M-theory S-Matrix from 3d SCFT

Silviu S. Pufu, Princeton University

Based on:

- arXiv:1711.07343 with N. Agmon and S. Chester
- arXiv:1804.00949 with S. Chester and X. Yin

Also:


OIST, June 26, 2018
Motivation

- Learn about gravity / string theory / M-theory from CFT.
- 3d maximally supersymmetric ($\mathcal{N} = 8$) CFTs w/ gravity duals: explicit Lagrangians; no marginal coupling; SUSY.
- Most well-understood example:
  M-theory on $\text{AdS}_4 \times S^7 \iff U(N)_k \times U(N)_{-k}$ ABJM theory at CS level $k = 1$.
- Last 10 years: progress in QFT calculations
  - using supersymmetric localization;
  - using conformal bootstrap in CFTs.
- What do these calculations tell us about M-theory?
  - Example: $S^3$ partition function of ABJM theory can be written as an $N$-dim’l integral. What info about M-theory does it contain?
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\textbf{This talk:} Reconstruct M-theory S-matrix \textit{perturbatively at small momentum} (scatter gravitons and superpartners).

Equivalently, reconstruct the derivative expansion of the M-theory effective action. Schematically,

\[ S = \int d^{11} x \sqrt{g} \left[ R + \text{Riem}^4 + \cdots + \text{(SUSic completion)} \right]. \]

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From the 4d point of view, we can scatter:
- graviton (1);
- gravitinos (8);
- gravi-photons (28);
- gravi-photinos (56);
- scalars ($70 = 35 + 35$)

At leading order in small momentum (i.e. momentum squared), scattering amplitudes are those in $\mathcal{N} = 8$ SUGRA at tree level. Amplitude depends on the type of particle, e.g.

$$A_{\text{SUGRA, tree}}(h^- h^- h^+ h^+) = \frac{\langle 12 \rangle^4 [34]^4}{stu},$$
$$A_{\text{SUGRA, tree}}(S_1 S_1 S_2 S_2) = \frac{tu}{s},$$

e tc.

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etc.

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Momentum expansion

- Momentum expansion takes a universal form (independent of the type of particle):
  \[ \mathcal{A} = \mathcal{A}_{\text{SUGRA, tree}} \left( 1 + \ell_p^6 f_{R^4}(s, t) + \ell_p^9 f_{1\text{-loop}}(s, t) \right. \\
  \left. + \ell_p^{12} f_{D^6 R^4}(s, t) + \ell_p^{14} f_{D^8 R^4}(s, t) + \cdots \right). \]

- \( f_{D^{2n} R^4} = \) symmetric polyn in \( s, t, u \) of degree \( n + 3 \)

- Known from type II string theory + SUSY [Green, Tseytlin, Gutperle, Vanhove, Russo, Pioline, …] :
  \[ f_{R^4}(s, t) = \frac{stu}{3 \cdot 2^7}, \quad f_{D^6 R^4}(s, t) = \frac{(stu)^2}{15 \cdot 2^{15}}. \]

- \( \ell_p^{10} f_{D^4 R^4} \) allowed by SUSY, but known to vanish.

- **This talk:** Reproduce \( f_{R^4} \) from 3d SCFT.
Flat space limit of CFT correlators

- **Idea**: Flat space scattering amplitudes can be obtained as limit of CFT correlators [Polchinski '99; Susskind '99; Giddings '99; Penedones '10; Fitzpatrick, Kaplan '11].

- **For a CFT\(_3\)** operator \(\phi(x)\) with \(\Delta_\phi = 1\),

  \[
  \left\langle \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4) \right\rangle_{\text{conn}} = \frac{1}{x_{12}^2 x_{34}^2} g(U, V)
  \]

  go to Mellin space

  \[
  g(U, V) = \int \frac{ds \, dt}{(4\pi i)^2} U^{t/2} V^{(u-2)/2} \Gamma^2 \left(1 - \frac{s}{2}\right) \Gamma^2 \left(1 - \frac{t}{2}\right) \Gamma^2 \left(1 - \frac{u}{2}\right) M(s, t)
  \]

  where \(s + t + u = 4\) and \(U = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}\), \(V = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}\).

- From the large \(s, t\) limit of \(M(s, t)\) one can extract 4d scattering amplitude \(A(s, t)\) [Penedones '10; Fitzpatrick, Kaplan '11].
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To obtain scattering amplitude of graviton + superpartners in M-theory, look at **stress tensor multiplet** in ABJM theory (ABJM theory is a 3d $\mathcal{N} = 8$ SCFT, and so it has $\mathfrak{so}(8)_R$ R-symmetry):

**focus on this**

<table>
<thead>
<tr>
<th>dimension</th>
<th>spin</th>
<th>$\mathfrak{so}(8)_R$</th>
<th>couples to</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>$35_c$</td>
<td>scalars</td>
</tr>
<tr>
<td>3/2</td>
<td>1/2</td>
<td>$56_v$</td>
<td>gravi-photinos</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
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<td>pseudo-scalars</td>
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Task: find the Mellin amplitude $M(s, t)$ corresponding to $\langle S_{IJ} S_{KL} S_{MN} S_{PQ} \rangle$ by solving superconformal Ward identity [Dolan, Gallot, Sokatchev '04] order by order in $\ell_p^2 \propto N^{-1/3} \propto c_T^{-2/9}$.

Here, $\langle T_{\mu\nu} T_{\rho\sigma} \rangle \propto c_T \propto N^{3/2}$.
Require: 1) at order $\ell_p^{2k}$, $M(s, t)$ should not grow faster than $(k + 1)^{st}$ power of $s, t, u$;
2) right analytic properties to correspond to a bulk tree-level Witten diagram.

Number of such solutions to Ward identity is:

<table>
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<tr>
<th>degree in $s, t, u$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>...</th>
</tr>
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<td>11D vertex</td>
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<td></td>
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</tr>
<tr>
<td># of params</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>...</td>
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<td></td>
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(degree 1 in [Zhou ’18]; degree $\geq 2$ in [Chester, SSP, Yin ’18].)
So:

- To determine $M(s, t)$ to order $1/c_T$ we should compute one CFT quantity.
- To determine $M(s, t)$ to order $1/c_T^{5/3}$ we should compute two CFT quantities.
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The CFT quantities can be, for instance, squared OPE coefficients appearing in the superconformal block decomposition. Schematically,

$$\langle S_{IJ} S_{KL} S_{MN} S_{PQ} \rangle = \frac{1}{x_{12}^2 x_{34}^2} \sum_{\mathcal{M}} \lambda^2_{\mathcal{M}} g_{\mathcal{M}}(U, V).$$

($\mathcal{M}$ is a superconformal multiplet appearing in the $S \times S$ OPE.)

$T_{\mu\nu}$ Ward identity gives $\lambda^2_{\text{stress}} = 256/c_T$. Using SUSY tricks, one can compute [Agmon, Chester, SSP '17]:

$$\lambda^2_{B,2} = \frac{32}{3} - \frac{1024(4\pi^2 - 15)}{9\pi^2} \frac{1}{c_T} + 40960 \left( \frac{2}{9\pi^8} \right)^{\frac{1}{3}} \frac{1}{c_T^{5/3}} + \cdots$$

where “stress” is the stress tensor multiplet, and “($B, 2$)” is a 1/4-BPS mutliplet appearing in the OPE $S \times S$. 

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Using these two expressions, we determined $M(s, t)$ to order $1/c_T^{5/3}$.

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OPE coefficients from SUSic localization

- It is hard to calculate correlation functions at separated points using SUSic localization. See however [Gerkchovitz, Gomis, Ishtiaque, Karasik, Komargodski, SSP ’16; Dedushenko, SSP, Yacoby ’16].

How were $c_T$ and $\lambda^2_{(B,2)}$ computed?

- From derivatives of the $S^3$ partition function with respect to an $\mathcal{N} = 4$-preserving mass parameter $m$, which can be computed using supersymmetric localization.

- (For $c_T$, see also [Chester, Lee, SSP, Yacoby ’14] for another method based on [Closset, Dumitrescu, Festuccia, Komargodski, Seiberg ’12].)
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Mass-deformed $S^3$ partition function

- For an $\mathcal{N} = 4$-preserving mass deformation of ABJM theory, $Z_{S^3}(m)$ is [Kapustin, Willett, Yaakov ’09] :

$$Z_{S^3}(m) = \int d^N \lambda \ d^N \mu \ e^{i k \sum_i (\lambda_i^2 - \mu_i^2)} \frac{\prod_{i<j} \sinh^2(\lambda_i - \lambda_j) \sinh^2(\mu_i - \mu_j)}{\prod_{i,j} \cosh(\lambda_i - \mu_j + m) \cosh(\lambda_i - \mu_j)}$$

- Small $N$: can evaluate integral exactly.
- Large $N$: rewrite $Z_{S^3}(m)$ as the partition function of non-interacting Fermi gas of $N$ particles with [Marino, Putrov ’11; Nosaka ’15]

$$U(x) = \log(2 \cosh x) - mx, \quad T(p) = \log(2 \cosh p).$$

Resummed perturbative expansion [Nosaka ’15] :

$$Z_{S^3}(m) \sim \mathrm{Ai} \left( f_1(m) N - f_2(m) \right)$$

for some known functions $f_1(m)$ and $f_2(m)$. ($\log Z \propto N^{3/2}$)
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Topological sector

- $3d \mathcal{N} = 4$ SCFTs have a 1d topological sector [Beem, Lemos, Liendo, Peelaers, Rastelli, van Rees ’13; Chester, Lee, SSP, Yacoby ’14; Dedushenko, SSP, Yacoby ’16] defined on a line $(0, 0, x)$ in $\mathbb{R}^3$.

- $\langle \mathcal{O}_1(x_1) \ldots \mathcal{O}_n(x_n) \rangle$ depends only on the ordering of $x_i$ on the line.

- Ops in 1d are 3d $1/2$-BPS operators $\mathcal{O}(\vec{x})$ placed at $\vec{x} = (0, 0, x)$ and contracted with $x$-dependent $R$-symmetry polarizations.

- The operators $\mathcal{O}(x)$ are in the cohomology of a supercharge $Q = “Q + S”$ cohomology s.t. translations in $x$ are $Q$-exact.

- The topological sector is defined either on a line in flat space or on a great circle of $S^3$.

- In ABJM, construct 1d operators $S_\alpha(x)$ from $S_{IJ}$, $\alpha = 1, 2, 3$. Their 2-pt function depends on $c_T$; their 4-pt function depends on $c_T$ and $\lambda^2_{(B,2)}$. 
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From $Z_{S^3}(m)$ to OPE coefficients

- One can show $Z_{S^3}(m) = Z_{S^1}(m)$, and so derivatives of the $Z_{S^3}$ w.r.t. $m$ corresponds to integrated correlators in the 1d theory.
- From 2 derivatives of $Z_{S^3}$ w.r.t. $m$ we can extract $c_T$.
- From 4 derivatives of $Z_{S^3}$ w.r.t. $m$ we can extract $\lambda_{(B,2)}^2$.
- So the (resummed) perturbative expansion of $c_T$, $\lambda_{B,2}^2$ can be written in terms of derivatives of the Airy function!
- Eliminating $N$ gives

$$\lambda_{(B,2)}^2 = \frac{32}{3} - \frac{1024(4\pi^2 - 15)}{9\pi^2} \frac{1}{c_T} + 40960 \left( \frac{2}{9\pi^8} \right)^{\frac{1}{3}} \frac{1}{c_T^{5/3}} + \cdots$$

- (Tangent: For 2d bulk dual of the 1d topological sector of ABJM theory, see [Mezei, SSP, Wang ’17]. The 1d theory is exactly solvable, and its 2d bulk dual is 2d YM.)
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- From 4 derivatives of $Z_{S^3}$ w.r.t. $m$ we can extract $\lambda^2_{(B,2)}$.

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$$\lambda^2_{B,2} = \frac{32}{3} - \frac{1024(4\pi^2 - 15)}{9\pi^2} \frac{1}{c_T} + 40960 \left( \frac{2}{9\pi^8} \right)^{\frac{1}{3}} \frac{1}{c_T^{5/3}} + \cdots$$

(Tangent: For 2d bulk dual of the 1d topological sector of ABJM theory, see [Mezei, SSP, Wang ’17]. The 1d theory is exactly solvable, and its 2d bulk dual is 2d YM.)
From $Z_{S^3}(m)$ to OPE coefficients

- One can show $Z_{S^3}(m) = Z_{S^1}(m)$, and so derivatives of the $Z_{S^3}$ w.r.t. $m$ corresponds to integrated correlators in the 1d theory.

- From 2 derivatives of $Z_{S^3}$ w.r.t. $m$ we can extract $c_T$.

- From 4 derivatives of $Z_{S^3}$ w.r.t. $m$ we can extract $\lambda_{(B,2)}^2$.

- So the (resummed) perturbative expansion of $c_T, \lambda_{(B,2)}^2$ can be written in terms of derivatives of the Airy function!

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$$\lambda_{(B,2)}^2 = \frac{32}{3} - \frac{1024(4\pi^2 - 15)}{9\pi^2} \frac{1}{c_T} + 40960 \left( \frac{2}{9\pi^8} \right)^{\frac{1}{3}} \frac{1}{c_T^{5/3}} + \cdots$$

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Beyond $f_{R^4}$?

Can one go beyond reconstructing $f_{R^4}$?

More SUSic localization results for ABJM theory are available: $Z_{S^3}$ as a function of three real mass parameters; partition function on a squashed sphere, etc.

Cannot use the 1d topological sector in this case, but it is very likely that this extra data will show $f_{D^4 R^4} = 0$ and maybe even determine $f_{D^6 R^4}$. (Work in progress with D. Binder and S. Chester.)

Another approach: conformal bootstrap.

Generally, we obtain bounds on various quantities.

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Known $\mathcal{N} = 8$ SCFTs

A few families of $\mathcal{N} = 8$ SCFTs:

With holographic duals:

- $\text{ABJM}_{N,1}: U(N)_1 \times U(N)_{-1} \leftrightarrow AdS_4 \times S^7$.
- $\text{ABJM}_{N,2}: U(N)_2 \times U(N)_{-2} \leftrightarrow AdS_4 \times S^7 / \mathbb{Z}_2$.
- $\text{ABJ}_{N,2}: U(N)_2 \times U(N + 1)_{-2} \leftrightarrow AdS_4 \times S^7 / \mathbb{Z}_2$.

Without known holographic duals:

- $\text{BLG}_k: SU(2)_k \times SU(2)_{-k}$. 
Bootstrap bounds [Agmon, Chester, SSP ’17]

- Bounds from conformal bootstrap applying to all $\mathcal{N} = 8$ SCFTs.

![Diagram with plot](image)

- SUGRA (leading large $c_T$) saturates bootstrap bounds.
- Conjecture: $\text{ABJM}_{N,1}$ or $\text{ABJM}_{N,2}$ or $\text{ABJ}_{N,2}$ saturate bound at all $N$ in the limit of infinite precision.
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Bound saturation \implies read off CFT data

- On the boundary of the bootstrap bounds, the solution to crossing should be unique \implies can find $\langle S_{IJ} S_{KL} S_{MN} S_{PQ} \rangle$ and solve for the spectrum !! [Agmon, Chester, SSP ’17]

Red lines are leading SUGRA **tree level** results [Zhou ’17; Chester ’18].

Lowest operators have the form $S_{IJ} \partial_{\mu_1} \cdots \partial_{\mu_\ell} S^{IJ}.$
\( \lambda_{(A,2)}^2 \) and \( \lambda_{(A,+)}^2 \) from extremal functional

Semishort \( (A, 2)_j \) and \( (A, +)_j \) OPE coefficients for low spin \( j \) in terms of \( \frac{16}{c_T} \) from extremal functional:

- Red line is tree level SUGRA result [Chester '18].
- \( \lambda_{(A,+)}^2 \) appears close to linear in \( 16/c_T \).
- More precision needed.
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Conclusion

- Can compute OPE coefficients in $\mathcal{N} = 8$ SCFTs with Lagrangian descriptions using supersymmetric localization.

- For ABJM theory, we can reproduce the $f_{R^4}(s, t) = \frac{stu}{3 \cdot 2^7}$ term in the flat space 4-graviton scattering amplitude.

- Bootstrap bounds are almost saturated by $\mathcal{N} = 8$ SCFTs with holographic duals.

For the future:

- Generalize to other dimensions, other 4-point function, less SUSY. (See [Chester, Perlmutter '18] on 6d as well as Shai Chester’s talk & poster.)

- Study other SCFTs from which one can compute scattering amplitudes of gauge bosons on branes. (?)

- Loops in AdS.
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