CHIRAL ALGEBRA AND BPS SPECTRUM OF ARGYRES-DOUGLAS THEORIES

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Based on:
D. Xie, WY and S-T Yau, 1604.02155
J. Song, D. Xie and WY, work in progress

SUPERCONFORMAL FIELD THEORIES IN 4D

Rapid development:

- New models, most with no Lagrangian description
- New techniques:

  Localization, integrability, effective action on moduli space and etc

In this talk, we present the study on the BPS spectrum of Argyres-Douglas theories engineered from M5 branes

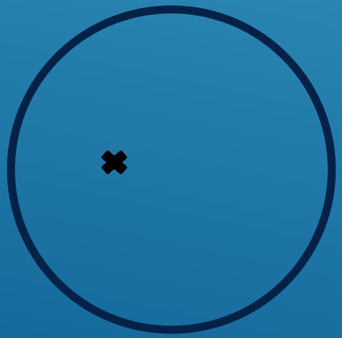
AD THEORIES FROM M5 BRANES

Intrinsically **strongly coupled**, original examples are isolated fixed points of certain supersymmetric gauge theories (pure SU(3)) [AD95]

- Coulomb branch operators: fractional dimension
- Wall-crossing phenomenon

M5 brane construction:

- ➢ Compactify 6d (2,0) type-J theory on a sphere with irregular puncture + at most one regular puncture (SCFT) [GMN09, BMT12, Xie13, WX15]



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BPS SPECTRUM OF AD THEORIES

- Our tool: (limit of) superconformal index

  Refined Witten index, only counts states saturate certain BPS condition, Invariant under RG

- Schur index for N=2 SCFTs

  $I = Tr(-1)^F q^{E-R}$, trace over BPS states $E - j_1 - j_2 - 2R = 0,\ r + j_1 - j_2 = 0$

  $\hat{C}$ (Stress tensor, higher spin current,…), $\hat{B}$ (moment maps,…),…

  No Coulomb branch operators

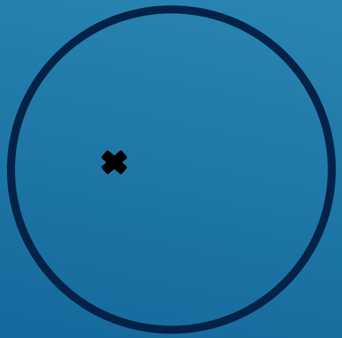
- Usually easy to compute for Lagrangian theories or theories dual to Lagrangian theories. Need new ways for AD theories. Guess work involved

SCHUR INDEX AND CHIRAL ALGEBRA

- Remarkable correspondence between 4d SCFTs and 2d chiral algebra [BLLPRR13, BPRR14, etc]
  - A BPS subsector of 4d N=2 SCFT forms a 2d chiral algebra
  - Meromorphic correlator of 2d chiral algebra = correlator of the BPS sector of 4d SCFTs
- We use two properties
  1. $c_{2d} = -12c_{4d}$, $k_{2d} = -k_F$
  2. The (normalized) vacuum character of 2d chiral algebra is the 4d Schur index

AD THEORIES WITHOUT FLAVOR SYMMETRY

- 6d (2,0) ADE theory compactified on a sphere with irregular puncture
- Irregular puncture: $\Phi \sim \frac{T}{z^{2+k/b}} + \cdots$
- Classified by $J^b[k]$, J=ADE



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AD THEORIES WITHOUT FLAVOR SYMMETRY

- AD theory $J^b[k]$
- 2d chiral algebra: **diagonal coset model**
- $A = \frac{g_l \oplus g_1}{g_{l+1}}$, with g=J, $l = -\frac{hk-b}{k}$
-
- $c_{2d} = \frac{l \dim J}{l+h} + \frac{\dim J}{1+h} - \frac{(l+1)\dim J}{l+1+h} = -12c_{4d}$
- h is dual Coexter number
- Schur index = vacuum character

$\mathcal{T}$	c_{4d}
$A_{N-1}^{N}[k]$	$\frac{(N-1)(k-1)(N+k+Nk)}{12(N+k)}$
$D_N^{2N-2}[k]$	$\frac{N(k-1)(-2-k+2N+2kN)}{12(-2+k+2N)}$
$E_6^{12}[k]$	$\frac{(k-1)(12+13k)}{2(12+k)}$
$E_6^{8}[k]$	$\frac{(3k-2)(13k+8)}{4(8+k)}$
$E_7^{14}[k]$	$\frac{(9k-7)(19k+14)}{12(14+k)}$
$E_8^{24}[k]$	$\frac{(5k-4)(24+31k)}{6(24+k)}$
$A_{N-1}^{N-1}[k]$	$\frac{(Nk-N+1)(N+k+Nk-1)}{12(N-1+k)}$
$D_N^{N}[k]$	$\frac{((N-1)2k-N)(N+k(2N-1))}{12(k+N)}$
$E_6^{9}[k]$	$\frac{(4k-3)(13k+9)}{6(9+k)}$
$E_7^{18}[k]$	$\frac{7(k-1)(19k+18)}{12(18+k)}$
$E_8^{30}[k]$	$\frac{2(k-1)(30+31k)}{3(30+k)}$
$E_8^{20}[k]$	$\frac{(3k-2)(20+31k)}{3(20+k)}$

AD THEORIES WITHOUT FLAVOR SYMMETRY

- AD theory $J^b[k]$
- 2d chiral algebra: diagonal coset model
- $A = \frac{g_l \oplus g_1}{g_{l+1}}$, with g=J, $l = -\frac{hk-b}{k}$
- When b=h, $W^J[h+k, h]$ minimal model
- Vacuum character = Schur index:

$$I = \frac{1}{\prod_{i=1}^{r}(q^{d_i}; q)} PE[-\frac{q^{b+k}}{1-q^{b+k}}\chi(q^{\lambda})]$$

$$PE[x] = \exp[\sum_{n=1}^{\infty}\frac{1}{n}x^n]$$

AD THEORIES WITHOUT FLAVOR SYMMETRY: EXAMPLE

- $A_{N-1}^{N}[k] = (A_{N-1}, A_{k-1})$
- 2d chiral algebra: W(k,k+N) minimal model
- Schur index = vacuum character = $PE\left[\frac{(q-q^k)(q-q^N)}{(1-q)^2(1-q^{k+N})}\right] = PE\left[\frac{(q^2+\cdots+q^N)}{1-q} - \cdots\right]$, with

  $PE[x] = \exp[\sum_{n=1}^{\infty}\frac{1}{n}x^n]$
-
- **Symmetric under the exchange of k and N**, two realization leads to the same SCFT
- The theory has N-1 operators with E-R=2,…, N. q^2 represents the Stress tensor.
- One can also read the relations between operators

AD THEORIES WITH FLAVOR SYMMETRY

- 6d (2,0) ADE theory compactified on a sphere with irregular puncture and regular puncture
- Irregular puncture: $\Phi \sim \frac{T}{z^{2+k/b}} + \cdots$
- Regular puncture: Y
- Classified by $(J^b[k], Y)$, J=ADE

$\times R^{3,1}$

AD THEORIES WITH FLAVOR SYMMETRY

- Consider only full puncture F
- Classified by $(J^b[k], \mathrm{F})$
- Kac-Moody algebra A_{-k_F}
- A=J, $c_{2d} = -\frac{k_F \dim A}{h-k_F} = -12\, c_{4d}$
- $k_F = h - \frac{b}{b+k}$
- Schur index = vacuum character (Kac-Wakimoto)

Theory	c_{4d}	k_F
$(A_{N-1}^N[k],F)$	$\frac{1}{12}(N+k-1)(N^2-1)$	$\frac{N(N+k-1)}{N+k}$
$(A_{N-1}^{N-1}[k],F)$	$\frac{(N+1)[N^2+N(k-2)+1]}{12}$	$\frac{(N-1)^2+kN}{N+k-1}$
$(D_N^{2N-2}[k],F)$	$\frac{1}{12}N(2N-1)(2N+k-3)$	$\frac{(2N-2)(2N+k-3)}{2N-2+k}$
$(D_N^N[k],F)$	$\frac{(2N-1)[2k(N-1)+N(2N-3)]}{12}$	$\frac{2k(N-1)+N(2N-3)}{N+k}$
$(E_6^{12}[k],F)$	$\frac{13(k+11)}{2}$	$\frac{12(k+11)}{k+12}$
$(E_6^9[k],F)$	$\frac{13}{6}(33+4k)$	$12-\frac{9}{k+9}$
$(E_6^8[k],F)$	$\frac{13}{4}(22+3k)$	$12-\frac{8}{k+8}$
$(E_7^{18}[k],F)$	$\frac{133}{12}(17+k)$	$\frac{18(k+17)}{k+18}$
$(E_7^{14}[k],F)$	$\frac{19}{12}(119+9k)$	$18-\frac{14}{k+14}$
$(E_8^{30}[k],F)$	$\frac{62}{3}(29+k)$	$\frac{30(k+29)}{k+30}$
$(E_8^{24}[k],F)$	$\frac{31}{6}(116+5k)$	$30-\frac{24}{k+24}$
$(E_8^{20}[k],F)$	$\frac{31}{3}(58+3k)$	$30-\frac{20}{k+20}$

AD THEORIES WITH FLAVOR SYMMETRY

- Classified by $(J^b[k], \mathrm{F})$
- Kac-Moody algebra A_{-k_F}
-
- A=J, $c_{2d} = -\frac{k_F \dim A}{h-k_F} = -12\, c_{4d}$
- At b=h, we have

$$I = PE[\frac{q - q^{b+k}}{(1-q)(1-q^{b+k})} \chi^F_{adj}(z)]$$

AD THEORIES WITH FLAVOR SYMMETRY: EXAMPLES

$A_4^5[-3] = D_2[SU(5)]$, Kac-Moody algebra is $su(5)_{-\frac{5}{2}}$

Schur index = $PE\left[\frac{q}{1-q^2}\chi_{adj}^{SU(5)}\right] = PE[\frac{q(1-q+q^2-q^3+\cdots)}{1-q}\mathbf{24} + q^2 - q^2]$

- Higgs branch operators O (moment maps) in adjoint (**24**) rep of SU(5); Stress tensor; etc
- $O^2_{\mathbf{24+1}} = 0$ which is the Joseph relation of the Higgs branch

Structure of the Higgs branch is **encoded** in the Schur index

- Direct obtain the Higgs branch from the chiral algebra

AD THEORIES WITH FLAVOR SYMMETRY: HIGGS BRANCH

AD theory $(J^b[k], \mathrm{F})$, Kac-Moody algebra A_{-k_F}

The Higgs branch is determined by the associated variety X of th Kac-Moody algebra A_{-k_F}

- For $k_{2d} = -h + \frac{p}{q}$, $p \geq h$, associated variety X is certain nilpotent orbit $\bar{O}_q$

  Details see Arakawa[2015], Beem-Rastelli

In our case, for b=h, the Higgs branch is determined by the nilpotent orbit $\bar{O}_q$

theory	k_{2d}	admissible?
$(A_{N-1}^{N}[k], F)$	$-N + \frac{N}{N+k}$	o
$(A_{N-1}^{N-1}[k], F)$	$-N + \frac{N-1}{N+k+1}$	x
$(D_N^{2N-2}[k], F)$	$-(2N-2) + \frac{2N-2}{2N-2+k}$	o
$(D_N^{N}[k], F)$	$-(2N-2) + \frac{N}{N+k}$	x
$(E_6^{12}[k], F)$	$-12 + \frac{12}{k+12}$	o
$(E_6^{9}[k], F)$	$-12 + \frac{9}{k+9}$	x
$(E_6^{8}[k], F)$	$-12 + \frac{8}{k+8}$	x
$(E_7^{18}[k], F)$	$-18 + \frac{18}{k+18}$	o
$(E_7^{14}[k], F)$	$-18 + \frac{14}{k+14}$	x
$(E_8^{30}[k], F)$	$-30 + \frac{30}{k+30}$	o
$(E_8^{24}[k], F)$	$-30 + \frac{24}{k+24}$	x
$(E_8^{20}[k], F)$	$-24 + \frac{30}{k+24}$	x

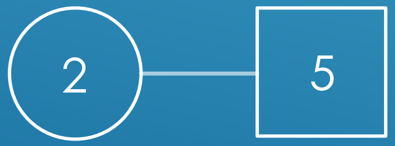
AD THEORIES WITH FLAVOR SYMMETRY: HIGGS BRANCH

- $(J^b[k], \mathrm{F})$, Kac-Moody algebra A_{-k_F}
- For $k_{2d} = -h + \frac{p}{q}$, $p \geq h$
  - $q < h$, nilpotent orbit is given by the table (Arakawa[2015])
  - $q \geq h$, nilpotent orbit is just the principal nilpotent orbit $\bar{O}_{prin}$
- $(J^h[k], \mathrm{F})$, $k = -h + \frac{h}{h+k}$
- $k \geq 0$, **all AD theories have the same Higgs branch**

g	q	$\bar{O}_q$
sl_n	any	$(q, \ldots, q, s), 0 \leq s \leq q-1$
so_{2n}	odd	$(q, \ldots, q, s), 0 \leq s \leq q,$ s odd, number of q odd; $(q, \ldots, q, s, 1), 0 \leq s \leq q-1,$ s odd, number of q even
	even	$(q+1, q, \ldots, q, s), 0 \leq s \leq q-1,$ s odd, number of q even; $(q+1, q, \ldots, q, q-1, s, 1), 0 \leq s \leq q-1,$ s odd, number of q even

AD THEORIES WITH FLAVOR SYMMETRY: HIGGS BRANCH: EXAMPLES

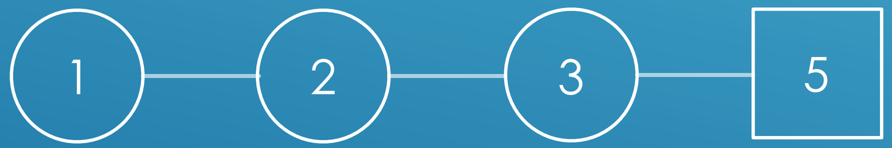
- $A_4^5[-3] = D_2[SU(5)]$, Kac-Moody algebra is $SU(5)_{-\frac{5}{2}}$, $k = -5 + \frac{5}{2}$, $q = 2$
  - Moment maps O in adjoint (**24**) rep of SU(5)
  - $O^2_{\mathbf{24+1}} = 0$ which is the Joseph relation of the chiral ring of Higgs branch
- The nilpotent orbit is given by (2,2,1)



- 2-instanton ADHM quiver for the gauge group SU(5)

AD THEORIES WITH FLAVOR SYMMETRY: HIGGS BRANCH: EXAMPLES

- $A_4^5[-1]$, Kac-Moody algebra is $SU(5)_{-\frac{15}{4}}$, $k = -5 + \frac{5}{4}$, $q = 4$
- The nilpotent orbit is given by (4,1)



- The Higgs branch of the above quiver gives the nilpotent orbit

SUMMARY

- Understand the BPS spectrum of AD theories using the index
  - Read off protected operators (current, Higgs branch operators…) and their relations
- Compute the index: chiral algebra vs. TQFT
  - AD theories without flavor symmetry: diagonal coset model
  - AD theoires with flavor symmetry: Kac-Moody algebra
  - Higgs branch = associated varieties of Kac-Moody algebra

FUTURE

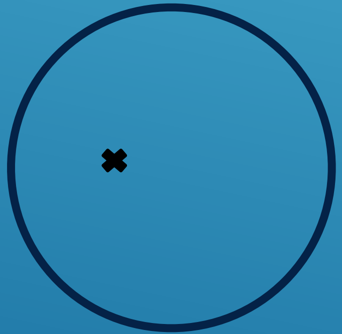
- Generalization
  - Arbitrary puncture: Drinfeld-Sokolov reduction
  - Twisted case: AD⇒BC
- Beyond spectrum:
  - Correlation functions between BPS operators
  - Combining with bootstrap and more constraints on AD theories [LL15]
- Thank you

INDEX AND TOPOLOGICAL FIELD THEORY

For class-S theories, indices enjoy a TQFT formulation [GPRR09, GRRY11, GRRY11, GRR12, etc]

- 6d (2,0) type-J theory compactified on a Riemann surface of genus g and s punctures
- 
- $I = \sum_\lambda \frac{1}{(\dim_q \lambda)^{2g-2+s}} \psi_{1,\lambda}(q)\psi_{2,\lambda}(q) \ldots \psi_{s,\lambda}(q)$

INDEX AND TOPOLOGICAL FIELD THEORY

- For AD theories, similar formula [BN15, Song15]
- 
- $I = \sum_{\lambda} (\dim_q \lambda) \varphi_{1,\lambda}(q)$

CHIRAL ALGEBRA VS TOPOLOGICAL FIELD THEORY

- Chiral algebra:
  - AD theories ⇒ 2d chiral algebra
  - Index = vacuum character
- TQFT:
  - Fix local contribution of each puncture to the index
  - Index = sum over representations
- Complementary to each other. Consistency check. BPS spectrum