \tilde{q}, \psi, \tilde{\psi}]$ is a quadratic action. So, at fixed σ, correlation functions are computed by Wick contractions.
  - Only $\mathcal{Q}^H$-invariant observables computed using (*) agree with those in SCFT.

Key observation #4

(*Not necessarily needed.*)

- Further Localize hyper by adding $t'\{\mathcal{Q}^H, \psi\{\mathcal{Q}^H, \psi\}\}$ to the action.
- Obtain 1d theory living on a great circle param by φ:

$$Z = \int_{\text{Cartan}} d\sigma \ \det{}_{\text{adj}}(2\sinh(\pi\sigma)) \prod_{\text{hypers } \mathcal{H} \text{ in irrep } \mathcal{R}} \mathbf{Z}_{\mathcal{H}}(\sigma)$$

where

$$\boxed{\mathbf{Z}_{\mathcal{H}} = \int DQD\tilde{Q} \exp\left[-\ell \int_{-\pi}^{\pi} d\varphi \ \text{tr}_{\mathcal{R}} \left(\tilde{Q}\frac{\partial Q}{\partial \varphi} + \tilde{Q}\sigma Q\right)\right]}$$

  - $Q \equiv \cos\frac{\varphi}{2} q_1 + \sin\frac{\varphi}{2} q_2$ tranforms in irrep $\mathcal{R}$ and $\tilde{Q} \equiv \cos\frac{\varphi}{2}\tilde{q}_1 + \sin\frac{\varphi}{2}\tilde{q}_2$ transforms in $\overline{\mathcal{R}}$. They are anti-periodic.
  - $\ell \equiv 4\pi r$, where r is the radius of S^3.

- We obtained a 1d Gaussian theory coupled to a matrix model.

Relation to previous work

- KWY result: Localizing w.r.t. $\mathcal{Q}^{\text{KWY}}$, the S^3 partition function can be written as

$$Z = \int_{\text{Cartan}} d\sigma \, \det{}_{\text{adj}}(2\sinh(\pi\sigma)) \prod_{\text{hypers } \mathcal{H} \text{ in irrep } \mathcal{R}} \frac{1}{\det{}_{\mathcal{R}}(2\cosh(\pi\sigma))} \, .$$

- Our result obtained after localization w.r.t. $\mathcal{Q}^H$ is that we simply expand the hyper factor into a 1d Gaussian theory:

$$\frac{1}{\det{}_{\mathcal{R}}(2\cosh(\pi\sigma))} = \int DQD\tilde{Q} \exp\left[-\ell \int_{-\pi}^{\pi} d\varphi \, \text{tr}_{\mathcal{R}}(\tilde{Q}\,\partial_\varphi Q + \tilde{Q}\,\sigma Q)\right]$$

- **Benefit: We can now insert twisted Higgs branch operators of the form $Q^n \tilde{Q}^m(\varphi)$ and calculate their correlation functions.**

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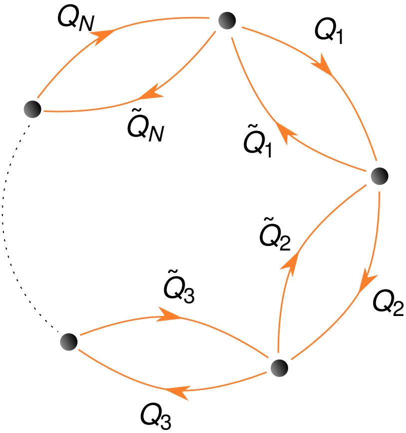
Application: necklace quiver

- Consider the necklace quiver w/ gauge group $U(1)^N/U(1)$ and N charged hypermultiplets.

- Twisted Higgs branch operators are products of

$$\mathcal{X} = Q_1 Q_2 \cdots Q_N \,,$$
$$\mathcal{Y} = \tilde{Q}_1 \tilde{Q}_2 \cdots \tilde{Q}_N \,,$$
$$\mathcal{Z} = \tilde{Q}_1 Q_1 = \ldots = \tilde{Q}_N Q_N \,,$$

with the relation $\mathcal{X}\mathcal{Y} = \mathcal{Z}^N$.



Application: necklace quiver

$$Z = \int \left(\prod_{j=1}^{N} d\sigma_j \right) \delta \left(\frac{1}{N} \sum_{j=1}^{N} \sigma_j \right) \int \left(\prod_{j=1}^{N} D\tilde{Q}_j \, DQ_j \right)$$
$$\times \exp \left[-\ell \int_{-\pi}^{\pi} d\varphi \sum_{j=1}^{N} \left(\tilde{Q}_j \partial_\varphi Q_j + \sigma_j \left(\tilde{Q}_j Q_j - \tilde{Q}_{j-1} Q_{j-1} \right) \right) \right]$$

- The relations
$$\tilde{Q}_1 Q_1 = \ldots = \tilde{Q}_N Q_N$$
are imposed by the Lagrange multipliers σ_j.
- At fixed σ_j, use Wick contractions with
$$\langle Q_i(\varphi) \tilde{Q}_j(0) \rangle_\sigma = \delta_{ij} \frac{\text{sgn}(\varphi) + \tanh(\pi(\sigma_i - \sigma_{i-1}))}{2\ell} e^{-(\sigma_i - \sigma_{i-1})\varphi}$$

Application: necklace quiver

- We find, for instance, if $\varphi_1 < \varphi_2 < \varphi_3$,

$$\langle \mathcal{Z}(\varphi_1)\mathcal{Z}(\varphi_2)\rangle = \frac{1}{\ell^2}\frac{1}{Z}\int d\tau \frac{1}{[2\cosh(\pi\tau)]^N}(i\tau)^2 \,,$$

$$\langle \mathcal{X}(\varphi_1)\mathcal{Y}(\varphi_2)\rangle = \frac{1}{\ell^N}\frac{1}{Z}\int d\tau \frac{1}{[2\cosh(\pi\tau)]^N}\left(i\tau - \frac{1}{2}\right)^N \,,$$

$$\langle \mathcal{Z}(\varphi_1)\mathcal{X}(\varphi_2)\mathcal{Y}(\varphi_3)\rangle = \frac{1}{\ell^{N+1}}\frac{1}{Z}\int d\tau \frac{1}{[2\cosh(\pi\tau)]^N}\left(i\tau\left(i\tau - \frac{1}{2}\right)^N\right)$$

  - These are initially $N-1$ dim'l integrals, but they look like the integral of the convolution of N functions. In Fourier space, they become a single integral, as above.
  - (Fourier transform trick also used in [Kapustin, Willet, Yaakov '10] . It implements mirror symmetry as in [Kapustin, Strassler '99] .)

The Coulomb branch

- To gain access to the Coulomb branch 1d topological theory, one should localize w.r.t. a supercharge $\mathcal{Q}^C$ instead of $\mathcal{Q}^H$.
- $\mathcal{Q}^C$ is also part of the massive $\mathcal{N} = 4$ algebra on S^3.
- In absence of defects (e.g. insertions of monopole operators), the YM action is $\mathcal{Q}^C$-exact.
- Can compute correlation functions of some twisted Coulomb branch operators (namely non-monopole operators) w/o any more work.
  - Including monopole operators should be possible as well.

The Coulomb branch: Example

- Consider SQED with N charged hypermultiplets (mirror dual of the necklace quiver).
- 1d Coulomb branch topological theory contains

$$\Phi(\varphi) \equiv \Phi_{\dot{1}\dot{1}} e^{i\varphi} + \Phi_{\dot{2}\dot{2}} e^{-i\varphi} + 2\Phi_{\dot{1}\dot{2}}$$

- Use KWY matrix model with $\Phi(\varphi) \to 2\sigma/r$:

$$\langle \Phi(\varphi_1)\Phi(\varphi_2)\rangle = \frac{64\pi^2}{\ell^2}\frac{1}{Z}\int d\sigma \frac{1}{[2\cosh(\pi\sigma)]^N}\sigma^2 .$$

- Compare with

$$\langle \mathcal{Z}(\varphi_1)\mathcal{Z}(\varphi_2)\rangle = \frac{1}{\ell^2}\frac{1}{Z}\int d\tau \frac{1}{[2\cosh(\pi\tau)]^N}(i\tau)^2$$

from before and conclude

Mirror symmetry: $\mathcal{Z}$ in N-node quiver $\longleftrightarrow$ $\frac{i\Phi}{8\pi}$ in SQED

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from before and conclude

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Summary

- New SUSic localization formula for correlation functions of 'twisted Higgs branch operators' in $\mathcal{N} = 4$ QFTs on S^3.
  - In SCFT, from 2- and 3-pt functions of twisted Higgs branch operators one can extract the 2- and 3-pt functions of (untwisted) Higgs branch operators.

Things I did not explain

- How hyper localization works in detail.
  - Similar in some ways to [Pestun '09] .
- Operator mixing on S^3. See also [Gerckhovitz, Gomis, Ishtiaque, Karasik, Komargodski, SSP '16]
- One can introduce Fayet-Iliopolous and real mass parameters.
  - Position independence in 1d is lost in some cases.
- Relation to Higgs branch chiral ring.
  - OPE in the 1d theory gives a non-commutative star product on the Higgs branch chiral ring. See also [Beem, Peelaers, Rastelli '16] .
- Inclusion of monopole operators or defects.