Traversable Wormholes

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Based on work in progress with:

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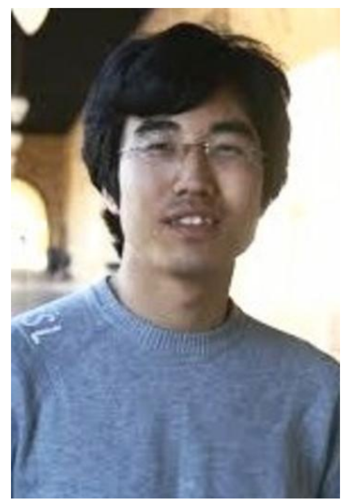
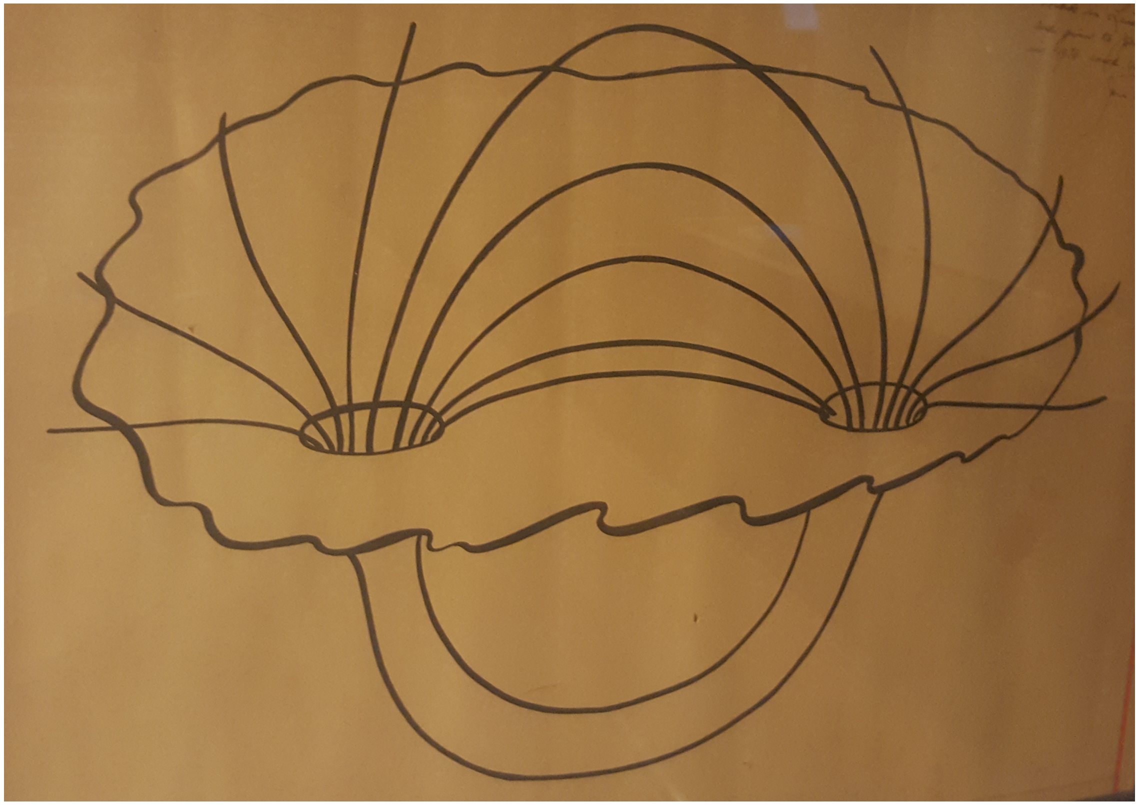
Related to previous work with Xiaoliang Qi



Inspired by work by Gao, Jafferis and Wall on “Traversable wormholes”

Drawing by John Wheeler, 1966



Charge without charge.
Mass without mass

Spatial geometry. Traversable wormhole

Recall classic results

There are no science fiction wormholes!

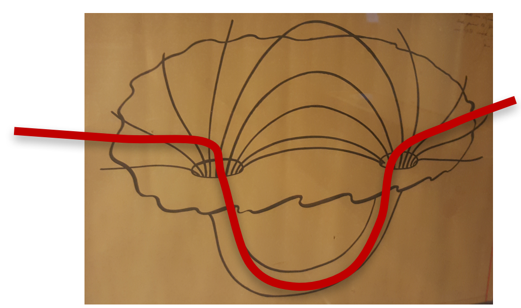
- No wormhole allows you to travel faster than the speed of light in the ambient space.
- Forbidden by:

Friedman Schleich, Witt, Galloway, Woolgar
Gao Wald

- I) Einstein equations.
- II) The Achronal Average Null Energy Condition

Not yet proven in a general spacetime, but believed to hold in QFT

$$\int dx^- T_{--} \geq 0$$



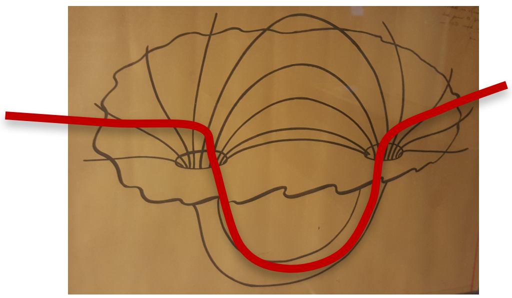
Achronal = points along null line are not timelike separated = fastest line

Longer wormholes?

- What if it takes longer to go through the wormhole ?
- Not possible in classical physics due to the Null Energy Condition.

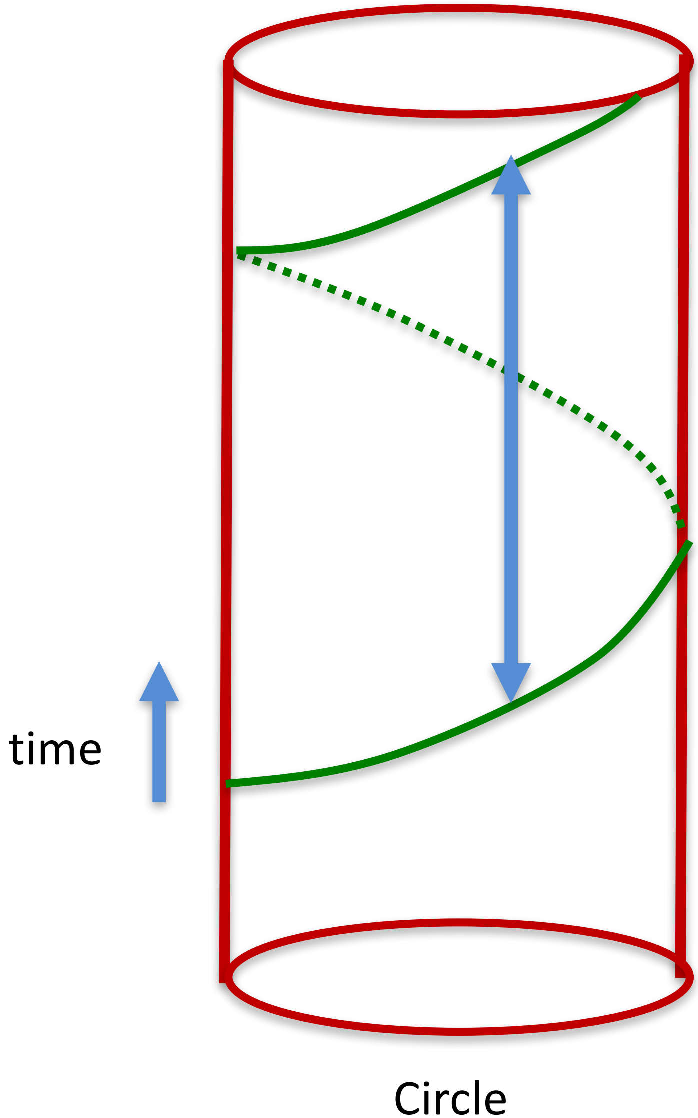
Topological censorship: Friedman Schleich, Witt, Galloway, Woolgar

- In classical physics → we can go to the covering space, which is forbidden by the previous case.
- → We need quantum effects to find a solution. Casimir-like energy.
- Can we do it in a controllable way ?



Negative null energy in QFT

Eg. Two spacetime dimensions



$$T_{--} < 0$$

$$E \propto -\frac{c}{L}$$

Negative Casimir energy

Quantum effect

The null energy condition does not hold for null lines that are not achronal!

Some necessary elements

- We need something looking like a circle to have negative Casimir energy.

- Large number of bulk fields to enhance the size of quantum effects.

- We will show how to assemble these elements in a few steps.

The theory

$$S = \int d^4x \left[R - F^2 + i\bar{\psi} \not{D}\psi\right]$$

Einstein + U(1) gauge field + massless charged fermion

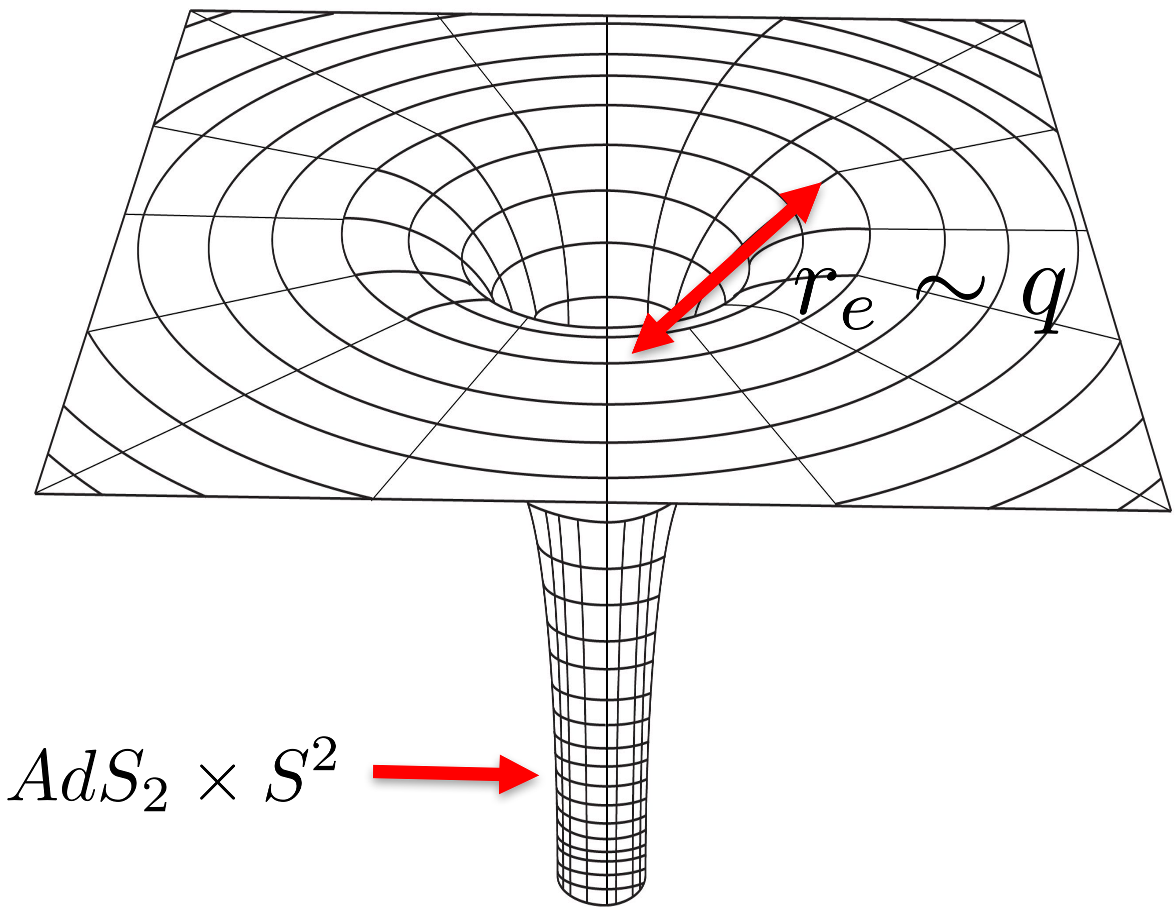
Could be the Standard Model at very small distances, with the fermions effectively massless. The U(1) is the hypercharge. SU(3) x SU(2) x U(1).

$l_{\text{Planck}} = 1$

The first solution: Extremal black hole

Magnetic charge q

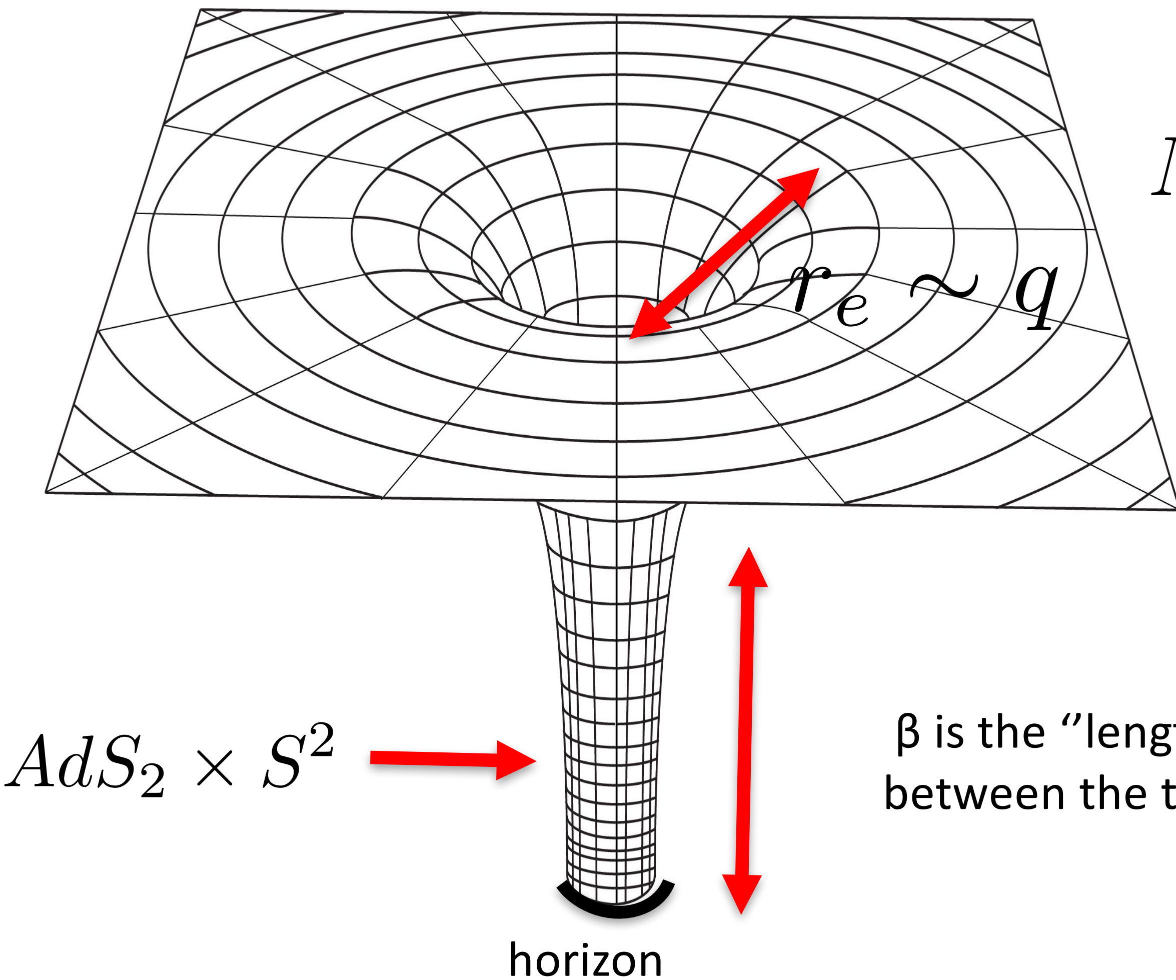
$$\int_{S^2} F = q = \text{integer}$$



$$M = q$$

$$l_{\text{Planck}} = 1$$

The next solution: Near Extremal black hole



$$M - q = q^3 T^2 = \frac{q^3}{\beta^2}$$

Very small

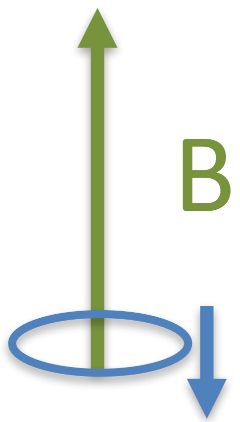
β is the "length" of the throat. Redshift factor between the top and the bottom

$l_{\text{Planck}} = 1$

Motion of charged fermions

- Magnetic field on the sphere.
- There is a Landau level with precisely zero energy.
- Orbital and magnetic dipole energies precisely cancel.
- Explained by an anomaly argument

Ambjorn, Olesen



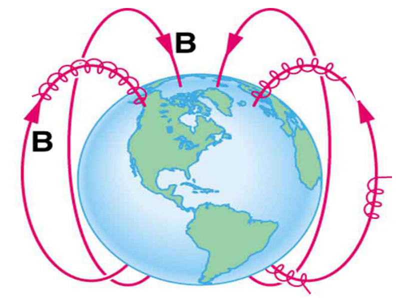
Massless fermions → U(1) chiral symmetry

4d anomaly → 2d anomaly → there should be massless fermions in 2d.

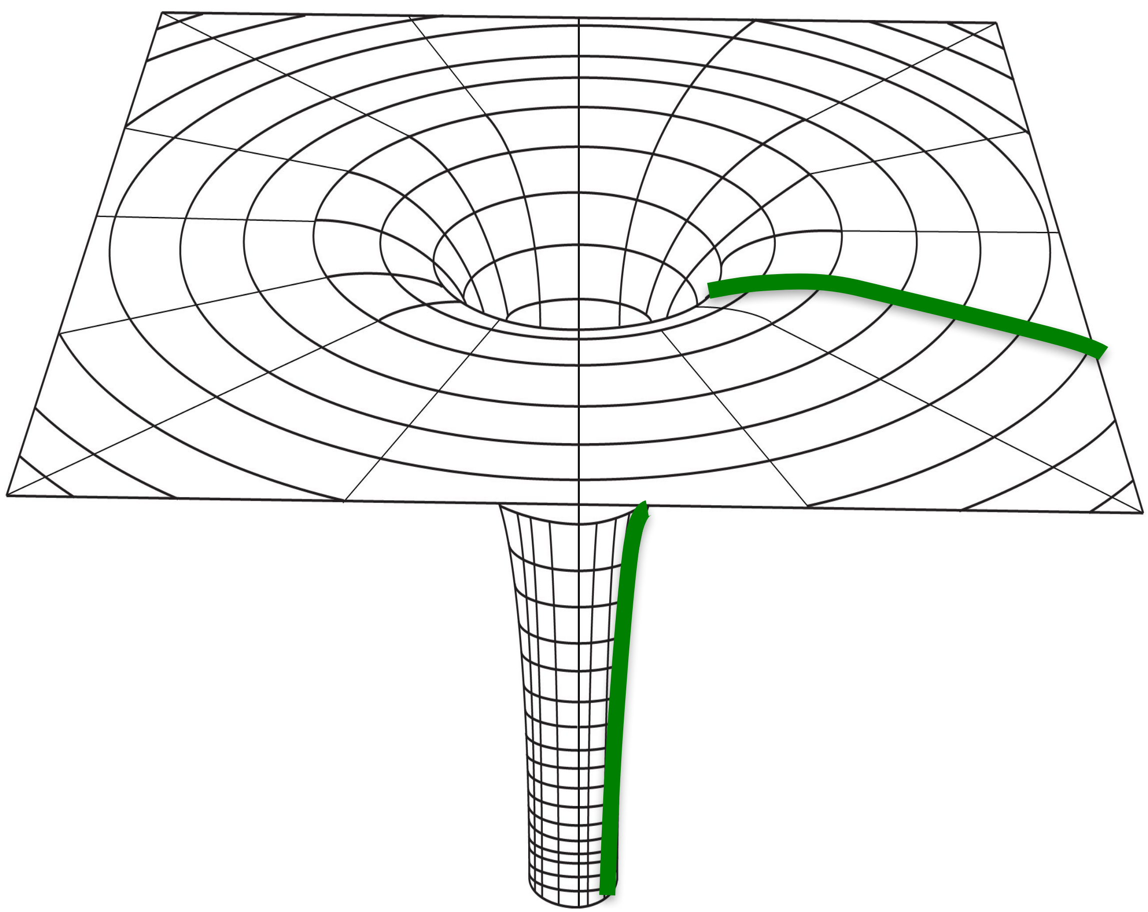
(Here we view F as non-dynamical).

Motion of charged fermions

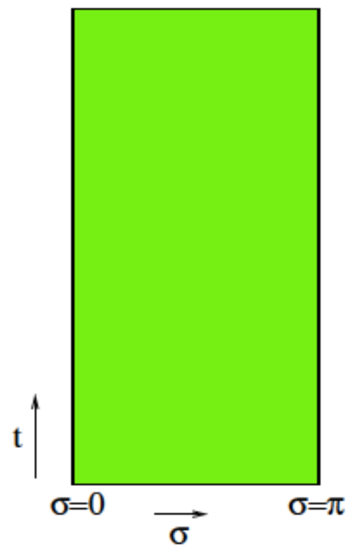
- Degeneracy = q = flux of the magnetic field on the sphere. Form a spin j, representation of SU(2), 2j +1 =q.
- We effectively get q massless two dimensional fermions along the time and radial direction.
- We can think of each of them as following a magnetic field line.

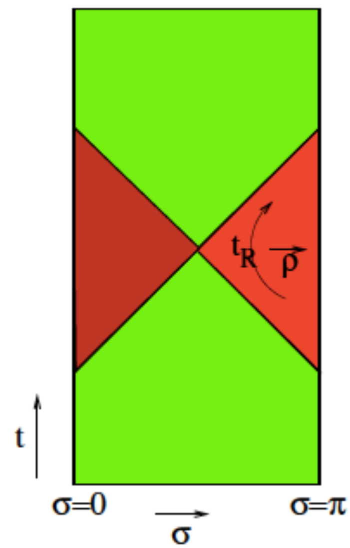


q massless two dimensional fields, along field lines.



AdS_2





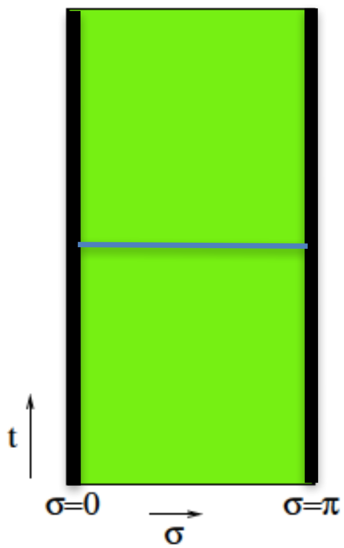
$$ds^2 = \frac{-dt^2 + d\sigma^2}{\sin^2 \sigma}$$

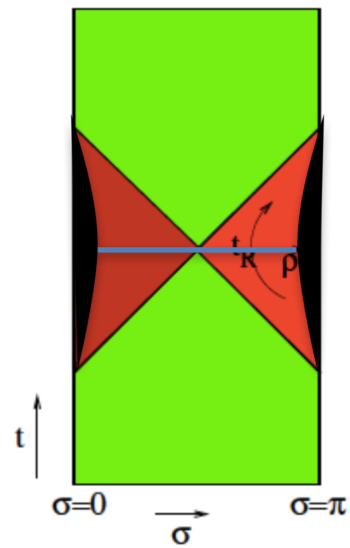
Global

$$ds^2 = -(r^2 - 1)d\tau^2 + \frac{dr^2}{(r^2 - 1)}$$

Thermal/Rindler

Nearly AdS_2





$$ds^2 = \frac{-dt^2 + d\sigma^2}{\sin^2 \sigma}$$

Global

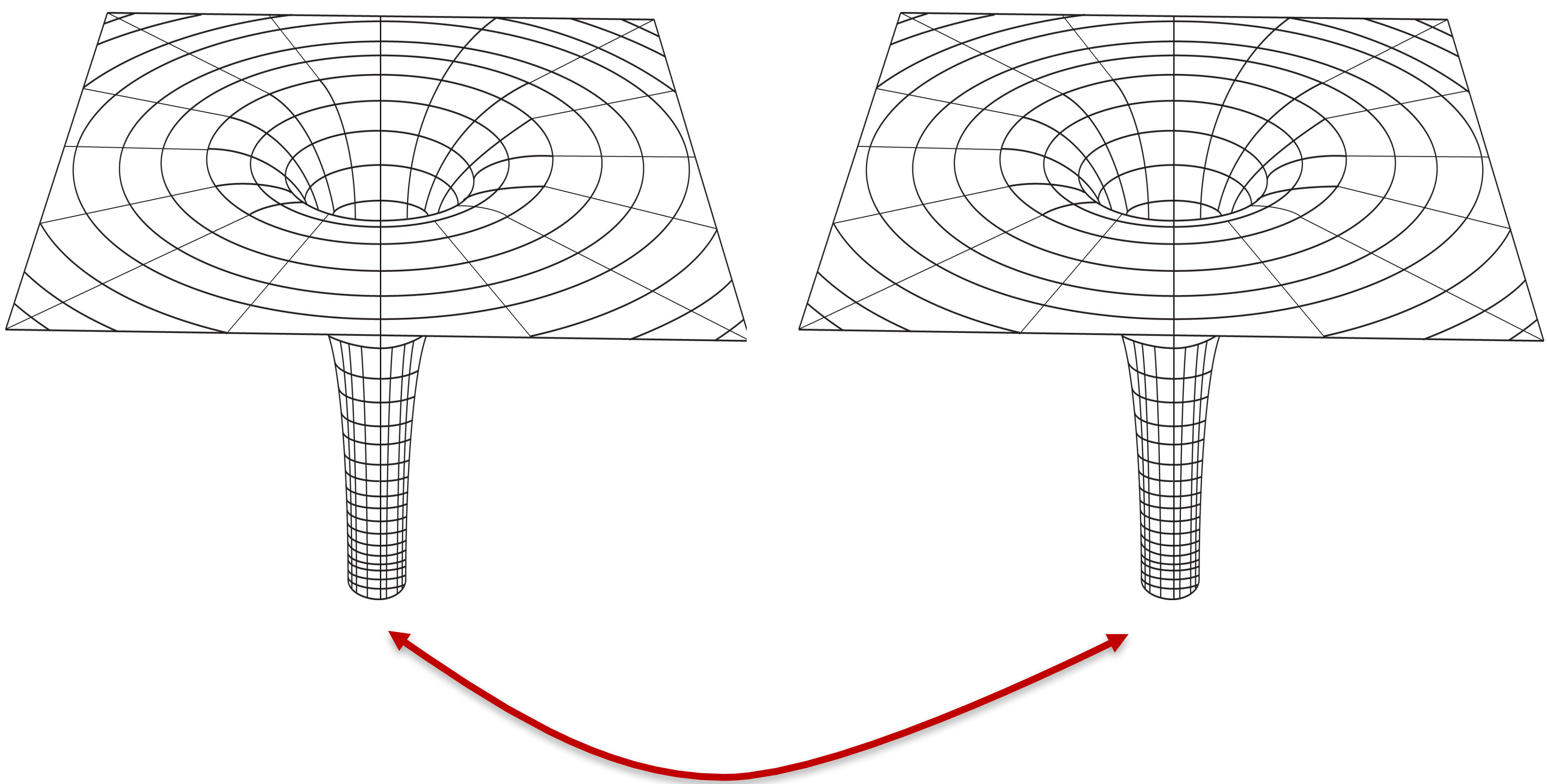
$$ds^2 = -(r^2 - 1)d\tau^2 + \frac{dr^2}{(r^2 - 1)}$$

Thermal/Rindler

Connect them to flat space, so that t is an isometry.
The acquire non-zero energy when the throat has finite length

$$M - q = q^3 T^2 = \frac{q^3}{\beta^2}$$

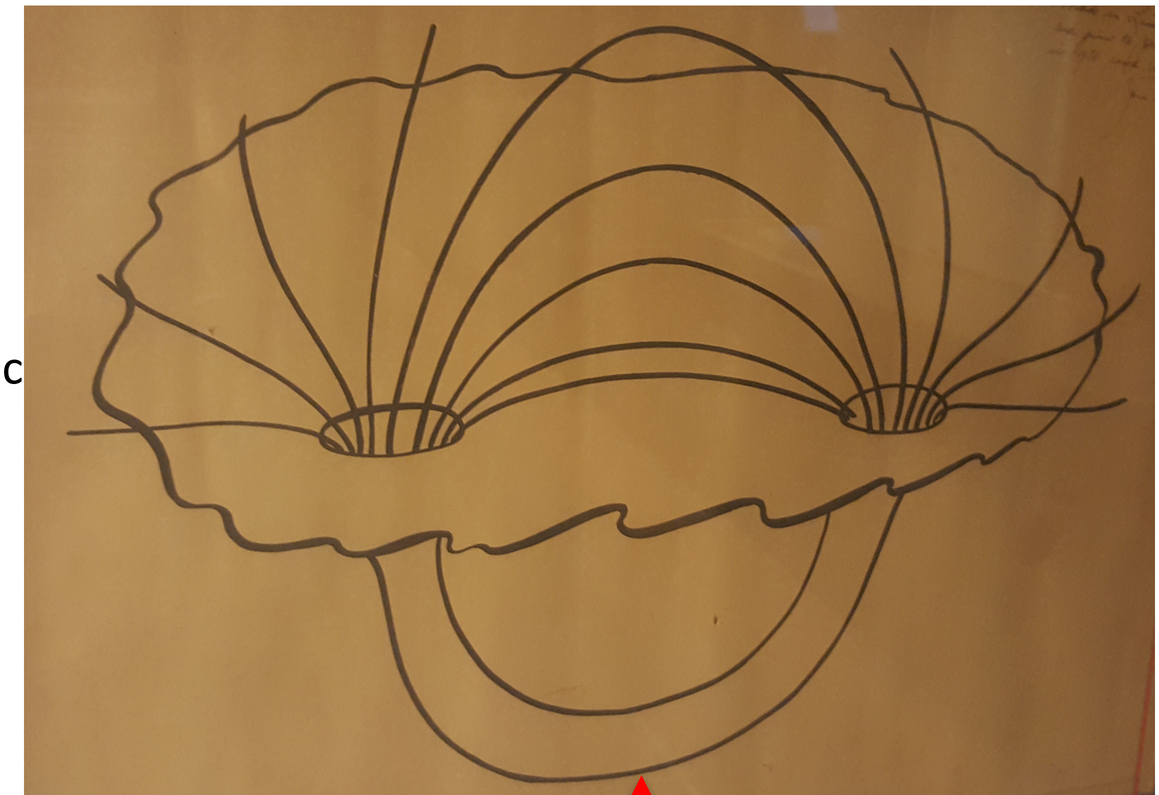
Connect a pair black holes



connect and in global AdS_2

Connect a pair black holes

Positive magnetic charge



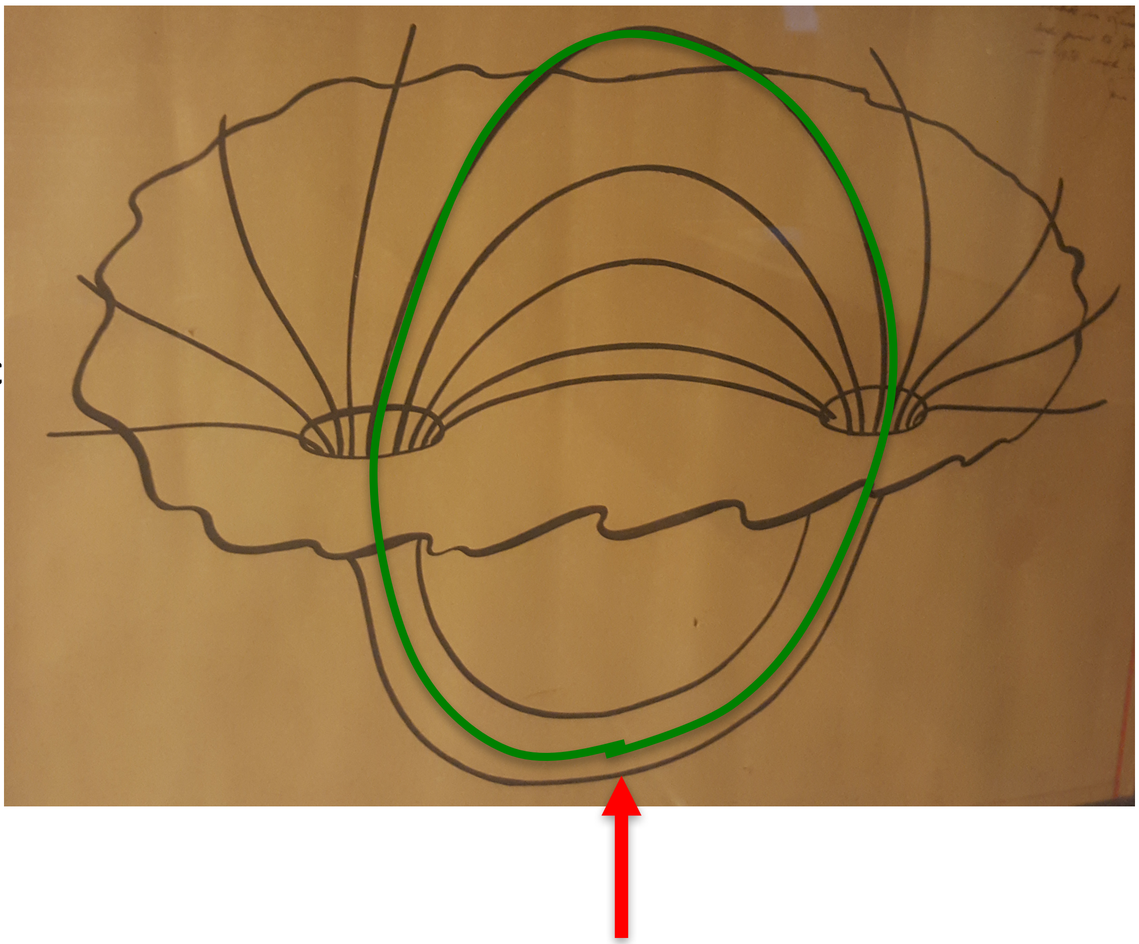
Negative magnetic charge

Not a solution yet.
Not a black hole.

Nearly $AdS_2 \times S^2$ wormhole of finite length

Fermion trajectories

Positive magnetic charge



Negative magnetic charge

Charged fermion moves along this closed circle.

Casimir energy

Assume: "Length of the throat" is larger than the distance.

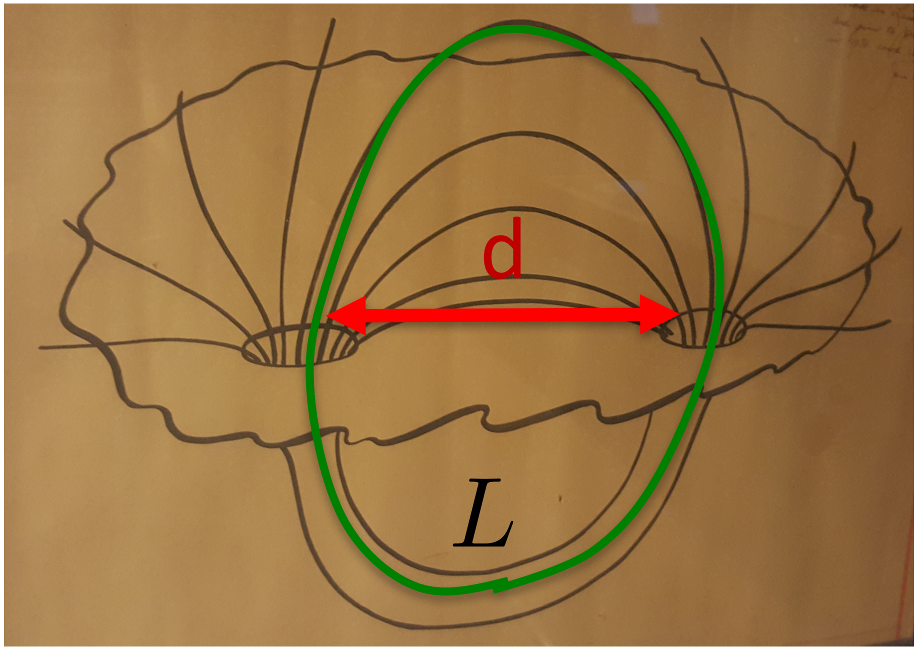
$$L \gg L_{out} > d$$

Casimir energy is of the order of

$$E \propto -\frac{q}{L}$$

Full energy also need to take into
Account the conformal anomaly because
AdS_2 has a warp factor.
That just changes the numerical factor.

L_{out}



Finding the solution

Solve Einstein equations with the negative quantum stress tensor

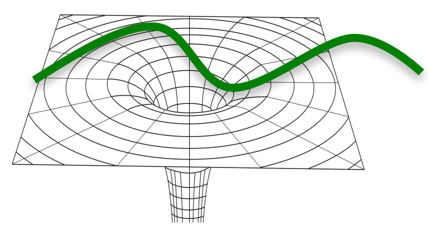
Balance the classical curvature + gauge field energy vs the Casimir energy.

$$M - q = \frac{q^3}{L^2} - \frac{q}{L}, \qquad \frac{\partial M}{\partial L} = 0 \longrightarrow L \sim q^2$$

Now the throat is stabilized. Negative binding energy.

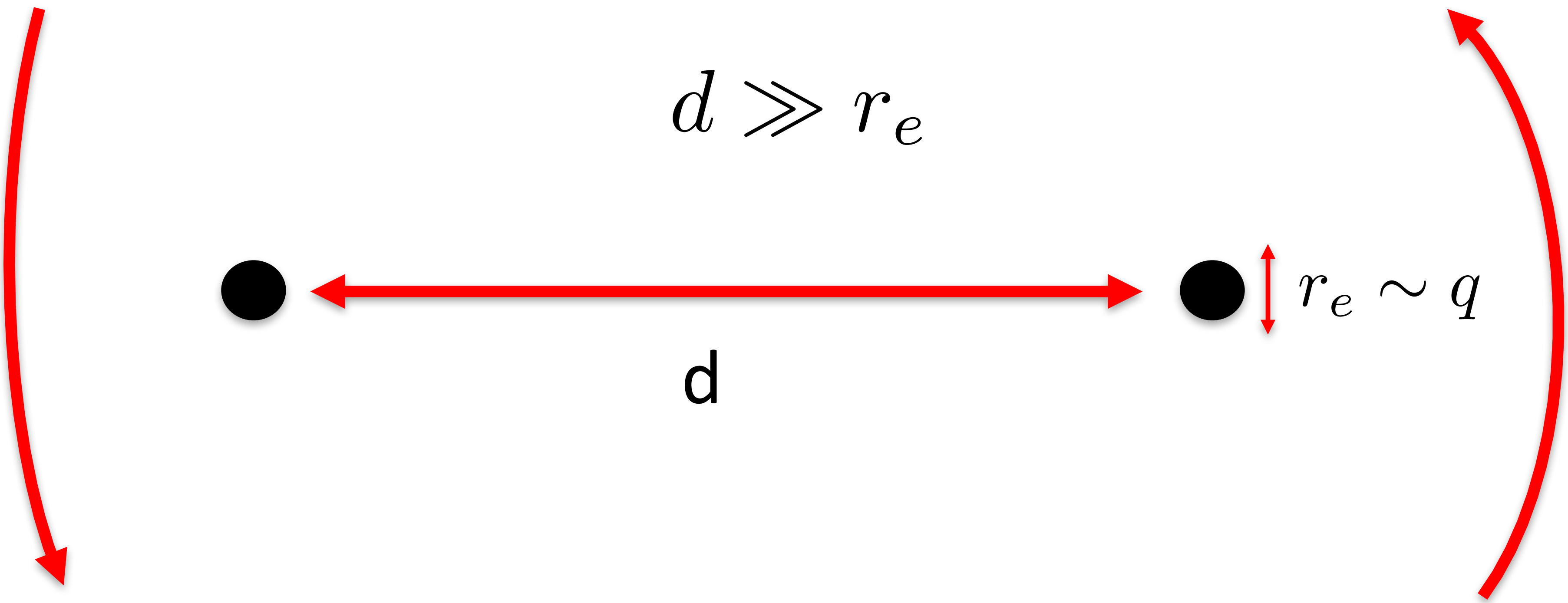
$$E_{\text{binding}} = M - q = -\frac{1}{q} = -\frac{1}{r_s}$$

Very small. Only low energy waves can explore it



This is not yet a solution: The two objects attract and would fall on to each other

Adding rotation



$$\Omega = \sqrt{\frac{r_e}{d^3}}$$

Kepler rotation frequency

Throat is fragile

- We must make sure not to start sending matter into the throat that can accumulate there and produce a black hole.
- Rotation → radiation → effective temperature: $T \sim \Omega$
- We need that Ω is smaller than the energy gap of the throat

$$\Omega \ll \frac{1}{L}$$

- The configuration will only live for some time, until the black holes get closer..
- These issues could be avoided by going to AdS_4 ...

Some necessary inequalities

$L \sim q^2$ From stabilized throat solution

$d \ll L \longrightarrow d \ll q^2$ Black holes close enough to that Casmir energy computation was correct.

$\sqrt{\frac{q}{d^3}} = \Omega \ll \frac{1}{L} \longrightarrow q^{\frac{5}{3}} \ll d$ Black holes far enough so that they rotate slowly compared to the energy gap.

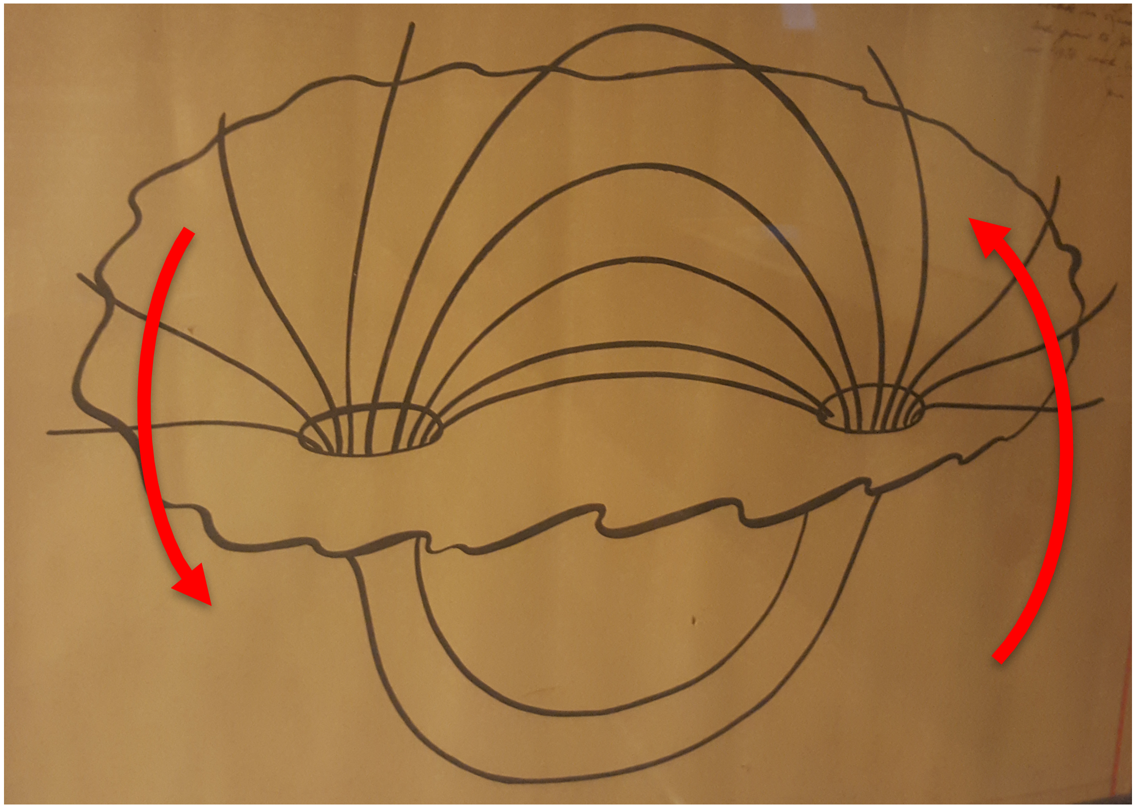
Kepler
rotation frequency

Unruh-like temperature less than energy gap

They are compatible $$q^{\frac{5}{3}} \ll d \ll q^2$$

Other effects we could think off are also small :
can allow small eccentricity, add electromagnetic and gravitational radiation, etc. Has a finite lifetime.

Final solution



Looks like the exterior of two near extremal black holes. But they connected.
But there is no horizon!. Zero entropy solution.
It has a small binding energy.

Rotation → temperature

$$T \sim \Omega \ll E_{\rm gap} \sim \frac{1}{L}$$

↑

Temperature does not create particles
in the throat

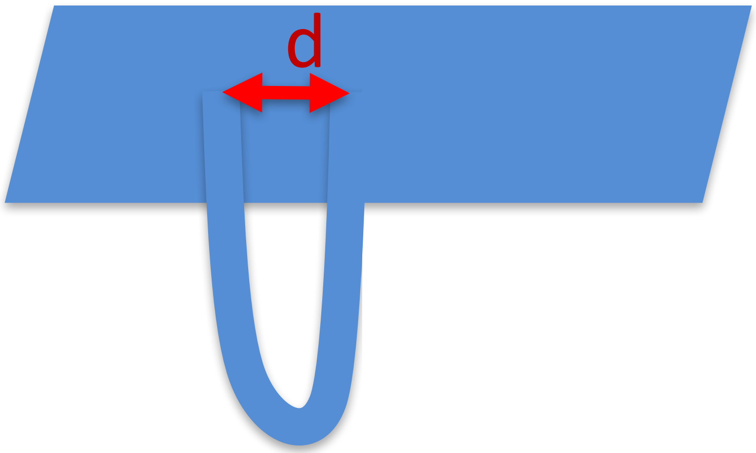
$$\text{Two Black Holes}: \quad F = -Tq^2$$

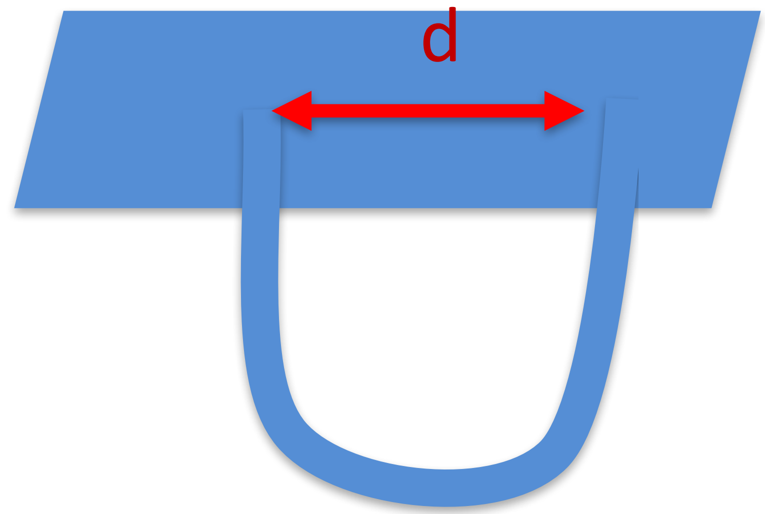
$$\text{Wormhole}: \quad F = -E_{\rm binding} = -\frac{1}{q}$$

Wormhole is the stable thermodynamic phase for $T < 1/q^3$

For the solution we described so far: Wormhole is metastable.

Length L as d increases

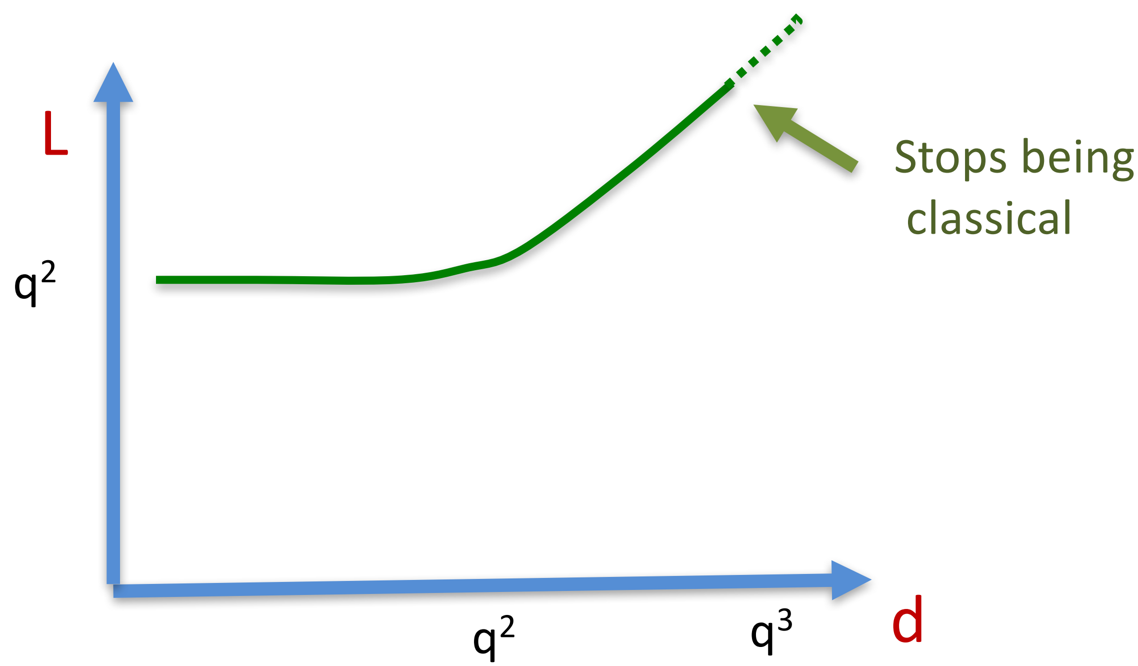




$$E_c \propto +\frac{1}{4}\frac{1}{L} - \frac{1}{L+d}$$

Conformal anomaly

Casimir cylinder



Wormholes in the Standard Model

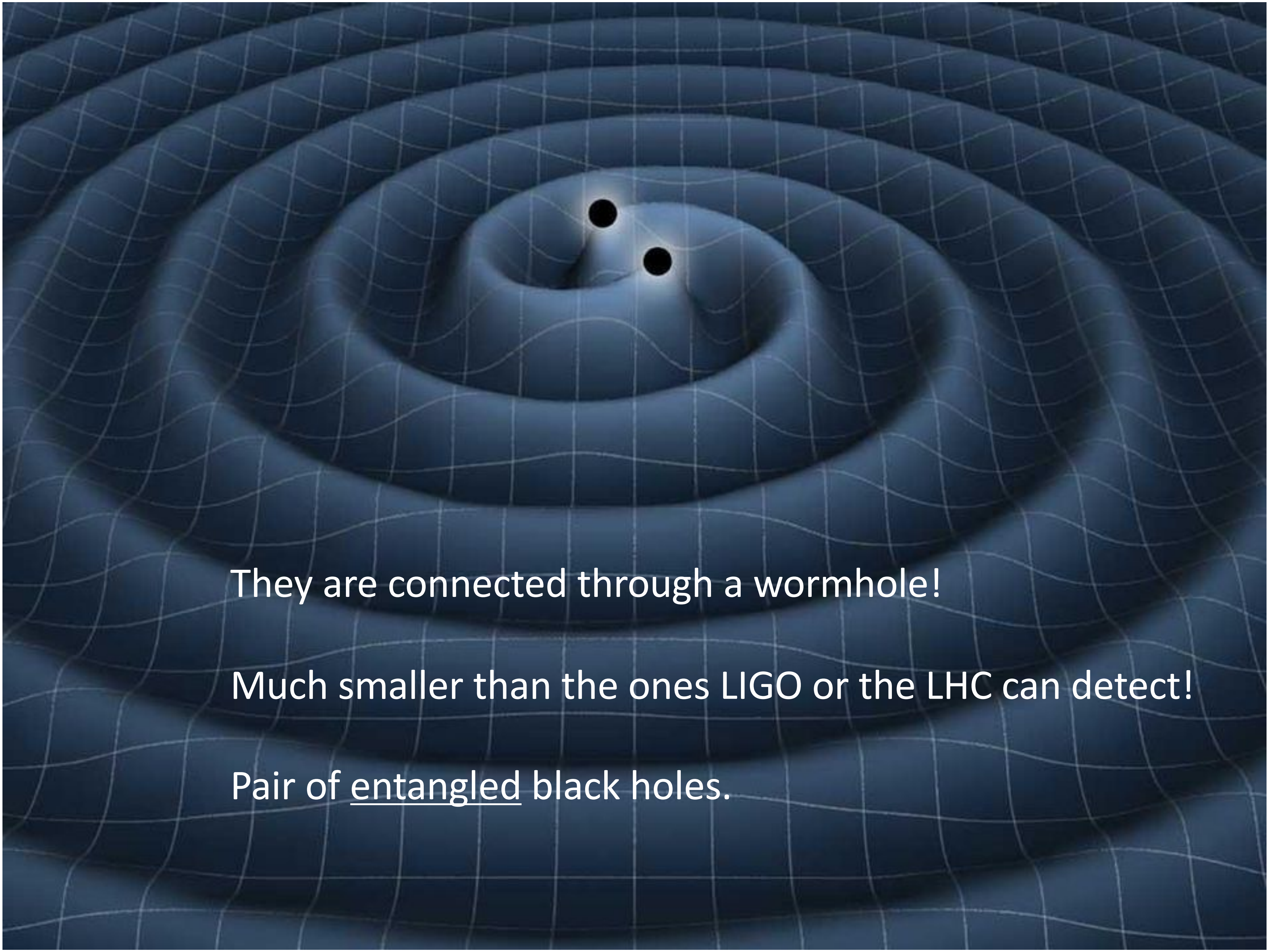
If nature is described by the Standard Model at short distances and d is smaller than the electroweak scale,

$$1 \ll q \ll 10^8$$

Distance d smaller than electroweak scale.

If the standard model is not valid → similar ingredients might be present in the true theory.

That it can exist, does not mean that it is easily produced by some natural or artificial process.



They are connected through a wormhole!

Much smaller than the ones LIGO or the LHC can detect!

Pair of entangled black holes.

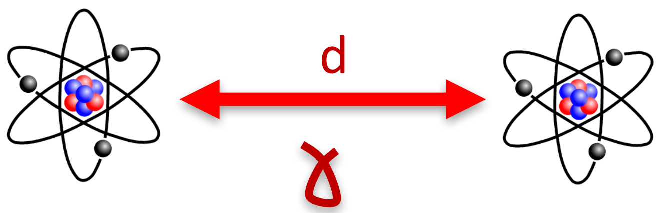
Entropy and entanglement

- Total spacetime has no entropy and no horizon.
- If we only look at one object → entanglement entropy = extremal black hole entropy
- Wormhole = two entangled black holes
- Total Hamiltonian $$H = H_L + H_R + H_{\rm int}$$

↑ Generated by fermions in exterior

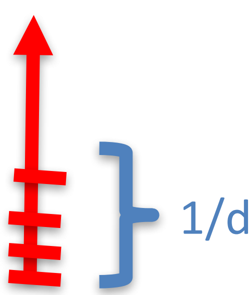
Van der Waals interaction

Two neutral atoms exchanging photons.

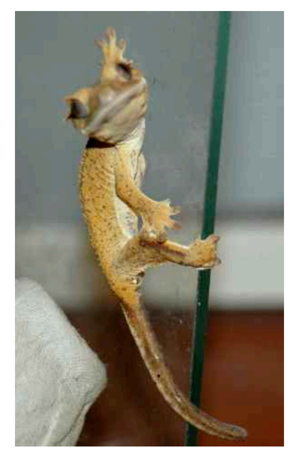


$$H_{int} \propto \frac{\vec{d}_L . \vec{d}_R}{d^3}$$

d small enough so that 1/d is larger than the gap between the ground state and the next states.

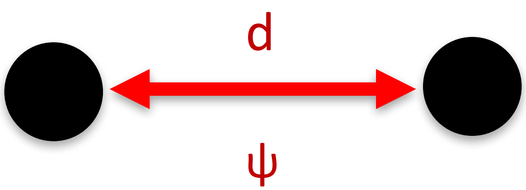


Entangle the two atoms.



Two black holes

Two black holes exchanging fermions.



$$H_{int} \propto \psi_L \psi_R$$

See Xiaoliang Qi's talk

Entangle the two black holes.

Conclusions

- We displayed a solution of an Einstein Maxwell theory with charged fermions.
- It is a traversable wormhole in four dimensions and with no exotic matter.
- It balances classical and quantum effects.
- It has a non-trivial spacetime topology, which is forbidden in the classical theory.
- It does <u>not</u> violate causality.
- It has no horizon and no entropy.
- Can be viewed as a pair of entangled black holes.

Questions

- If we start from disconnected near extremal black holes: Can they be connected quickly enough ? → topology change.
- Could we turn it into a prediction from quantum gravity ?