Nonplanar Correlators in N=4 SYM From Integrability

Shota Komatsu

IAS

Strings 2018

1711.05326 [Bargheer, Caetano, Fleury, Vieira, SK]
+1807.xxxxxx [Bargheer, Caetano, Fleury, Vieira, SK]

AdS = CFT

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- Deep insights into quantum gravity & interacting QFT.
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- Learn from solvable examples…?

N=4 Super Yang-Mills

- Solvable at large N using integrability

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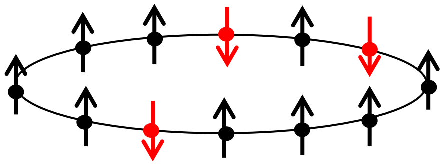
N=4 SYM

Map between "single-trace operators" and spin chains. [Minahan, Zarembo 2002]

$$\mathcal{O} \sim \mathrm{Tr} \ldots ZYZ \ldots ZYZ \ldots$$

$$=$$



Spin chain turns out to be integrable

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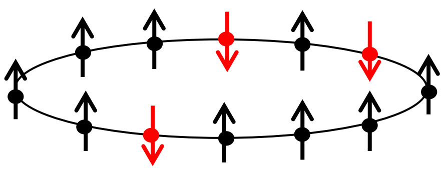

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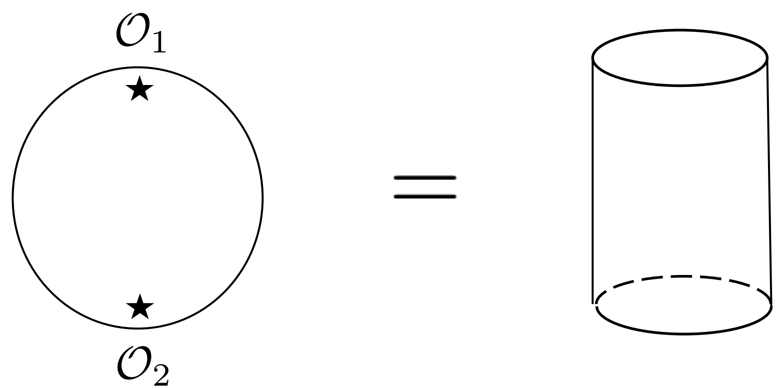
String theory

Sigma model in AdS is integrable (classically) [Bena, Polchinski, Roiban 2003]



Integrable QFT on a cylinder : solved by Thermodynamic Bethe Ansatz

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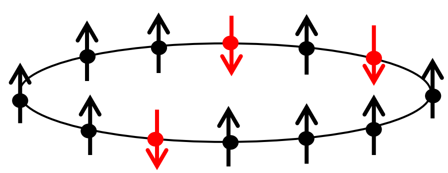

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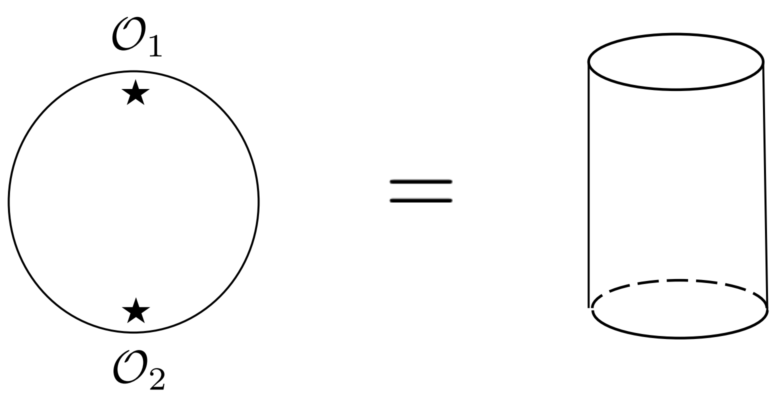


Integrable QFT on a cylinder : solved by Thermodynamic Bethe Ansatz

Quantum spectral curve

[Gromov, Kazakov, Leurent, Volin 2013]

Planar higher-pt functions

$$\langle \mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_3 \rangle$$

$$\langle \mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_3 \mathcal{O}_4 \rangle$$

For simplicity, I will focus on the correlation functions of ½ BPS operators.

$$\mathcal{O}_i = \mathrm{Tr}\left[(Y_i \cdot \Phi)^{L_i}\right]$$

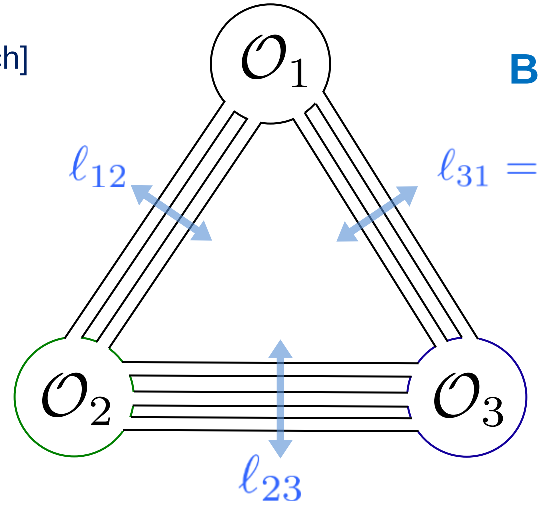
$$Y_i : \text{ Null 6-component vector} \qquad Y_i \cdot Y_i = 0$$

although many things can be generalized to non-BPS operators.

3pt = a pair of pants

N=4 SYM at zero coupling $\lambda = 0$

[Alday et al], [Okuyama, Tseng.], [Roiban, Volovich]
[Escobedo et al.] and many others



Bridge length

$$\ell_{31} = \frac{L_3 + L_1 - L_2}{2}$$

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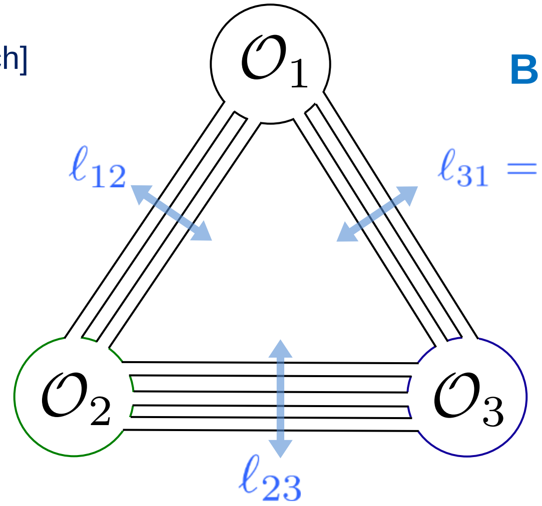
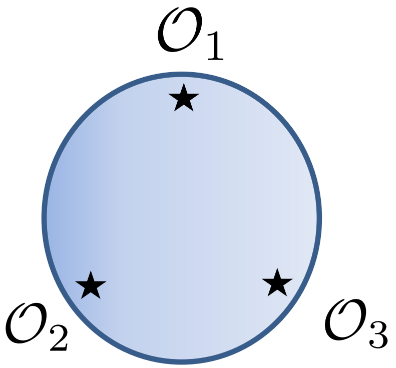
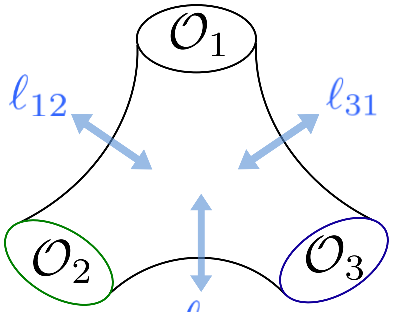


Planar surface for 3pt functions



or equivalently



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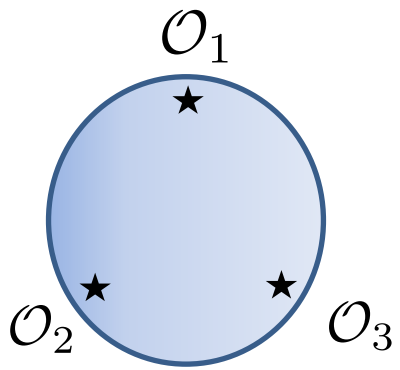

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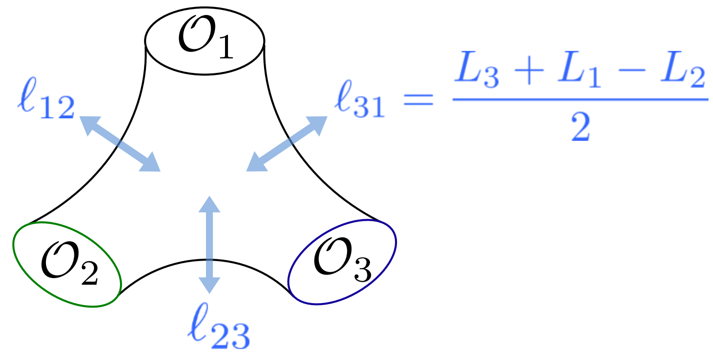
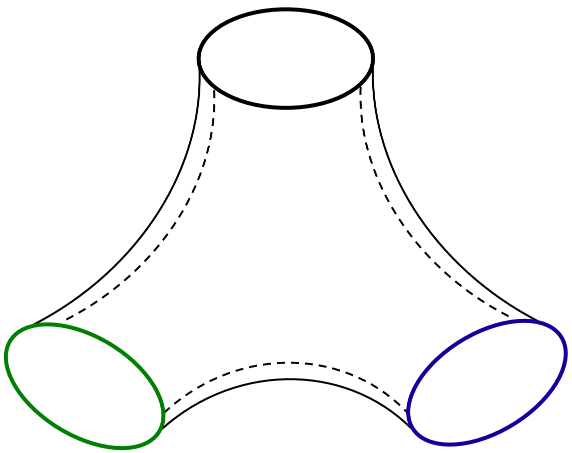


- To use integrability, one has to consider integrable models on a pair of pants.

- Never studied before in the literature.

3pt = (Hexagon)^2



[Basso, SK, Vieira 2015]

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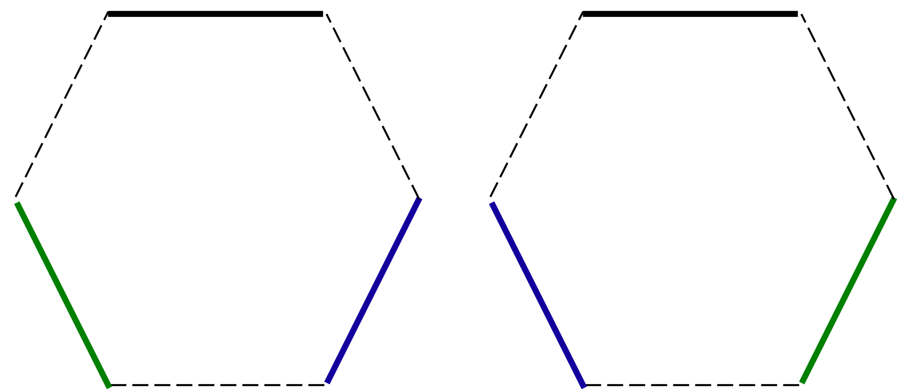
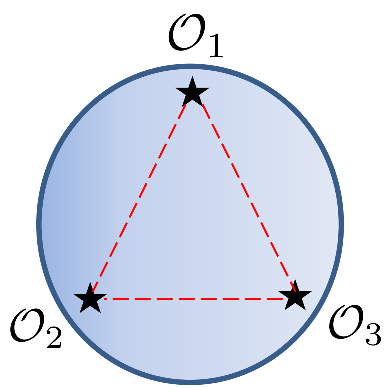
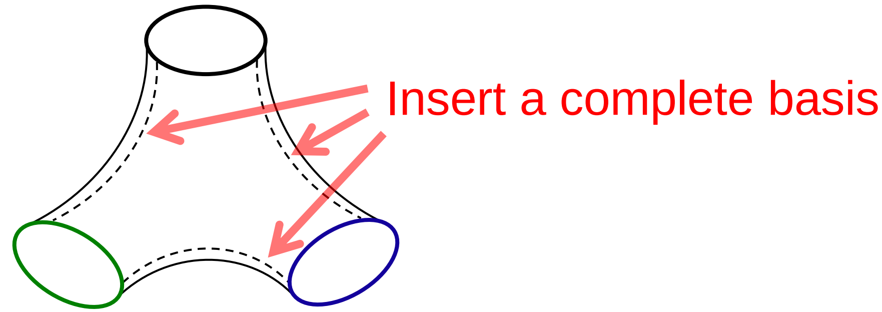




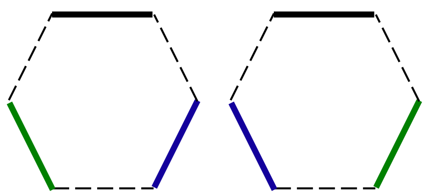
triangulation of the worldsheet

3pt = (Hexagon)^2

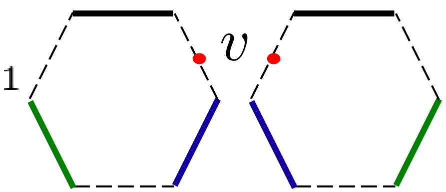
Insert a complete basis of states on the dashed lines.



$$\langle \mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_3 \rangle =$$



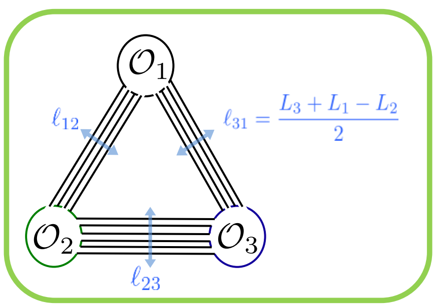
$$+ \int dv\, \mu(v) e^{-\tilde{E}\ell_{31}}$$



$$+ \cdots$$

measure

propagation factor

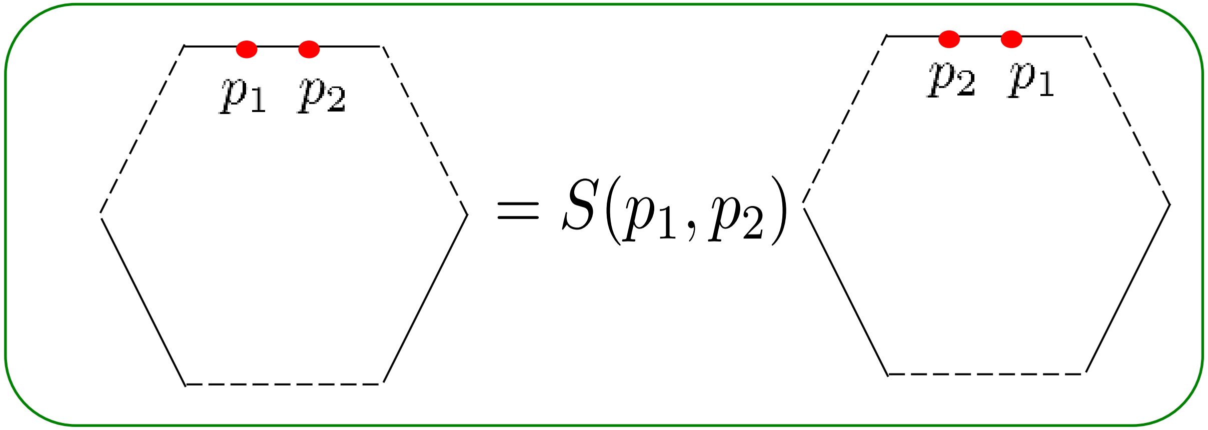


- Similar to sewing construction of 2d CFT.
- The computation of 3pt boils down to the computation of “hexagons”.

Hexagon and integrability

Symmetry (psl(2|2)) and integrability determine the contributions from each hexagon.

"Form factor bootstrap" Cf. [Smirnov]



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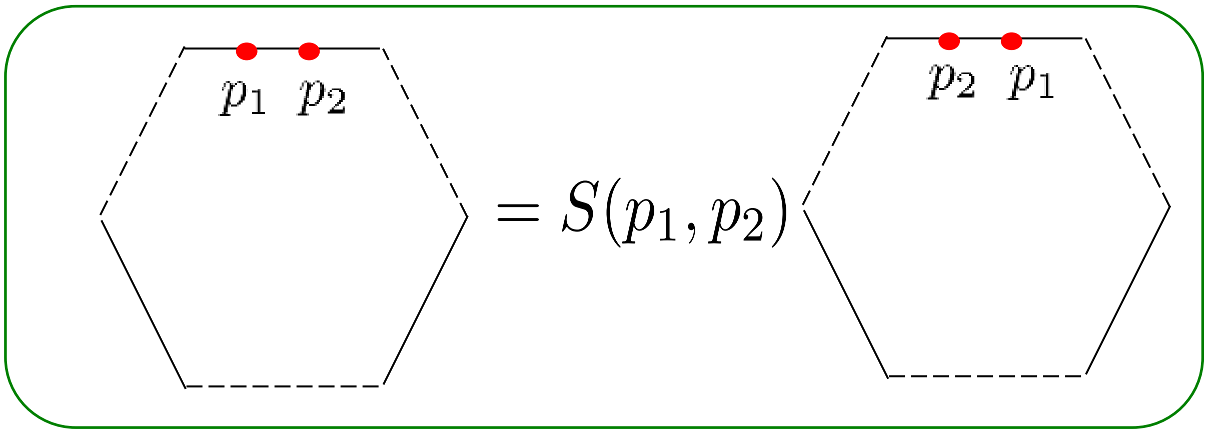

Simplest 2-particle contribution: $h(u_1, u_2) = \frac{x_1^- - x_2^-}{x_1^- - x_2^+} \frac{1 - 1/x_1^- x_2^+}{1 - 1/x_1^+ x_2^+} \frac{1}{\sigma_{12}}$

$$\frac{\lambda}{16\pi^2}\left(x^{\pm} + \frac{1}{x^{\pm}}\right) = u \pm \frac{i}{2}$$

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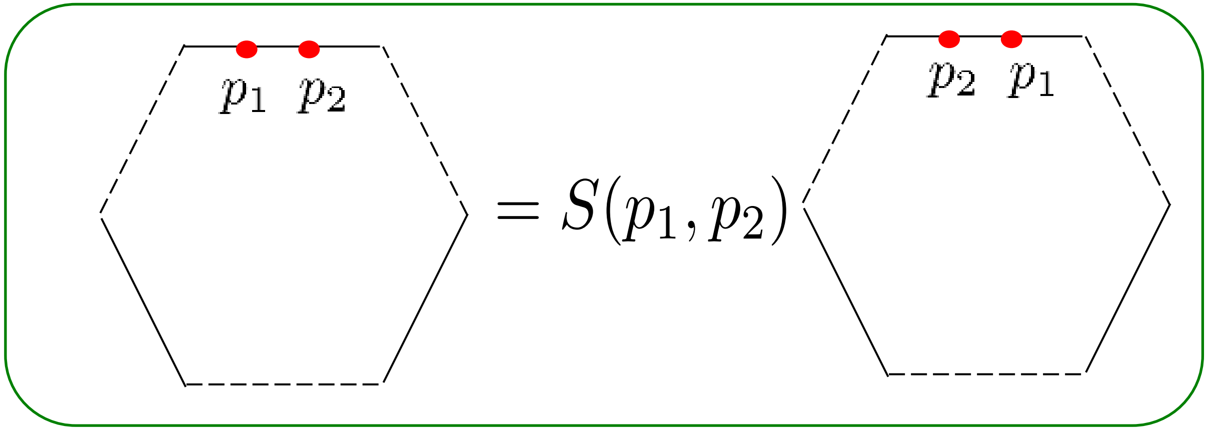

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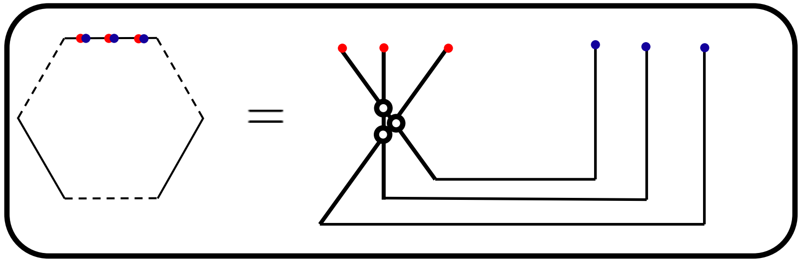

Factorization structure resembling Yang-Baxter.

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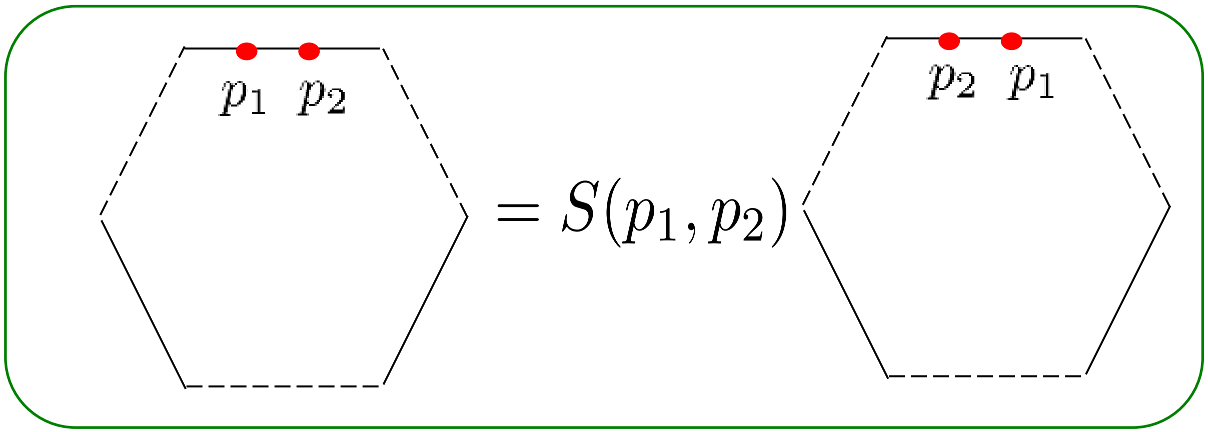

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The result beautifully reproduces the perturbative computation

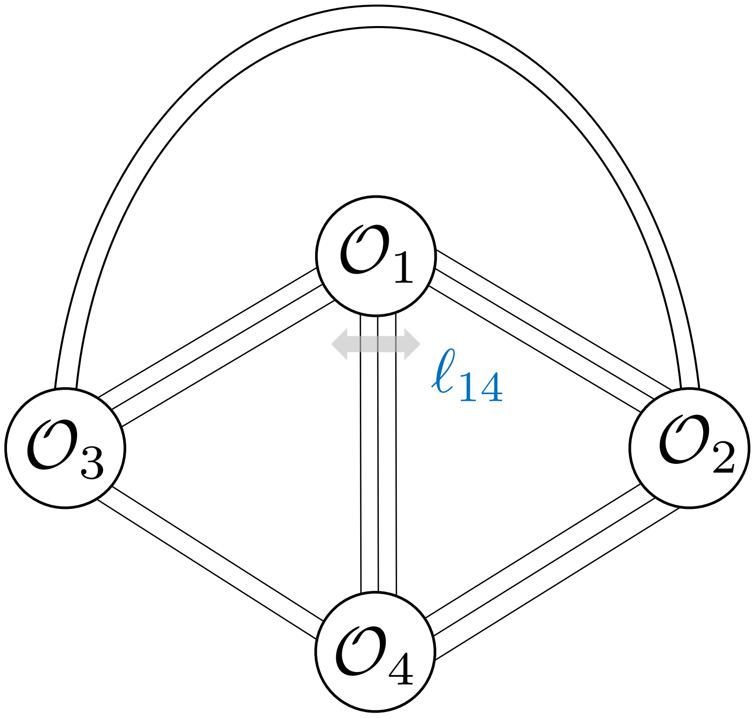
(checked up to four loops and partially at strong coupling) [Jiang, Kostov, Serban, SK], [Eden, Sfondrini] [Basso, Goncalves, Vieira SK], [Basso, Goncalves, SK]

Generalization to higher pt

All these can be generalized to higher point functions.

1) List up all possible planar graphs.
2) Compute contributions from each graph.

$$\sum_{\{\ell_{ij}\}}$$



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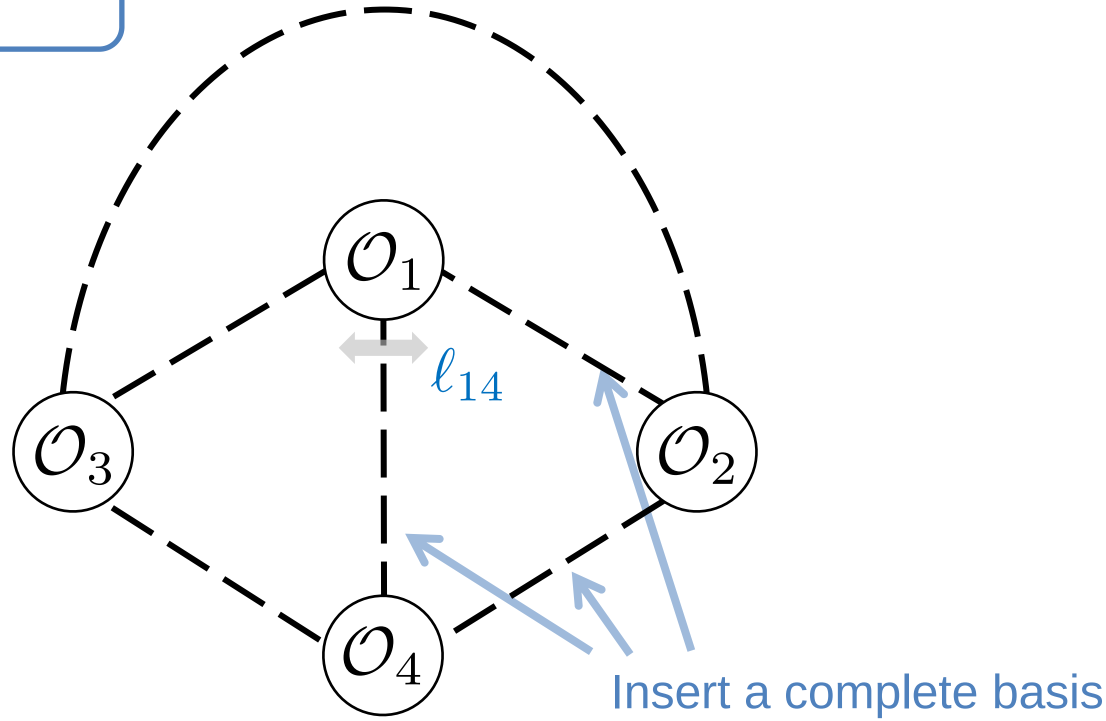
Idea: Insert a complete basis and cut it into hexagons.

$$\sum_{\{\ell_{ij}\}} \sum_{\{\psi_{ij}\}}$$



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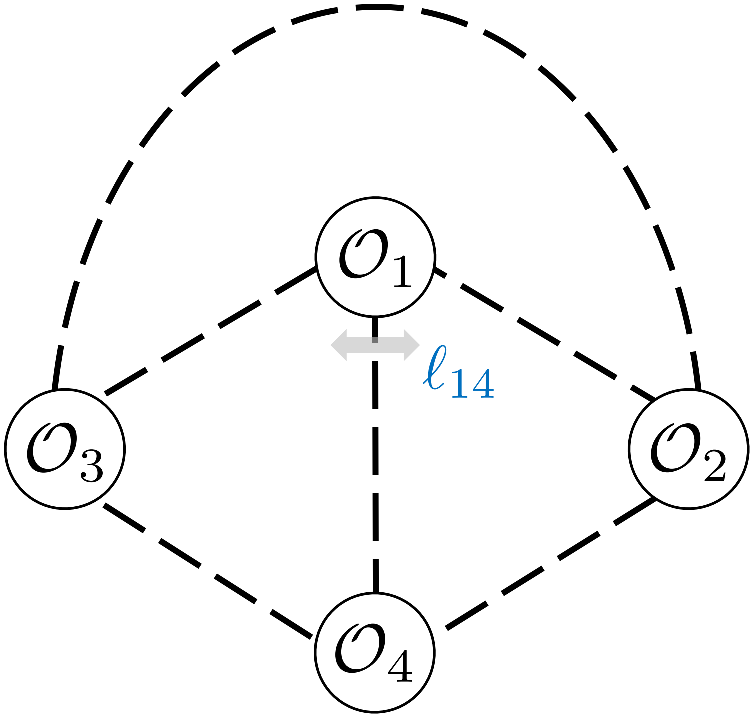
“Moduli integral”



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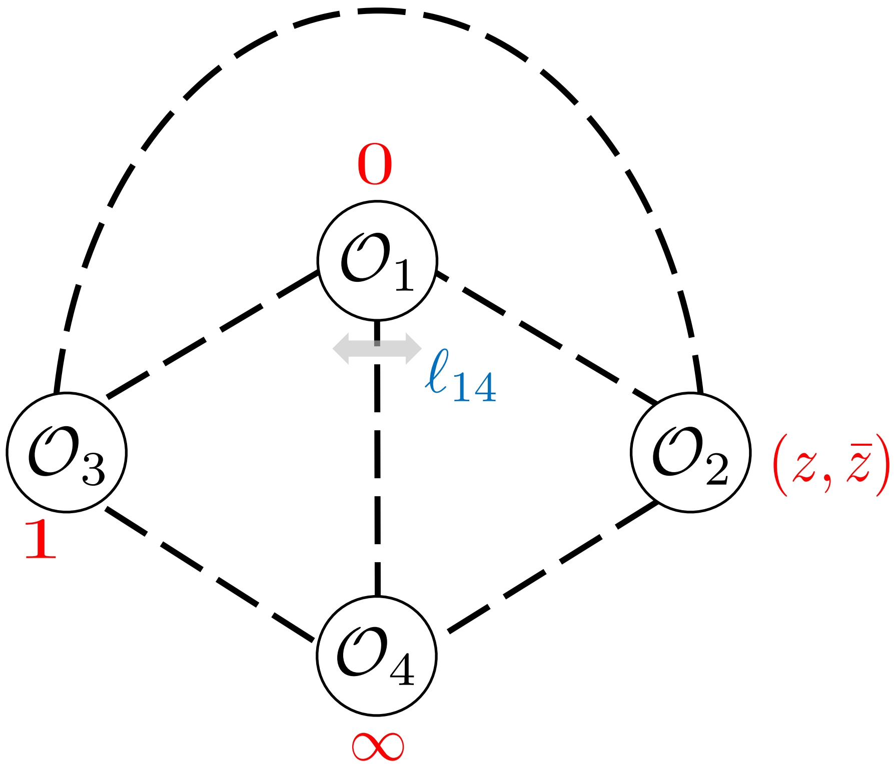

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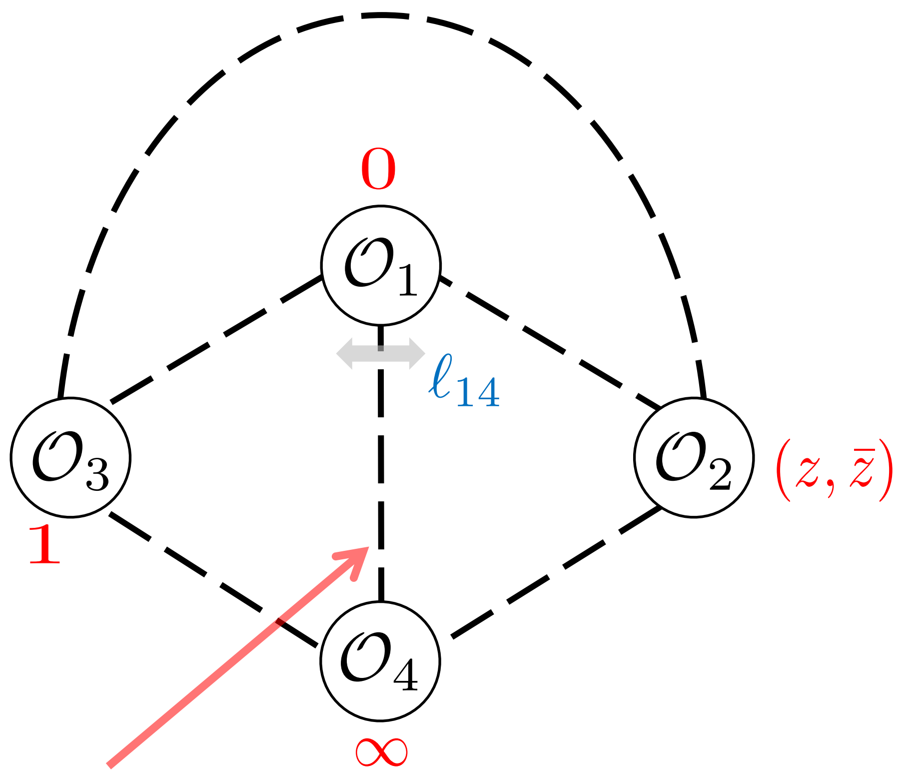

Weight factor $\mathcal{W}(z,\bar{z})$ (purely kinematical)

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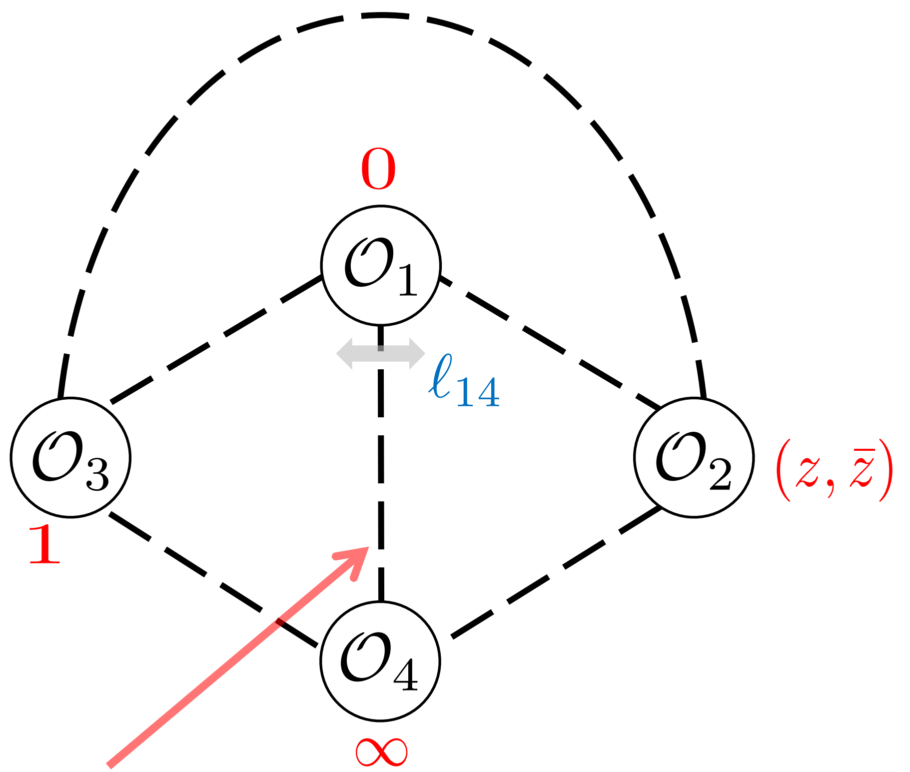

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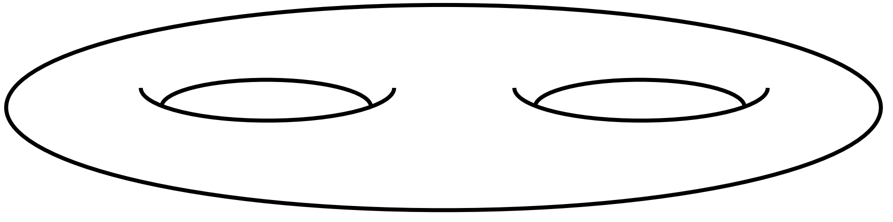
Reproduces the perturbative computation (1-loop four pt, 1-loop five pt, higher loops in special kinematics)

[Fleury, SK 2016, 2017], [Fleury, Goncalves, SK in progress]

See also [Eden, Sfondrini 2016]

Can we generalize this to nonplanar surfaces?



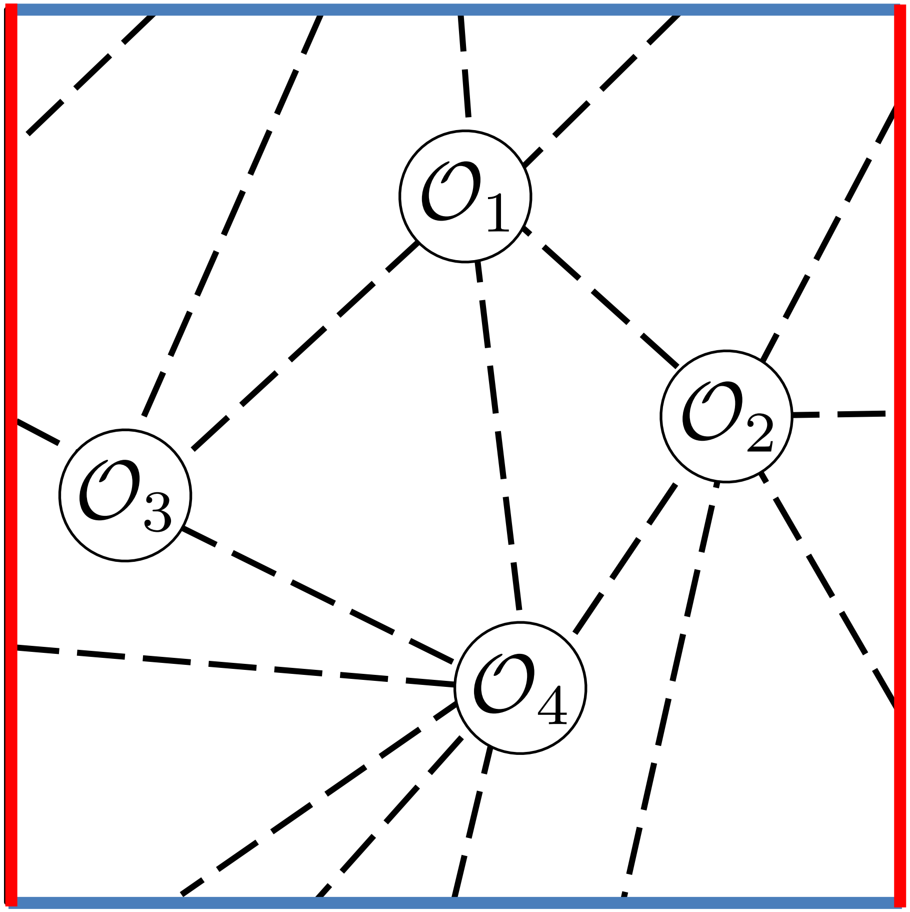
Decomposing the surface with hexagons

First guess

$$\sum_{\substack{\text{tree-level}\\ \text{graphs}}} \sum_{\psi} (\texttt{Weight}) \prod (\texttt{Hexagon})$$

Decomposing the surface with hexagons

First guess [Bargheer, Caetano, Fleury, Vieira, SK] also [Eden, Jiang, Sfondrini]



Cut the surface into **planar** hexagons, insert a complete basis, and sum over the graph.

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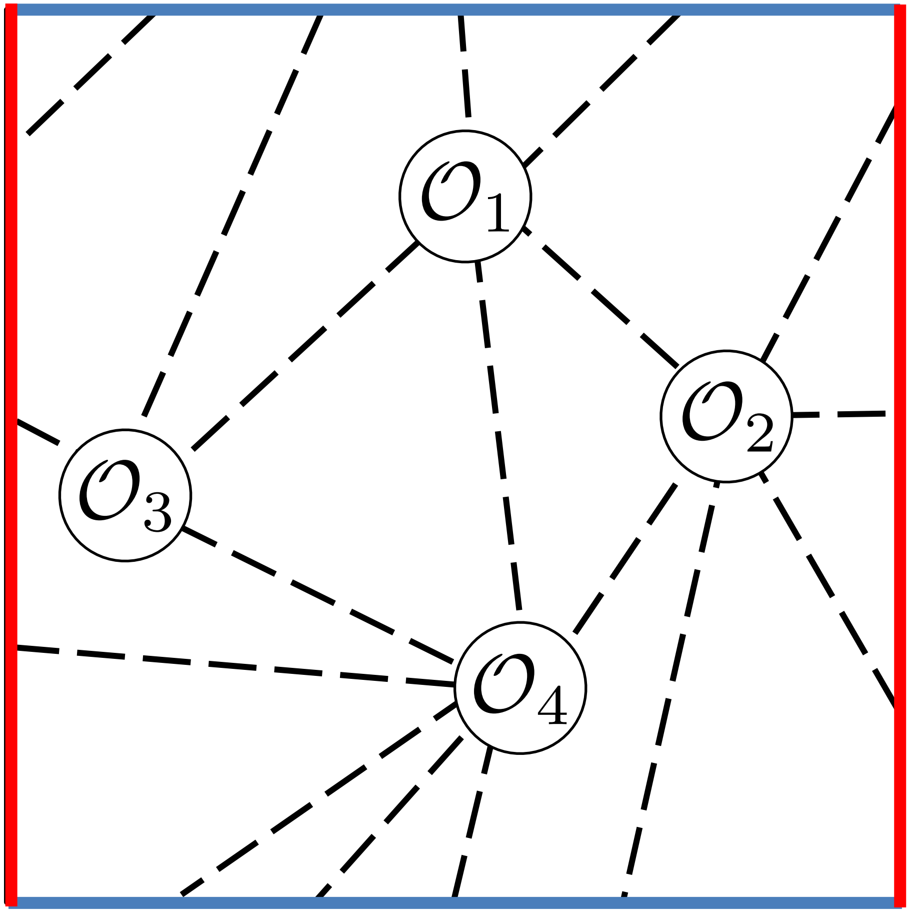

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8 hexagons, complicated...

Simplification for large-length operators

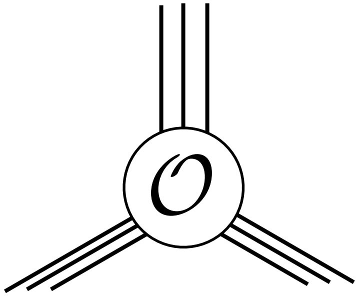
Combinatorial enhancement:

When the operators are very long,

$$L_i \gg 1$$

the maximally connected graphs will be combinatorially dominant.

# of ways to split L propagators to n groups = $\begin{pmatrix} L+n \\ n \end{pmatrix} \sim L^n$



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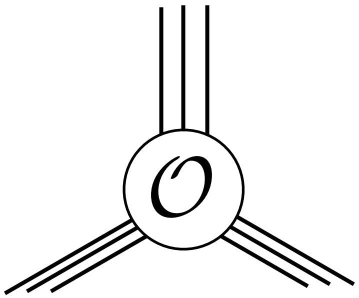
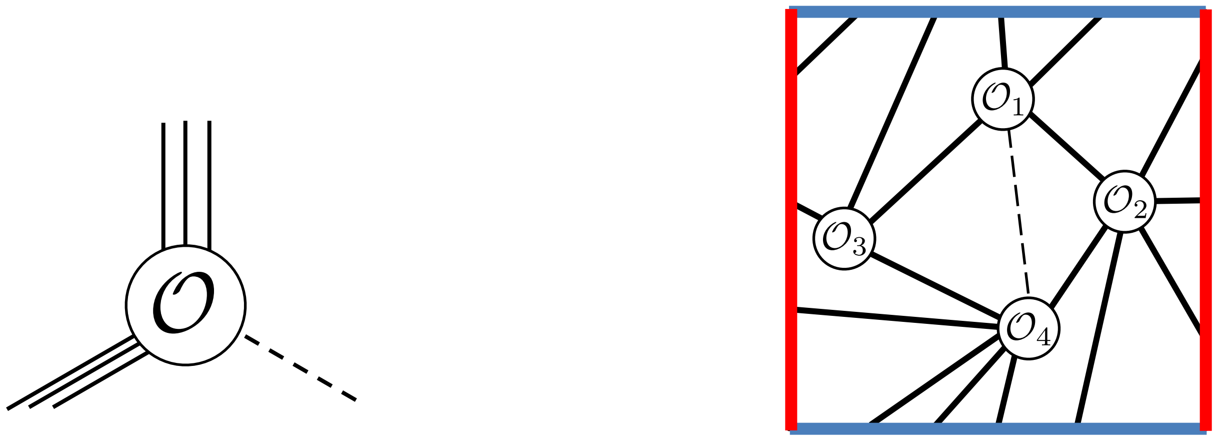


Dynamical suppression:

If all ℓ_{ij} are large, the contributions are exponentially suppressed.

$$e^{-E\ell_{ij}} \ll 1$$

Simplification for large-length operators



Submaximal graphs, obtained from the maximal graphs by erasing a few connections, will dominate!
(Locally look the same as planar four-point function.)

$$(\texttt{4pt at } \frac{1}{N_c^4})\Big|_{L\gg 1} = \lambda [\cdots] F^{(1)} + \lambda^2 \left([\cdots] F^{(2)} + [\cdots] (F^{(1)})^2 \right)$$

match!

$$+ \lambda^3 \left([\cdots] F^{(3)} + [\cdots] F^{(2)} F^{(1)} + [\cdots] (F^{(1)})^3 \right)$$

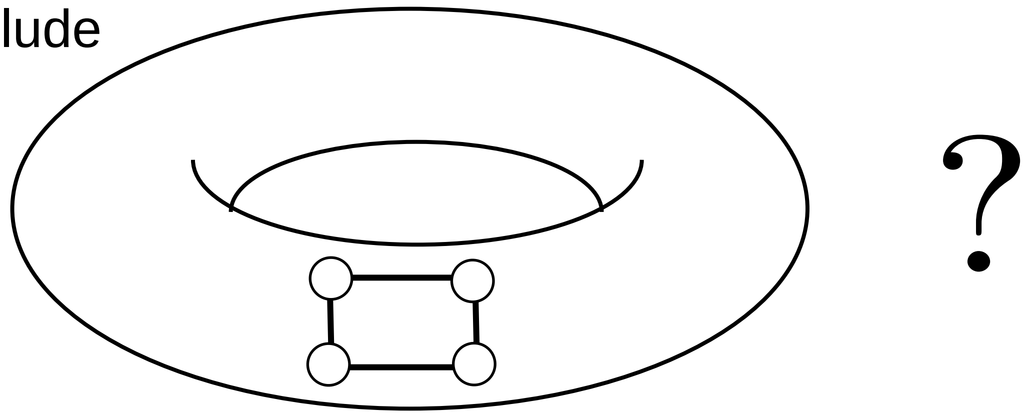
prediction

Finite L's

At $O(\lambda)$, we can also try to study the correlators of finite-length operators.

Puzzle:

- Our basic strategy : Start from tree-level graphs, cut it into hexagons and dress them with magnons.
- Should we include

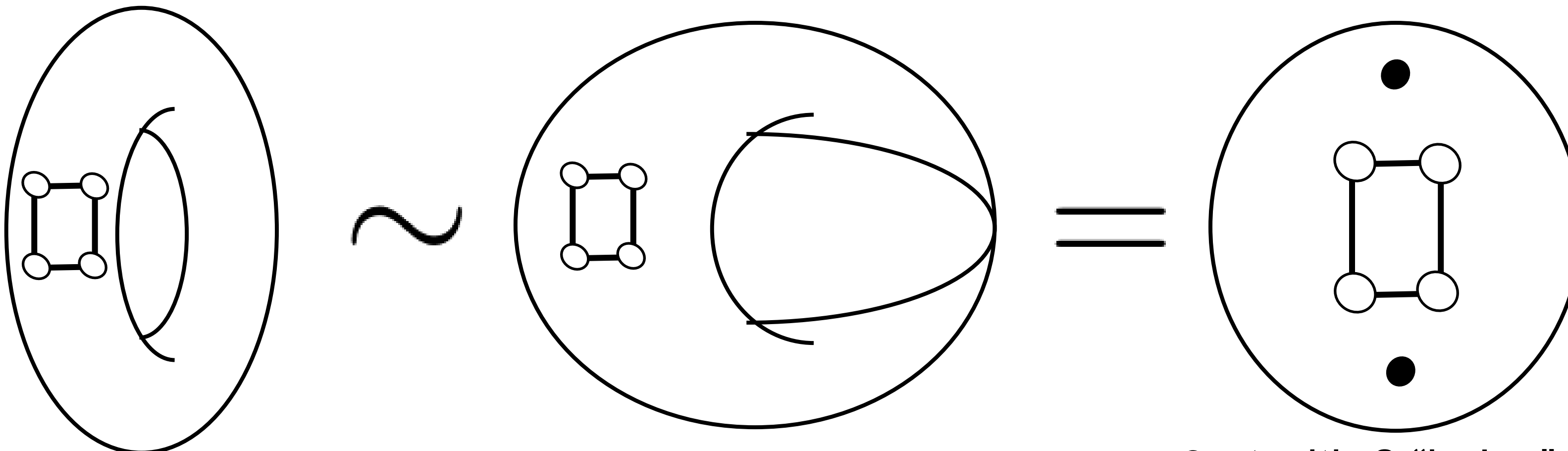


- **Not at tree level**. But we cannot simply throw it away since this can include genuine nonplanar contributions (when dressed with magnons).

Resolution: Use the analogy between the sum over graphs and the moduli space of Riemann surface.

“Stratification”

- The graphs that are secretly planar are the analogues of the degenerate Riemann surfaces.



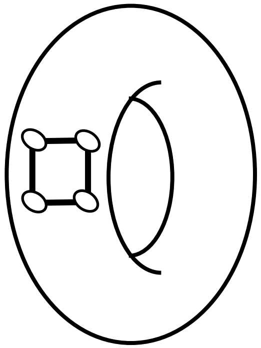
6-pt with 2 “holes”.

- They correspond to the boundaries of the torus moduli space.

“Stratification”

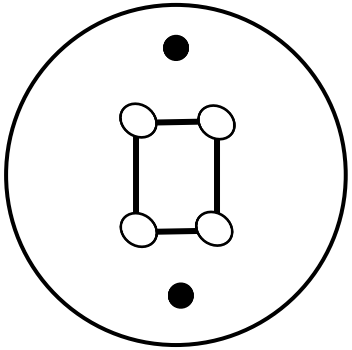
- **Prescription**: Sum all the graphs and subtract the “boundaries of moduli”.

[Deligne-Mumford]
[Chekhov]



All the tree-level graphs on a torus

$-$



All the tree-level graphs on a sphere with two holes.

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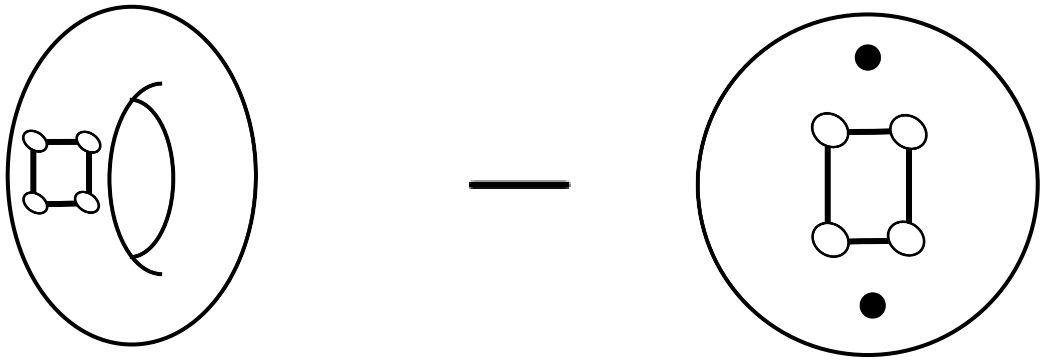
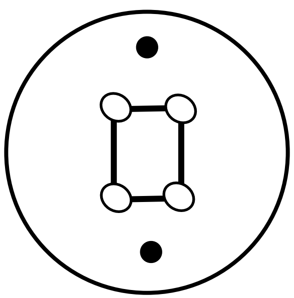


All the tree-level graphs on a torus

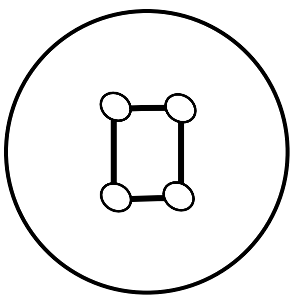
All the tree-level graphs on a sphere with two holes.

- No dynamics at the holes. They are just the holes of the double-line Feynman diagram.

  ➡ Same as adding probe D3’s



$$\sim \frac{\partial^2}{(\partial N_c)^2}$$



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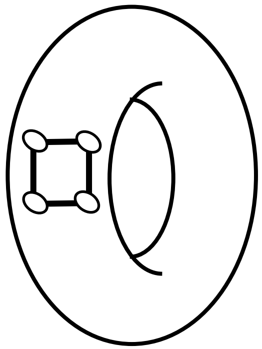

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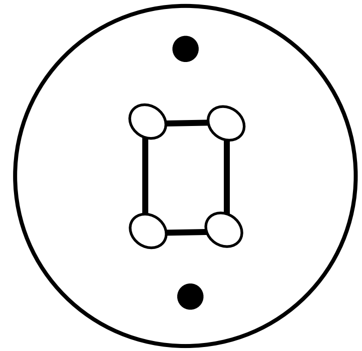

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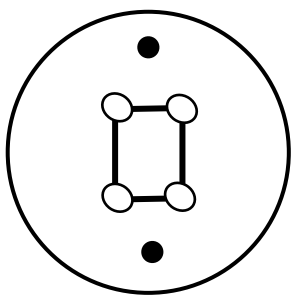
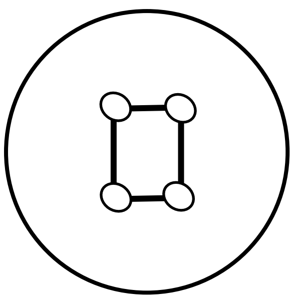
- No dynamics at the holes. They are just the holes of the double-line Feynman diagram.

➡ Same as adding probe D3’s



$$\sim \frac{\partial^2}{(\partial N_c)^2}$$



- Correctly reproduces the finite L answers at $O(\lambda)$

Summary and outlook

Summary:

- One can use integrability to study non-planar quantities.
- Sum over graphs, cut them into planar hexagons.
- Finite L $\rightarrow$ “Stratification”.

Outlook:

- Better understand the relation between the graphs and the moduli space, and the stratification. Cf. [Gopakumar 2003] [Razamat 2008] [Gopakumar, Pius 2012]
- Try to resum 1/N series in some kinematics? Cf. [Gross Mende] [Mende Ooguri]
- Find ways to efficiently resum magnons. Quantum spectral curve?