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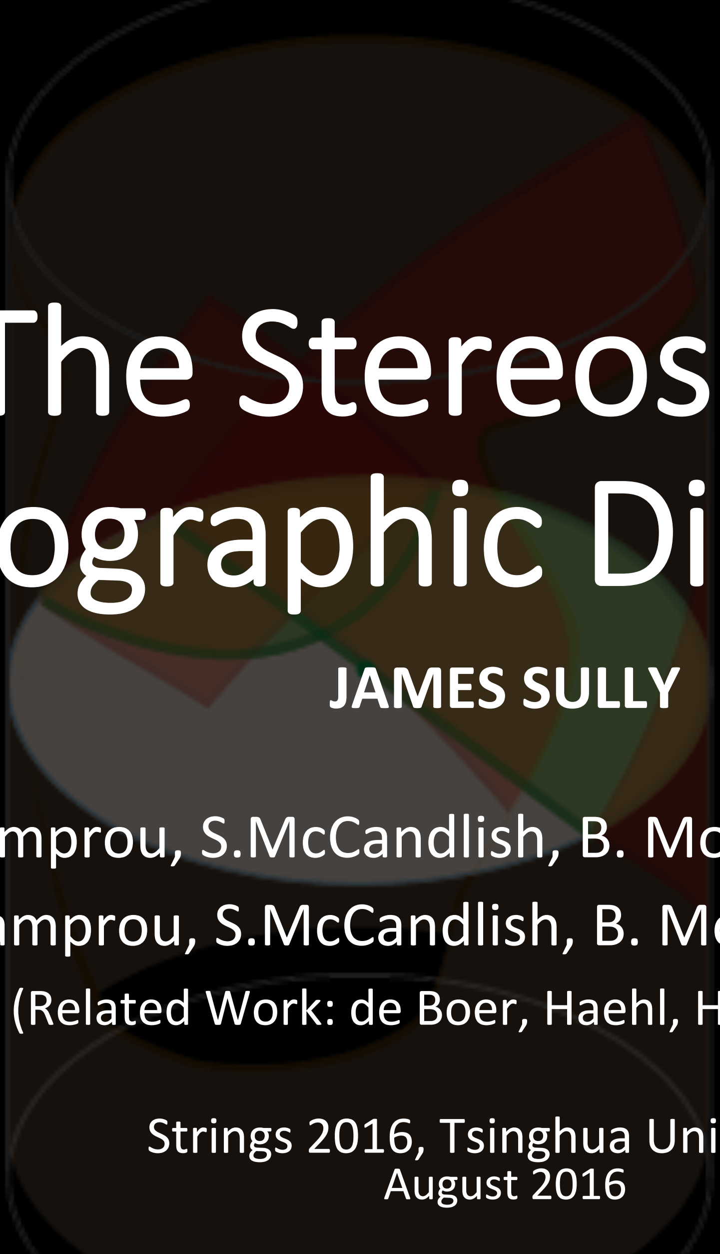


The Stereoscopic Holographic Dictionary

JAMES SULLY

B. Czech, L. Lamprou, S.McCandlish, B. Mosk, JS   arXiv: 1604.03110
B. Czech, L. Lamprou, S.McCandlish, B. Mosk, JS   arXiv: 160X.XXXX
(Related Work: de Boer, Haehl, Heller, Myers)

Strings 2016, Tsinghua University
August 2016

Motivation

There is an uncomfortable asymmetry in our use of the holographic dictionary:

Boundary: *Gauge-invariance is sacrosanct.*

- We always use gauge-invariant observables to probe the bulk and vice-versa.

Bulk: *Diff-invariance is required… in the same way that flossing is required.*

- When we visit an expert twice a year, we acknowledge the importance of diff-invariance.
- We really only worry about it once a month or so.

Worrying about **diff-invariance** is largely seen as adding **unpleasant (and unnecessary) complications**

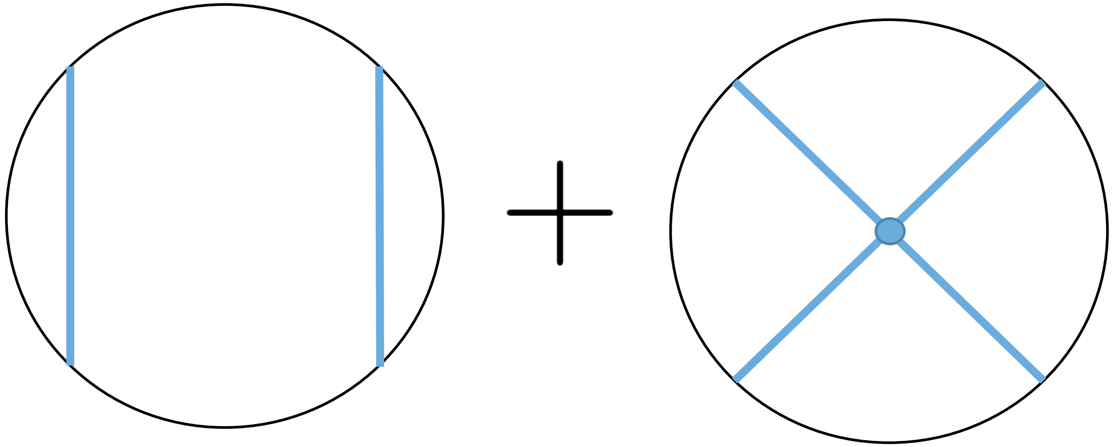
So, instead, we just work with the 'local' effective operators.

What has locality done for us lately?

Using these non-invariant effective observables is **meant to be an easier** way to do bulk calculations:

Gravity:

$$\langle \mathcal{O}_1\mathcal{O}_2\mathcal{O}_3\mathcal{O}_4\rangle =$$



$$+ \ldots$$

$$= \textbf{hard integrals ...}$$

How do we compare?

CFT:

$$\langle \mathcal{O}_1\mathcal{O}_2\mathcal{O}_3\mathcal{O}_4\rangle = \underbrace{\sum_k C_{12k}C_{k34}}_{\textbf{Dynamical Parameters}} \; \underbrace{g_k^{1234}(x_i)}_{\textbf{Conformal Kinematics: All integrals hidden here!}}$$

Standard picture is much easier?

Not really....

… then, are diff-invariant (non-local) variables really that bad?

Aim of this talk: NO.

Questions:

1. What is an **improved gauge/gravity dictionary** that is **gauge-invariant** on one side and **manifestly diff-invariant** on the other?
2. Can we find a **better connection to the natural CFT variables** that make the OPE so simple?

Answer:

Stereoscopic Dictionary:

Surface Operators in AdS
(Non-local, diff-invariant)

↔

Partial Waves of the OPE
(building blocks of the OPE)

The Kinematic Dictionary

This will be a **powerful framework**. It brings together many familiar ideas in holography, including:

1. The **Entanglement First Law** and **Einstein's Equations** [Faulkner, Guica, Hartman, Lashkari, McDermont, Myers, Swingle, Van Raamsdonk]
2. **Geodesic Witten diagrams** and **conformal blocks** [Hijano, Kraus, Perlmutter, Snively]
3. The **HKLL** construction of interacting **'local' bulk fields** [Hamilton, Kabat, Lifschytz, Lowe; Kabat, Lifschytz]
4. **de Sitter dynamics** for the variations of **EE** [de Boer, Heller, Myers, Neiman] [Nozaki, Numasawa, Prudenziati, Takayanagi], [Bhattacharya, Takayanagi]

Lesson from gauge theories

You might be familiar with a similar story:

We typically formulate a gauge theory in terms of a **gauge potential** $A_{\mu}^{a}(X)$ and the corresponding **field strength** $F_{\mu\nu}^{a}$, where the equation of motion is

$$(\nabla^{\mu} F_{\mu\nu})^{a} = 0$$

But the inclusion of the gauge field is just a **redundancy of the description.** It's unfortunate to have to use such an **inefficient representation** of the physics.

However, we do have a **complete set of gauge-invariant observables**

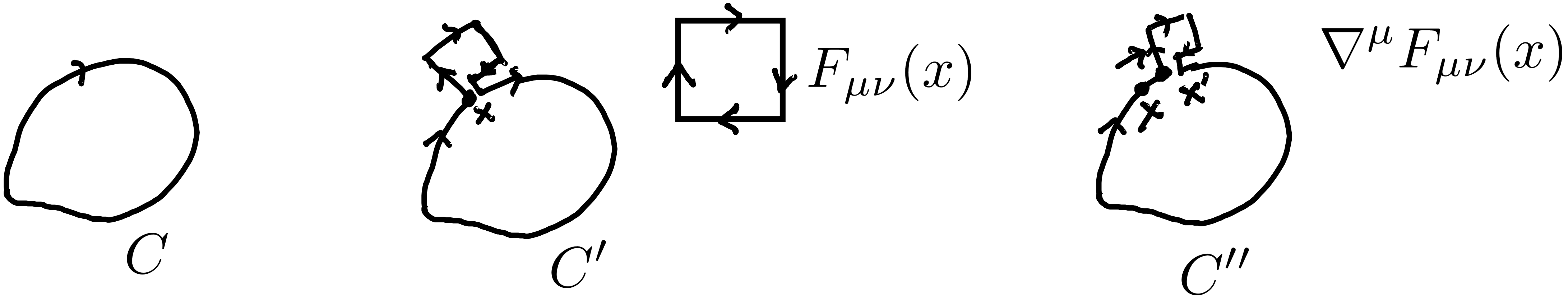
$$\Psi(C) = \mathrm{Tr}\left[\mathrm{P}\exp(\oint_{c} A_{\mu} dx^{\mu})\right] \qquad W(C) = \langle \Psi(C) \rangle$$

Can we express the dynamics of the theory in terms of these variables alone? (That is, can we write an equivalent EOM in the **space of loops?)**

Loops

[Migdal, Makeenko;
Polyakov; Halpern,
Makeenko;…]

How do we insert the equation of motion into a Wilson loop?



Loop Equation: $\Box_L W = 0$ (Classically)

- Can also define a field space Laplacian $\Box \downarrow F$, and view Wilson loop as a ‘loop transform’ ***W*** of the gauge field. *At large N*:

$$\Box_L W\,[A] = -W\,[\Box_F A]$$

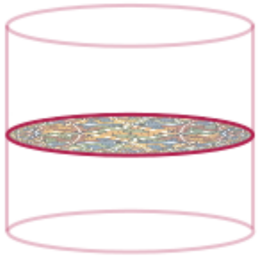
Intertwining Operators

Requirements for a new language

The loop equations are **analogous** to the organization of gravitational physics we have been looking for.

We might re-express our search for the correct language:

1. What is the right **loop space for gravity**?
2. What are the **loop operators** and their **loop equations for bulk physics**?
3. What do the loop equations and loop operators **look like in the gauge theory**?



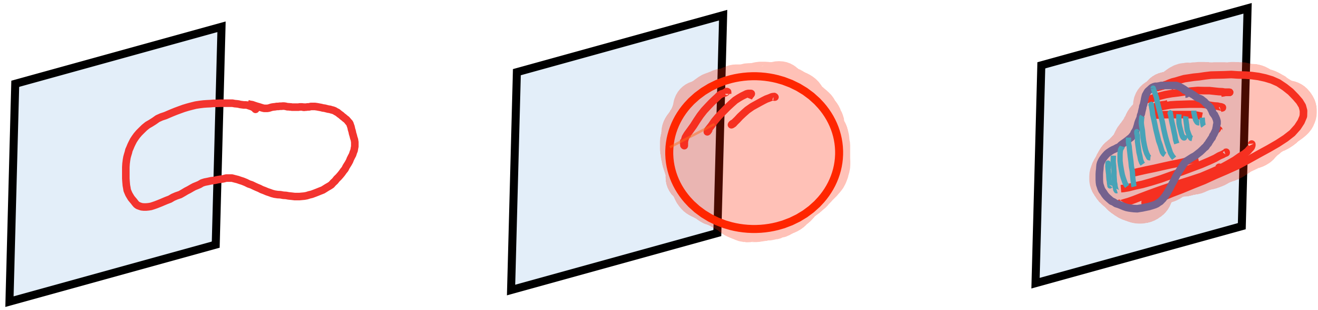
$\mathcal{M}$

$|\psi\rangle_{\partial\mathcal{M}}$

Loop Space

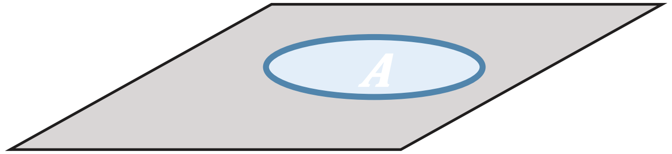
Loop Space = ?

We need to understand: **what is the right loop space for gravitational physics in AdS/CFT?**

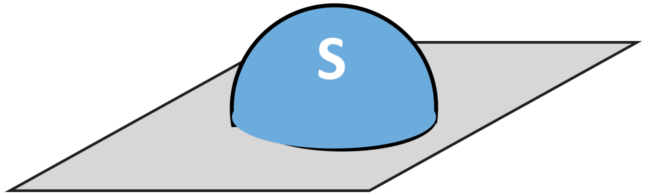


Thankfully an example of the type of construction we're searching for has already been found: **Ryu-Takayanagi Proposal.**

The **Ryu-Takayanagi (RT/HRT) proposal** connects:



The **entanglement entropy** of a region A on the boundary.



The **area of a minimal surface** S in the gravitational dual

Integral Geometry

What is the lesson from Ryu-Takayanagi?

The CFT sees the bulk geometry naturally through codimension-2 minimal surfaces.

- Boundary conditions are codimension-2 surfaces on the CFT cylinder

One could define the relevant **loop space** to be **all possible spacelike surfaces.**

- Probably **enormously redundant:** **infinite-dimensional space**

Let's define our loop space to be the space of minimal surfaces with spherical boundary conditions (in any frame)

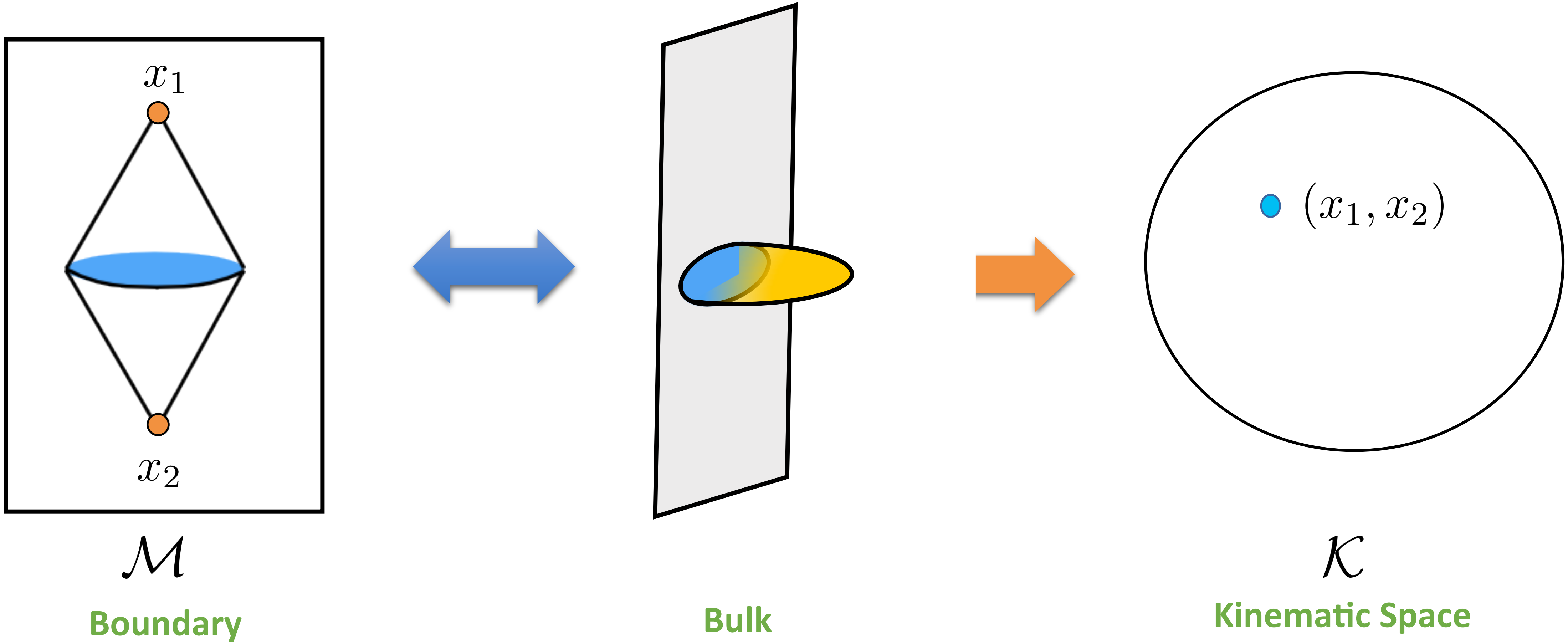
We will give it a new name: **Kinematic Space**

- Familiar object of study for some mathematicians: it is a primary object in the field of **integral geometry.**

Kinematic Space

What does kinematic space look like?

- Consider an **ordered pair of timelike separated points** on the boundary:



Kinematic Space

Can we assign a metric to kinematic space?

- For the **AdS vacuum**, the metric on K is **uniquely fixed by isometries** of the geometry:

$$ds^2 = \frac{I_{\mu\nu}(l^\mu)}{l^2} \underbrace{(d\chi^\mu d\chi^\nu - dl^\mu dl^\nu)}_{\text{Signature: (d,d)}}$$

χ: Center of mass

l: Separation

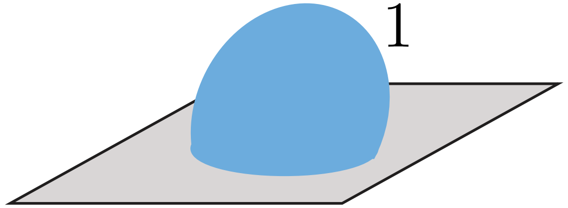
- In some cases, we understand how to specify a metric on kinematic space even when it's not fixed by symmetry
- Geometries are equivalent ⇔ **map from real space to kinematic space** is **invertible**.
  - This is an active area of mathematical research (*Boundary Rigidity / Lens Rigidity*)

Kinematic Fields

We want to generalize the Ryu-Takayanagi example to describe all of the bulk physics. We also need the right ‘Wilson Loops’ in addition to the right space.

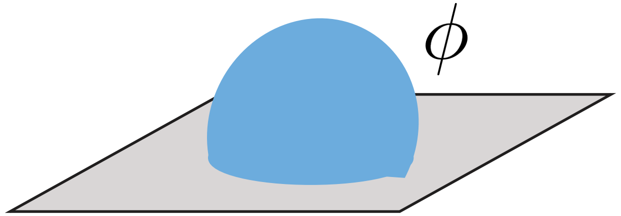
- We can think of their areas as integrating the unit operator over the minimal surface:

$$A = \int d^n x \sqrt{h}(1)$$



- A natural generalization then is:

$$\tilde{\phi} = \int d^n x \sqrt{h}(\phi)$$

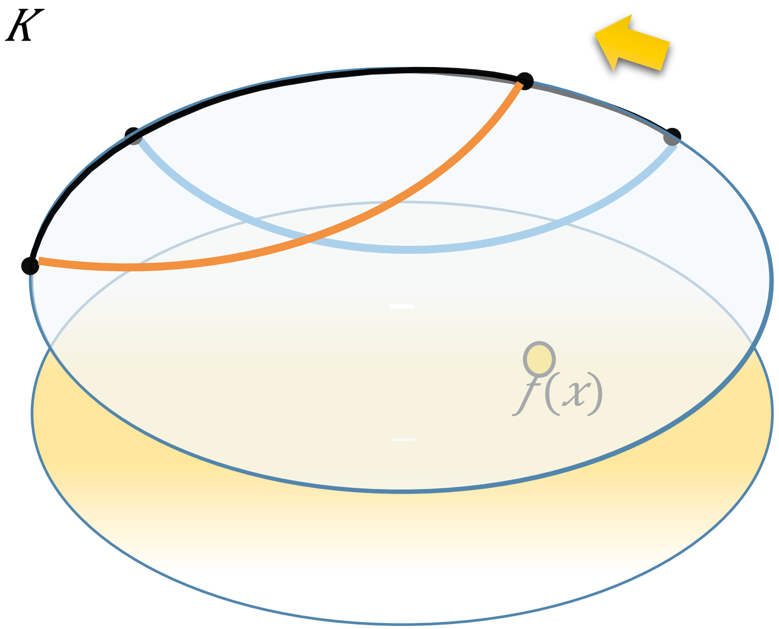


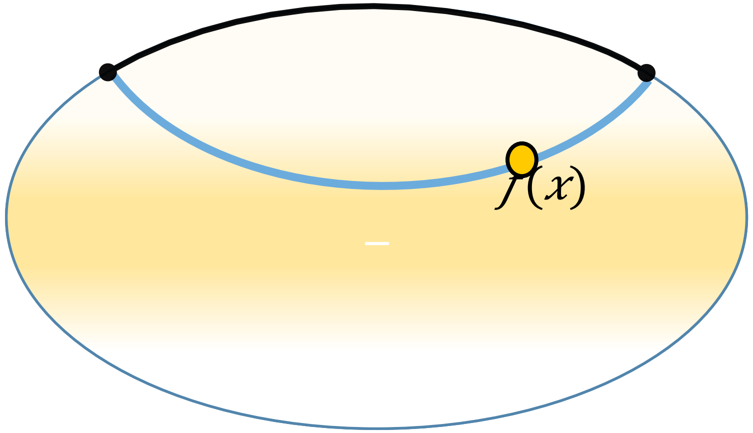
Radon Transform: $\boldsymbol{R[\phi]:=\phi}$

- This bulk surface/geodesic operator is a **non-local** and **diff-invariant** bulk

Radon Transform

The Radon transform has nice properties under isometries of the geometry:





$$\Box_{KS} R f = -R \Box_{AdS} f$$

“Intertwining Operators”

Loop Equations in kinematic space

Intertwinement allows us to **rewrite the dynamics of the gravitational theory** in terms of **dynamics on kinematic space**:

Free Scalar Field:

$$\begin{tikzcd}
\left(\Box_{AdS} - m^2\right)\phi(x) = 0 \arrow[r, leftrightarrow] & \Box_{KS} R f = -R\Box_{AdS} f \arrow[dl, leftrightarrow] \\
\boxed{(\Box_{KS} + m^2)\tilde{\phi}(\gamma) = 0} &
\end{tikzcd}$$

Loop EOM for free scalar

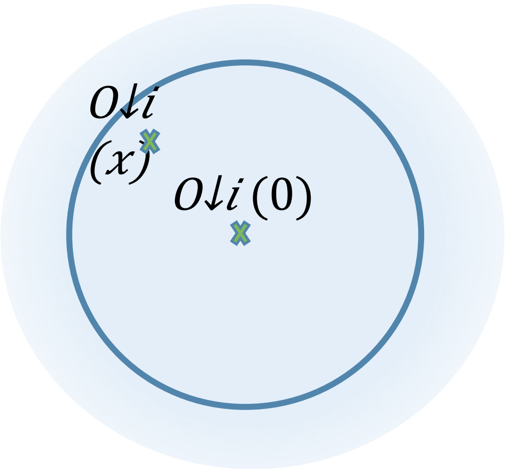
= Kinematic Free scalar EOM

The Operator Product Expansion

We've found a **natural set of gravitational variables** that are non-local, and that contain the information normally specified using local 'differential' description.

We **haven't succeeded** unless these appear **naturally in the dual gauge theory**:

- Consider a quasi-primary operator *O↓i (x)* of dimensions *Δ↓i* . We can expand the product of two such operators using a local basis of operators:



$$\mathcal{O}_i\left(x\right)\mathcal{O}_i\left(0\right)=\underbrace{\sum_k C_{iik}}_{\text{Dynamical Parameters}}\underbrace{\left|x\right|^{\Delta_k-2\Delta_i}\left(1+b_1\,x^\mu\partial_\mu+b_2\,x^\mu x^\nu\partial_\mu\partial_\nu+\ldots\right)\mathcal{O}_k\left(0\right)}_{\text{Conformal Kinematics}}$$

- Let us introduce a more compact notation for this expansion

$$\mathcal{O}_i\left(x\right)\mathcal{O}_i\left(y\right)=\left|x-y\right|^{-2\Delta_i}\sum_k C_{iik}\mathcal{B}_k\left(x,y\right)$$

- We will call *B↓k↑ (x,y)* the '**OPE Block**'

OPE Blocks as Kinematic Fields

- The OPE block carries coordinates of two points *(x,y)*, so we might naturally identify it with a **'loop operator' living in our kinematic space**.

Consider a scalar block $(\Delta\downarrow k\ ,l=0)$. Let's characterize this field:

1) What type of field is an OPE block on KS?

- Consider a conformal transformation $x\to x\uparrow'$. Then $\mathcal{B}_k\left(x,y\right) \to \mathcal{B}_k\left(x',y'\right)$ **So the OPE block is a scalar operator.**

2) What is its equation of motion?

- Eigen-operator of the conformal Casimir: $\left[L^2, \mathcal{B}_k\left(x,y\right)\right] = C_{\mathcal{O}_k}\mathcal{B}_k\left(x,y\right)$

$$C_{\mathcal{O}_k} = -\Delta\left(\Delta - d\right)$$

- We represent this as

$$\boxed{\mathcal{L}^2_{(B)} = 2\Box_{\mathrm{KS}}}$$

OPE Blocks as Kinematic Fields

We thus find an equation of motion for the OPE block

$$\left[\Box_{\text{KS}} + m^2_{\Delta_k}\right] \mathcal{B}_k\left(x, y\right) = 0$$

$$m^2_{\Delta_k} = -C_{\Delta_k}$$

It is the same wave equation obeyed by a bulk geodesic operator dual to the operator *𝑶↓𝒌* :

	Gravity	CFT
EOM	$\left[\Box_{KS} + m^2_{\Delta_k}\right] \tilde{\phi}_k(\gamma) = 0$	$\left[\Box_{\text{KS}} + m^2_{\Delta_k}\right] \mathcal{B}_k\left(x, y\right) = 0$
BCs	$\lim_{x\to 0} \tilde{\phi}_k(x, 0) = x^{\Delta_k} \mathcal{O}_k\left(0\right)$	$\lim_{x\to 0} \mathcal{B}_k(x, 0) = x^{\Delta_k} \mathcal{O}_k\left(0\right)$
Constraint	‘John’s Equations’	‘Spin’

***Generic theory on KS has infinite tower of spins…**

The Kinematic Dictionary

We have now established a **new organization of the holographic dictionary**

$$\boxed{\mathcal{B}_k\left(x,y\right) = \tilde{\phi}(\gamma)}$$

between **OPE Blocks** and **geodesic operators** (and extensions to surface operators).

Requirements for a new language

We might re-express our search for the correct language:

1. What is the right **loop space for gravity**?

   A: Kinematic space of minimal surfaces

2. What are the **loop operators** and their **loop equations for bulk physics**?

   A: Surface operators. Their loop equations take the form of simple equations of motion in kinematic space.

3. What do the loop equations and loop operators **look like in the gauge theory**?

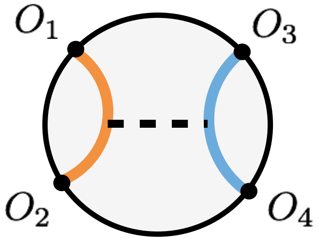
   A: The gauge theory sees the loops equations through the decomposition of the operator product expansion.

Some Applications

(see paper for details)

1. Geodesic Witten Diagrams

$$g_k\left(z,\bar{z}\right)=\langle 0|\mathcal{B}_k\left(x_1,x_2\right)\mathcal{B}_k\left(x_3,x_4\right)|0\rangle$$

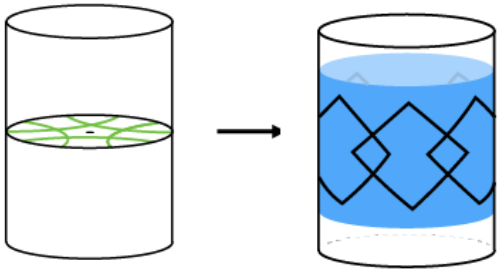


[Hijano, Kraus, Perlmutter, Snively]

2. Local Operators

- The radon transform is invertible: $\phi\left(\gamma\right)\rightarrow\phi(x)$
- Recover the spacelike smearing function of HKL

$$\phi(x,z)=\int d^dx'K(x,z|x')\mathcal{O}(x')$$



[Hamilton, Kabat, Lifschytz, Lowe]

3. Interactions

- Usefulness of construction isn't just kinematic—allows for **simpler solution to bulk dynamics**.
- For example, much of difficulty of finding interacting bulk fields comes from locality, not interactions!

$$\phi(x,z)=\int d^dx'K(x,z|x')\mathcal{O}(x')+\frac{1}{N}\sum_n a_n^{CFT}\int d^dx'K_n(x,z|x')\mathcal{O}_n(x')$$

$$\tilde{\phi}(\gamma)=\mathcal{B}_\Delta(\gamma)+\frac{1}{N}\sum_n a_n^{CFT}\mathcal{B}_n(\gamma)$$

[Kabat, Lifschytz]

4) Linearized Einstein Equations

- We have the linearized EOM: $\delta R_{\mu\nu}(x) = T_{\mu\nu}(x) - \frac{1}{d-1} T g_{\mu\nu}$
- Can define Radon Transforms:
$$\begin{aligned} R_{||}[s_{\mu\nu}] &= \int_B \sqrt{h} h^{ab} \delta g_{ab} \\ R_{\perp}[s_{\mu\nu}] &= \int_B \sqrt{h} (g^{ab} - h^{ab}) \delta g_{ab} \end{aligned}$$

  and prove an analogous intertwining relation: $R_{||}[2\delta R_{\mu\nu}] = \Box_K R_{||}[\delta g_{\mu\nu}]$
- We find the Kinematic version of the EOM:

$$\boxed{\left(\Box_{KS} + 2d\right) R_{||}[\delta g] = -2 R_{\perp}[T]}$$

Cf. [de Boer, Heller, Myers, Neiman]
[Nozaki, Numasawa, Prudenziati, Takayanagi], [Bhattacharya, Takayanagi]

Cf. [Faulkner, Guica, Hartman, Lashkari, McDermont, Myers, Swingle, Van Raamsdonk]

What about the CFT?

$$H_{\text{mod}} = 2\pi \int_{|x|<R} \frac{R^2 - r^2}{2R} T_{00} \quad \propto \mathcal{B}_{T_{00}}$$

H↓mod **is just an OPE Block!**

- Casimir Eqn: $\left(\Box_{KS} + 2d\right) H_{\text{mod}} = 0$
- Using the 1/N expansion

$$H_{\text{mod}} = A + H_{\text{mod}}^{Bulk} \qquad H_{\text{mod}}^{Bulk} = \int_{\Sigma} d\Sigma T_{\mu\nu} \xi^{\mu} n^{\nu}$$

- Then for a perturbation around the vacuum:

$$\boxed{\left(\Box_{KS} + 2d\right) H_{\text{mod}} = 0} \;\Leftrightarrow_T\; \boxed{\left(\Box_{KS} + 2d\right) R_{||}\left[\delta g\right] = R_{\perp}\left[T\right]}$$

- Read in the opposite direction, this is a **derivation of the bulk expression for *H↓mod***

Summary

We found a set of **non-local, gauge-invariant building blocks** both in the bulk and on the boundary to build a **‘better’ holographic dictionary.**

- *On the boundary:* **OPE blocks**
- *In the bulk*: **Geodesic/Surface Operators**

Finding the right **kinematic variables** isn’t ***just*** kinematics**:**

- Our improved dictionary **simplifies** many **dynamical bulk calculations.**

Where next?

- Must extend formalism **away from small perturbations** about vacuum.