# Modular flow for bulk reconstruction and the QNEC

Strings 2018

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[arXiv:1706.09432](arXiv:1706.09432)

**arXiv:1806.XXXXX**

# Modular flow

- Consider a bi-partite quantum system:

$$|\psi\rangle \in \quad \mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_{\bar{A}}$$

- Observations restricted to $\mathcal{O}_A$ described by $\rho_A$

- Assuming $\rho_A$ is invertible, define Modular flow:

$$\mathcal{O}_A(s) = \rho_A^{is} \mathcal{O}_A \rho_A^{-is}$$

- Why? in some situations $-\ln \rho_A$ ~ Hamiltonian, e.g. a thermal/Gibbs state

# Obscure …

- Still seems like an obscure operation … maximally mixed state?

- We will study it for several reasons:

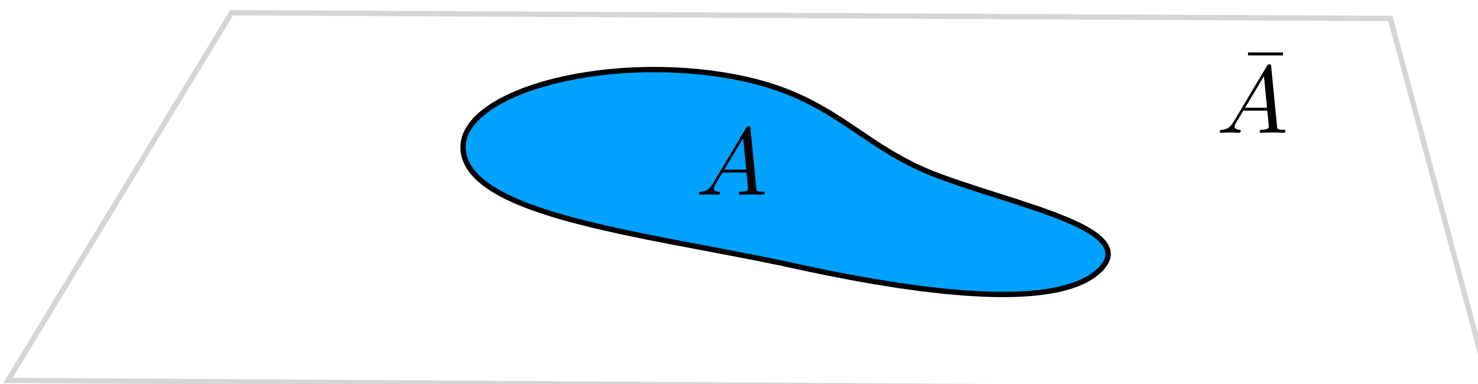
    - Universality in QFT ~ like a boost generator close to the entangling surface for any state

    - Satisfies powerful constraints - analyticity and unitarity

    - For AdS/CFT ~ tool for revealing bulk locality and causality from the boundary

# In QFT

- Consider a geometric partition:



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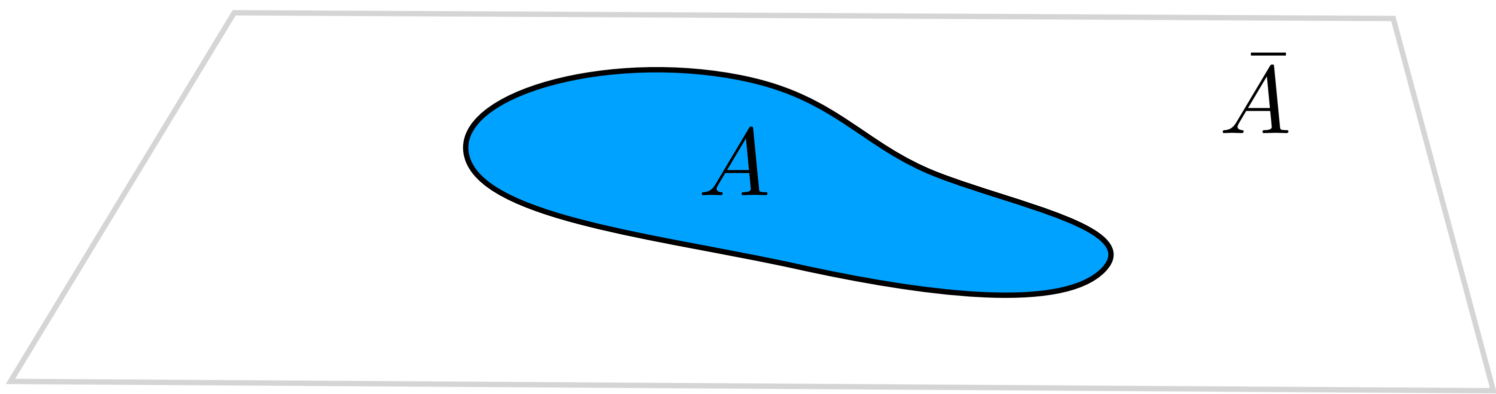


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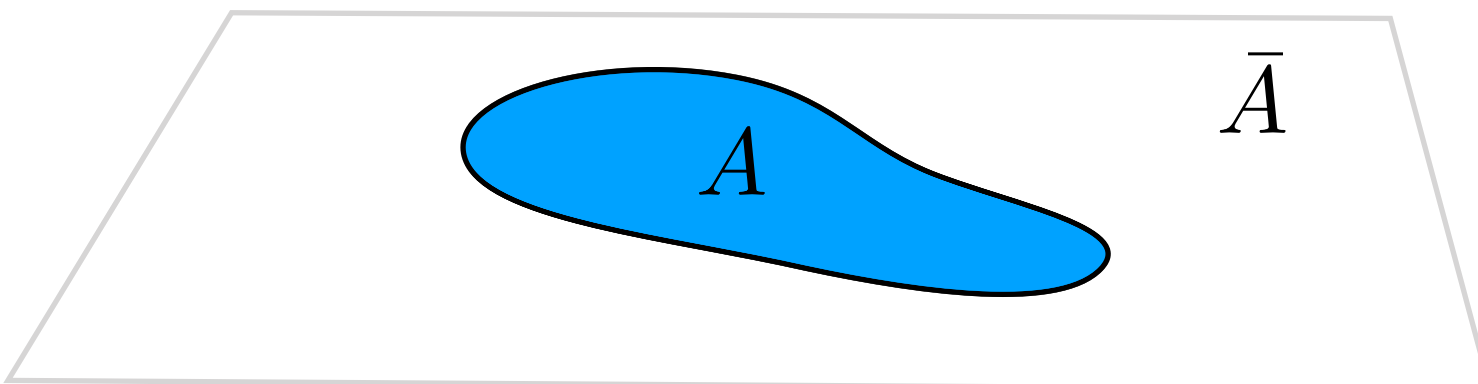
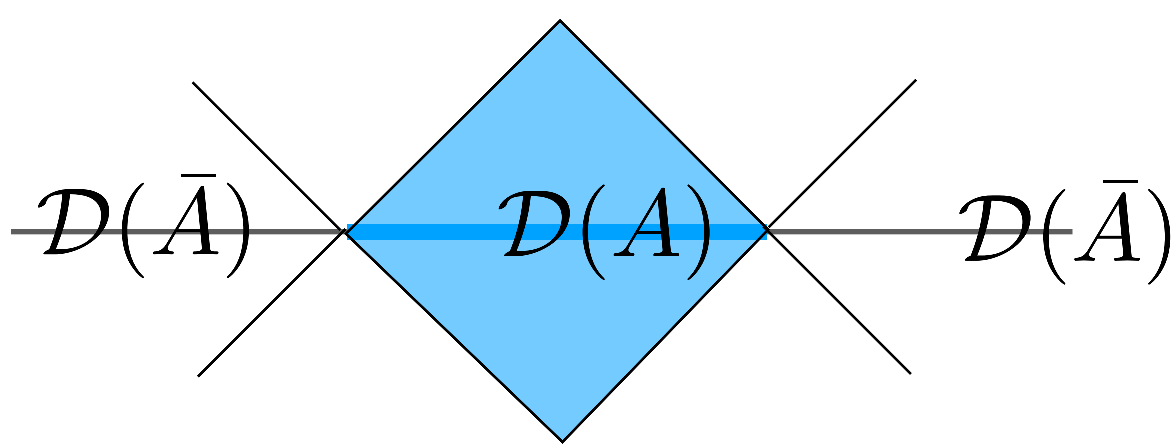



- Not really a tensor factorization $S_{EE} = \infty$
- Rather think about algebra of operators in spacetime regions:



# In QFT

- Modular flow still well defined, associated to some $|\psi\rangle$

**Modular operator:** $\Delta_A (= \rho_A \otimes \rho_{\bar{A}}^{-1})$ (Tomita-Takesaki)

$$\mathcal{O}_A(s) = \Delta_A^{is} \mathcal{O}_A \Delta_A^{-is} \quad \text{in } \mathcal{D}(A)$$

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**Modular conjugation:** $J$

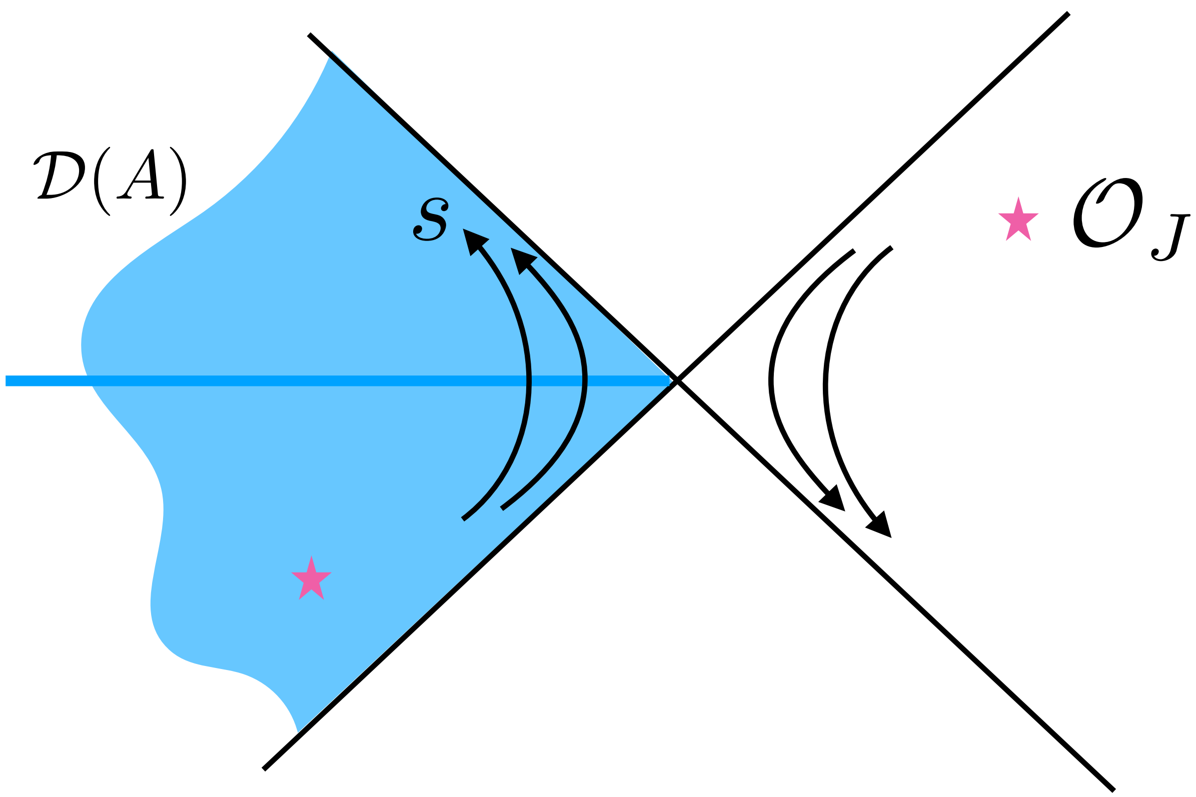
$$\mathcal{O}^J \equiv J \mathcal{O}_A J \quad \text{in } \mathcal{D}(\bar{A})$$

$$J_A |\psi\rangle = |\psi\rangle$$

# In QFT

(Bisognano-Wichmann)

- For example for a half space cut (Rindler):

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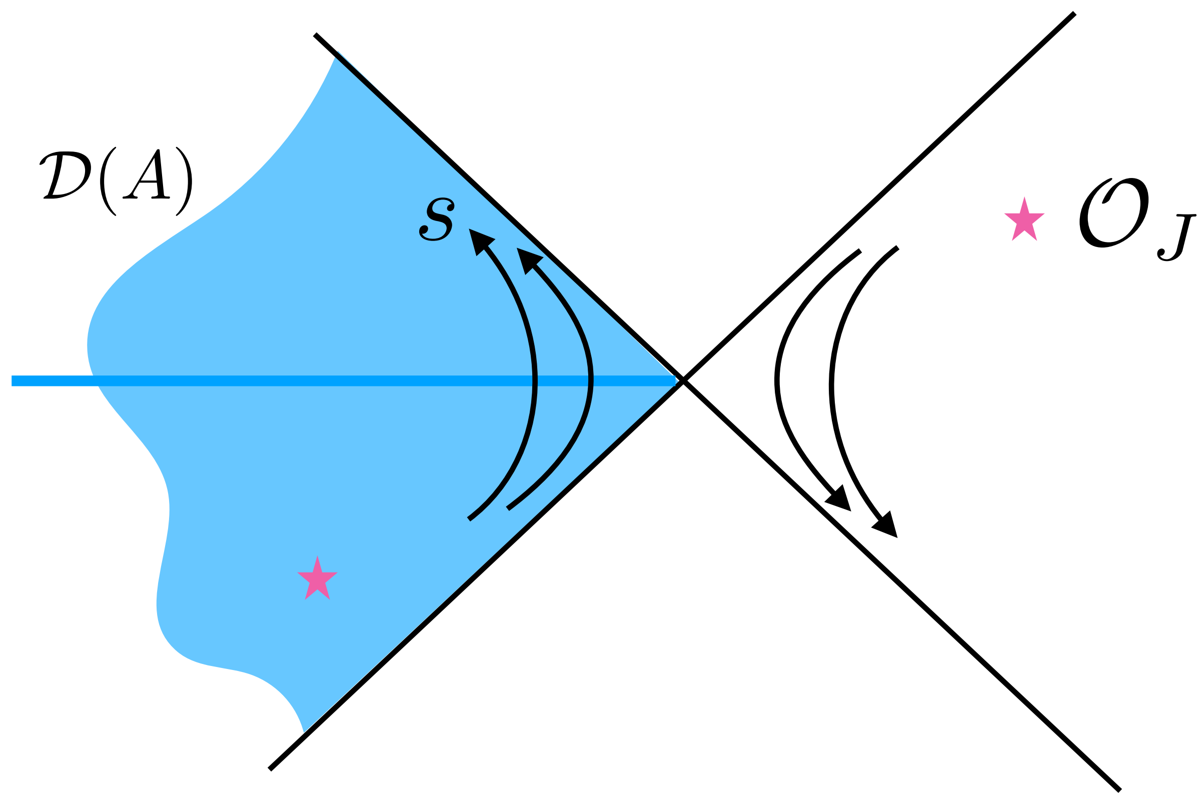


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$$J\Delta^{1/2}\mathcal{O}_A\left|\psi\right\rangle = \mathcal{O}_A^\dagger\left|\psi\right\rangle$$

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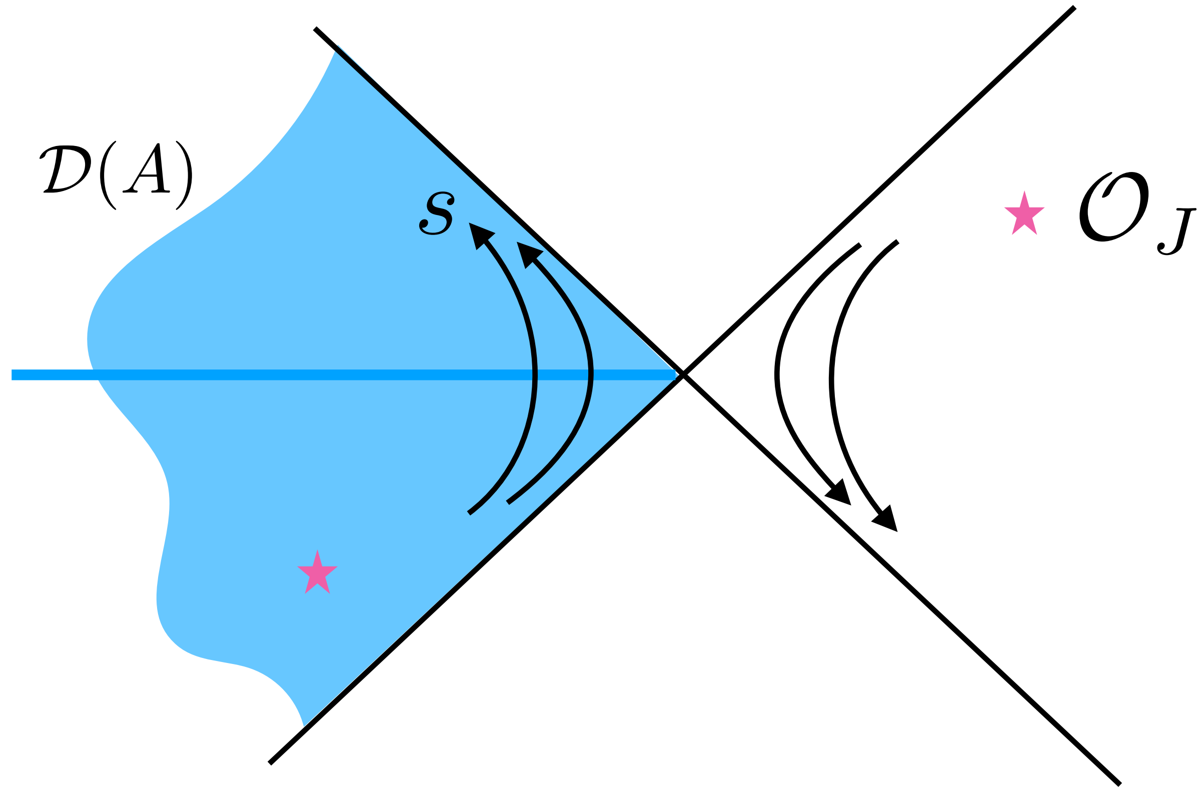


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- For more general states, UV structure of entanglement is the same near the cut. So expect modular flow has universal geometric description at least acting on operators close to the cut ….

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- Analyticity of correlation functions: $\langle\psi|\,\mathcal{O}_A\Delta^{is}\mathcal{O}'_A\,|\psi\rangle$

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$\beta = 1$

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- Unitarity. e.g bounds:

$$\langle\psi|\,\mathcal{O}_A\Delta^{1/2}\mathcal{O}_A^{\dagger}\,|\psi\rangle = \langle\psi|\,\mathcal{O}_A\mathcal{O}_A^{J}\,|\psi\rangle \geq 0$$

**etc.**

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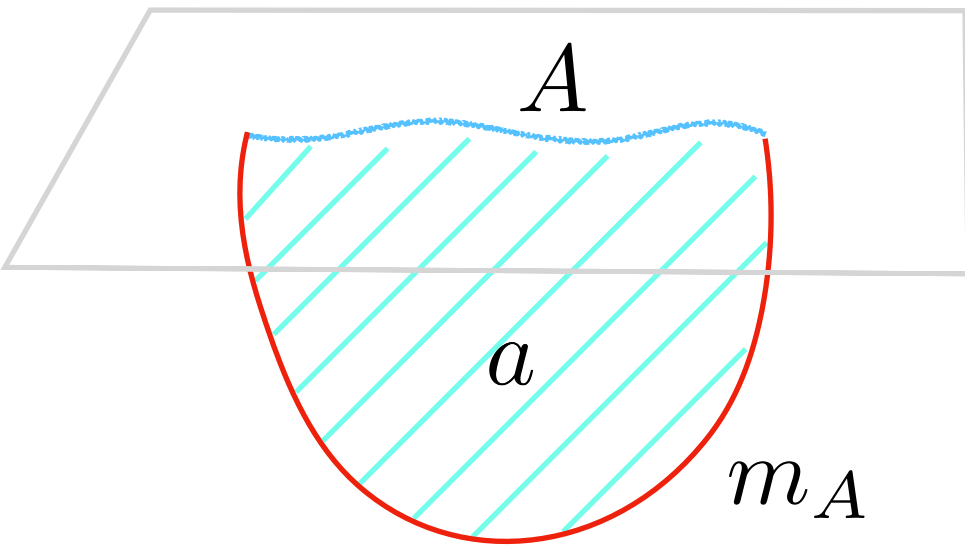
# In AdS/CFT …

- We start with the holographic entanglement entropy formula (RT/HRT/LM):

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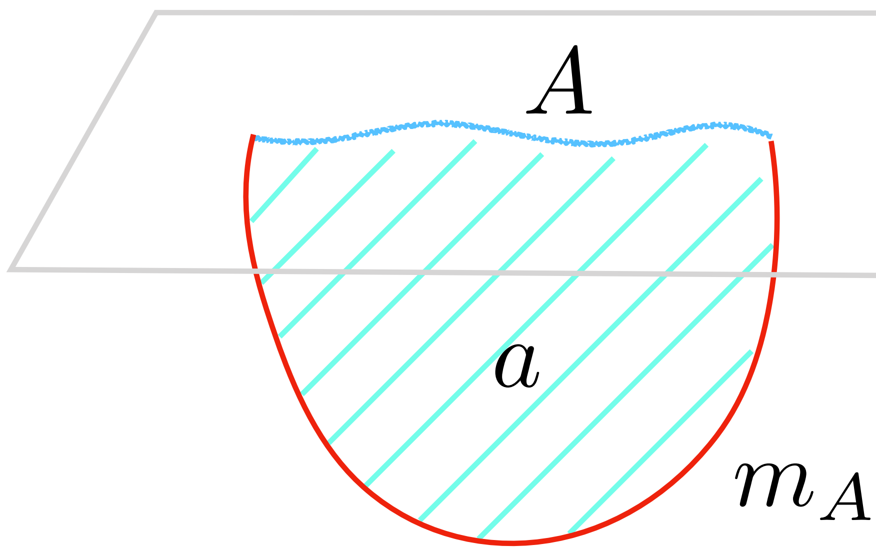


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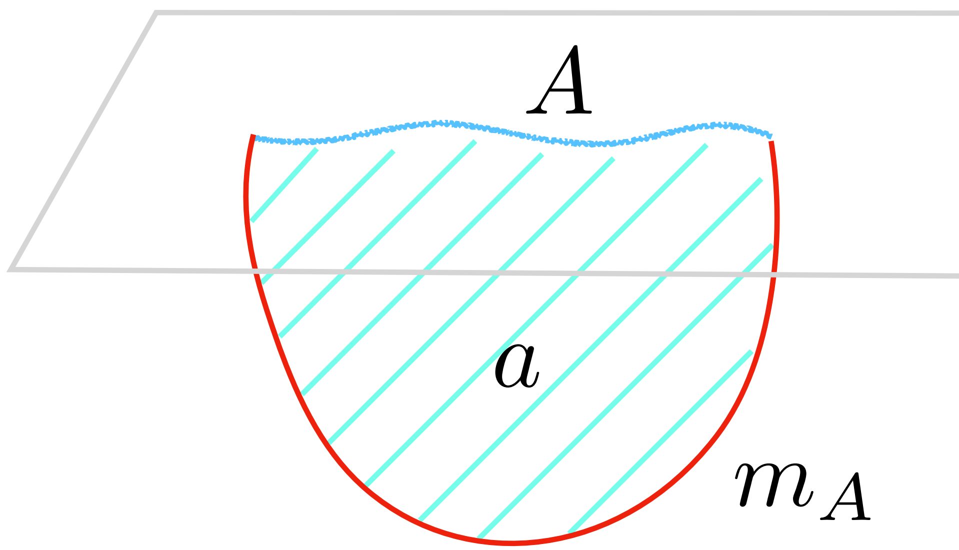


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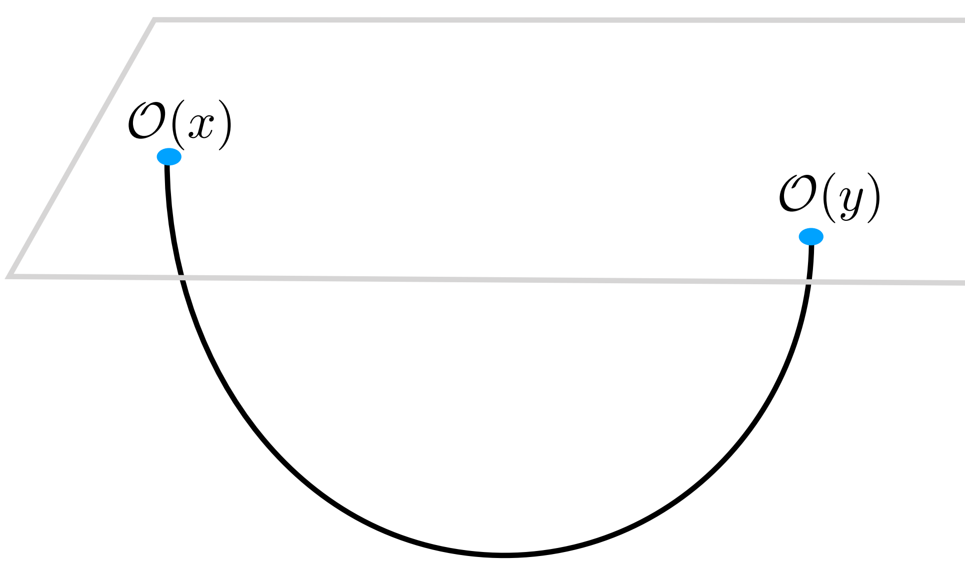
**Entanglement wedge:** $\mathcal{E}_a \equiv \mathcal{D}(a)$

# Heavy probe operators

- To get some handle on this we will study correlation functions of heavy probe operators. These are usually determined by spacelike geodesics:

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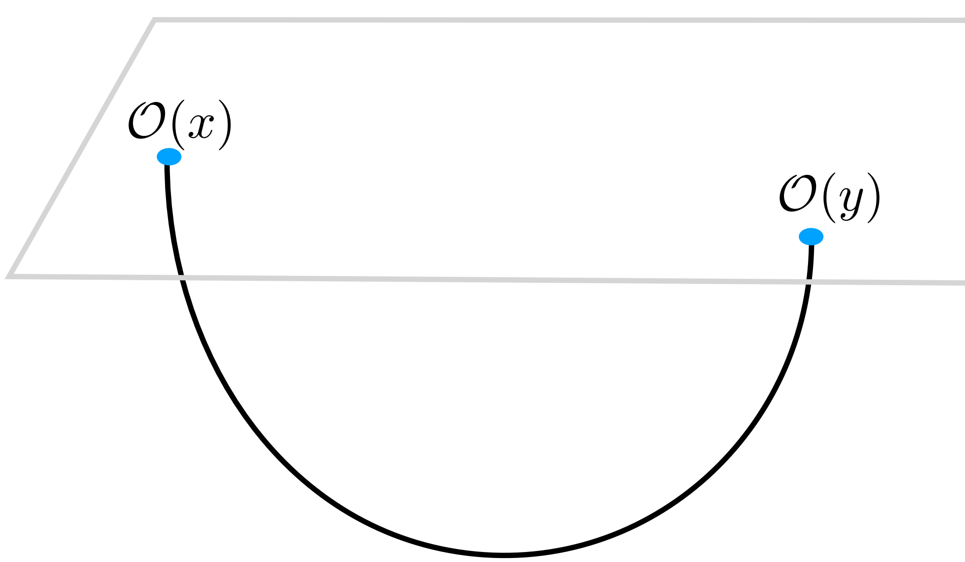


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- In the presence of modular flow expect still semiclassical answer:

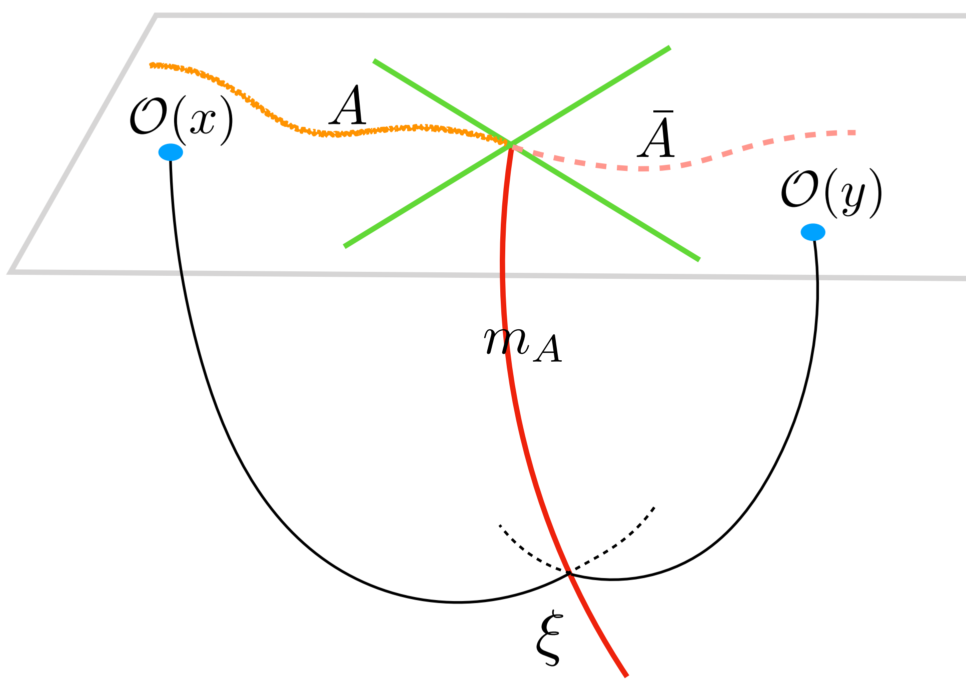
$$\Big\langle \mathcal{O}(x)\Delta^{is}\mathcal{O}(y)\Big\rangle \sim e^{-m\ell_s(x,y)}??$$

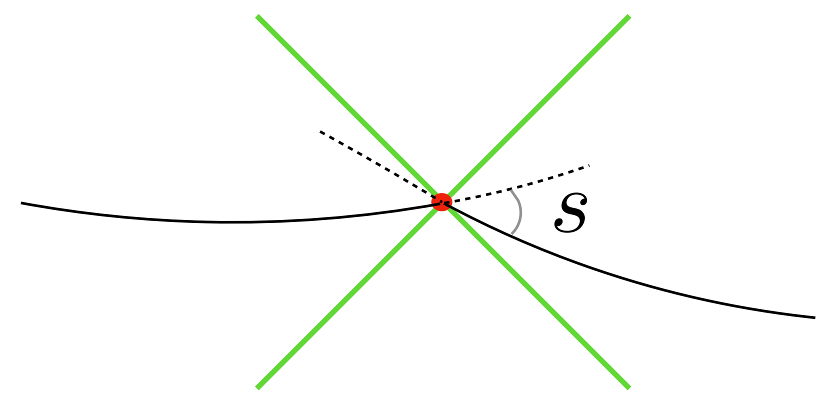
# Rules for geodesics

(TF, Li, Wang ) See also: (Chen, Dong, Lewkowycz,Qi )

- We give some rules for when such correlators can be computed:

$$\langle \mathcal{O}(x)\Delta^{is}\mathcal{O}(y)\rangle$$



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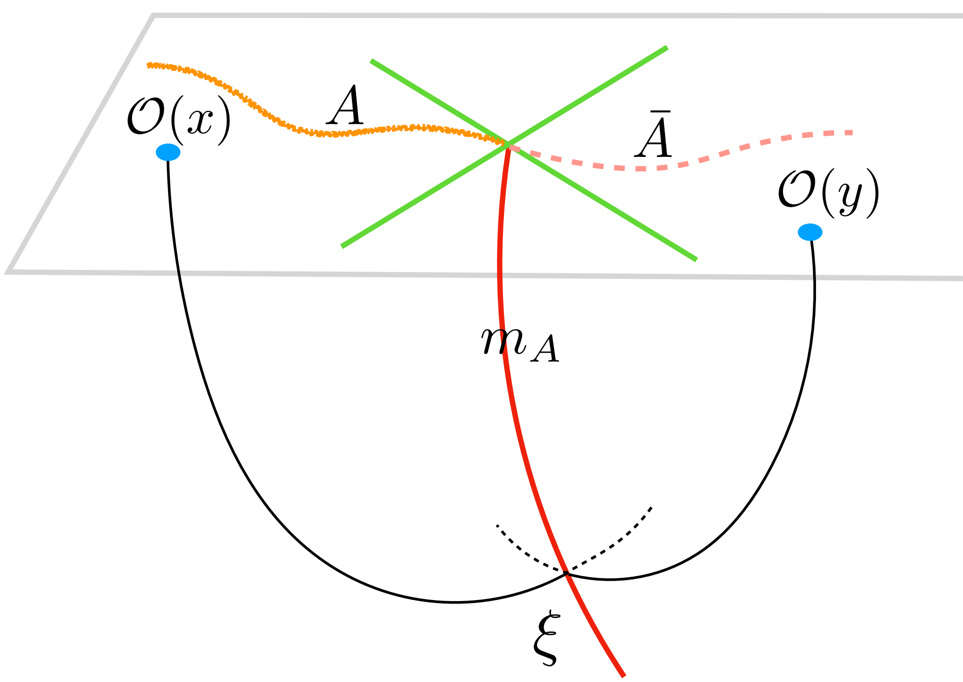


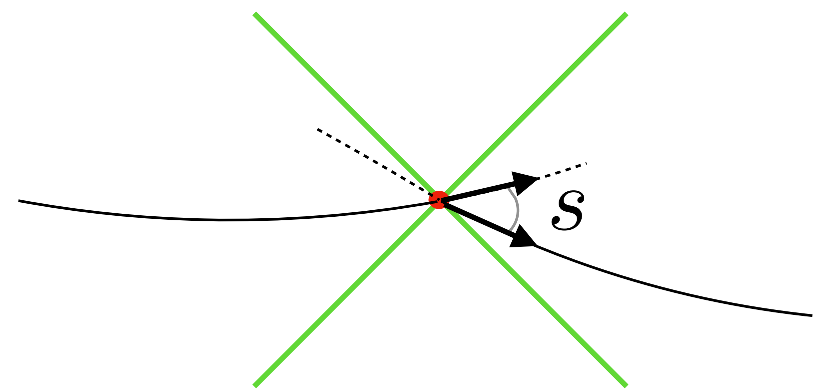


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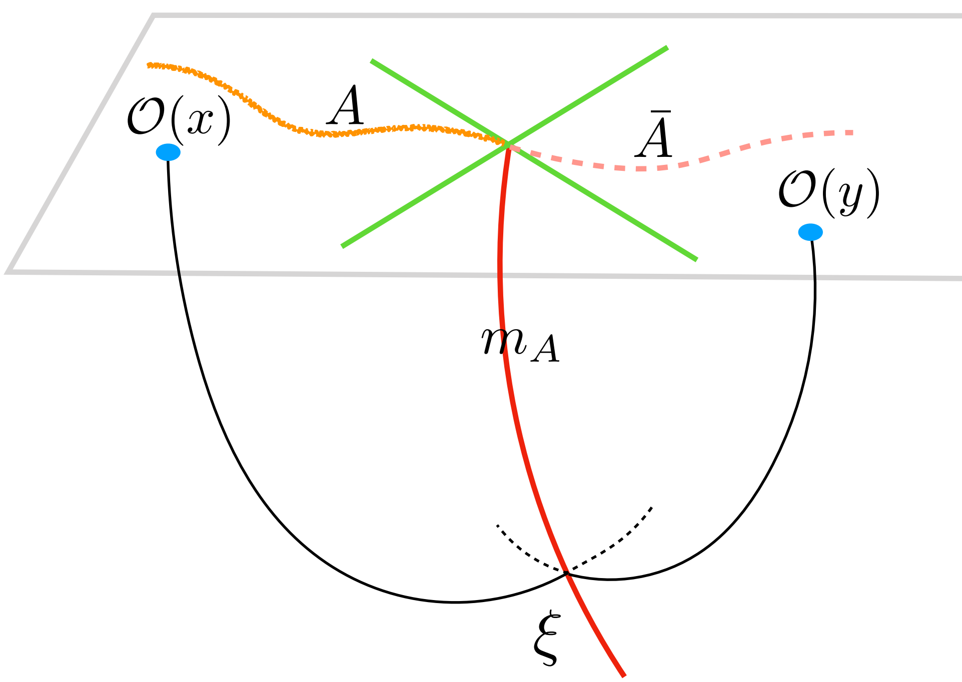


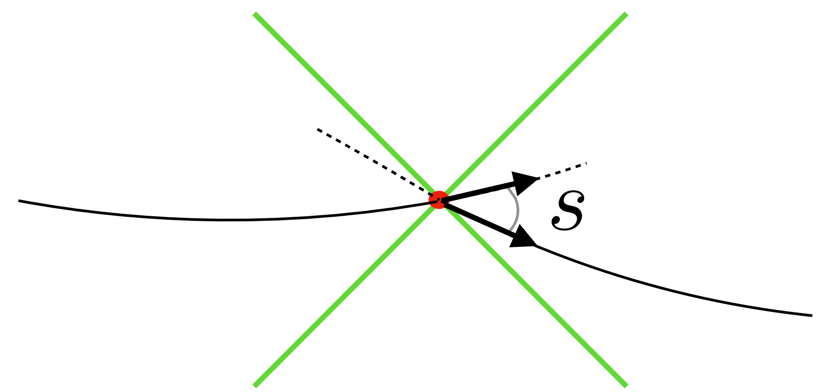


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$$= \exp(-m\ell(x,\xi) - m\ell(y,\xi))$$

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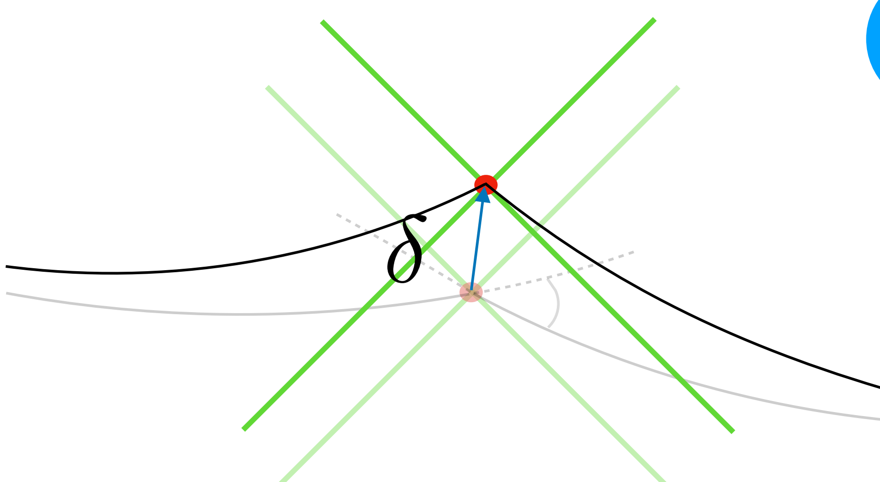
- We will also need for linear deformations thereof:




$$\exp(-m(\ell(x, \xi) + \ell(y, \xi) + \delta^2))$$

# Some intuition:

- Following Jafferis, Suh (2014): $|\psi_s\rangle = \rho_A^{is} |\psi\rangle = \rho_{\bar{A}}^{is} |\psi\rangle$

**Using ``boost invariance”:** $\Delta_A |\psi\rangle = \rho_A \otimes \rho_{\bar{A}}^{-1} |\psi\rangle = |\psi\rangle$

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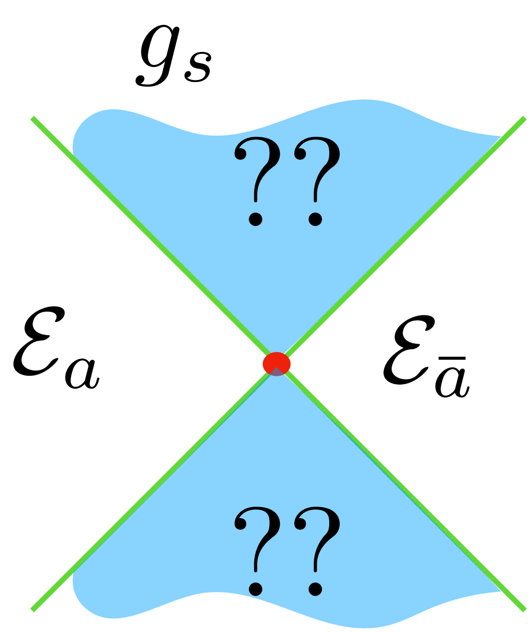
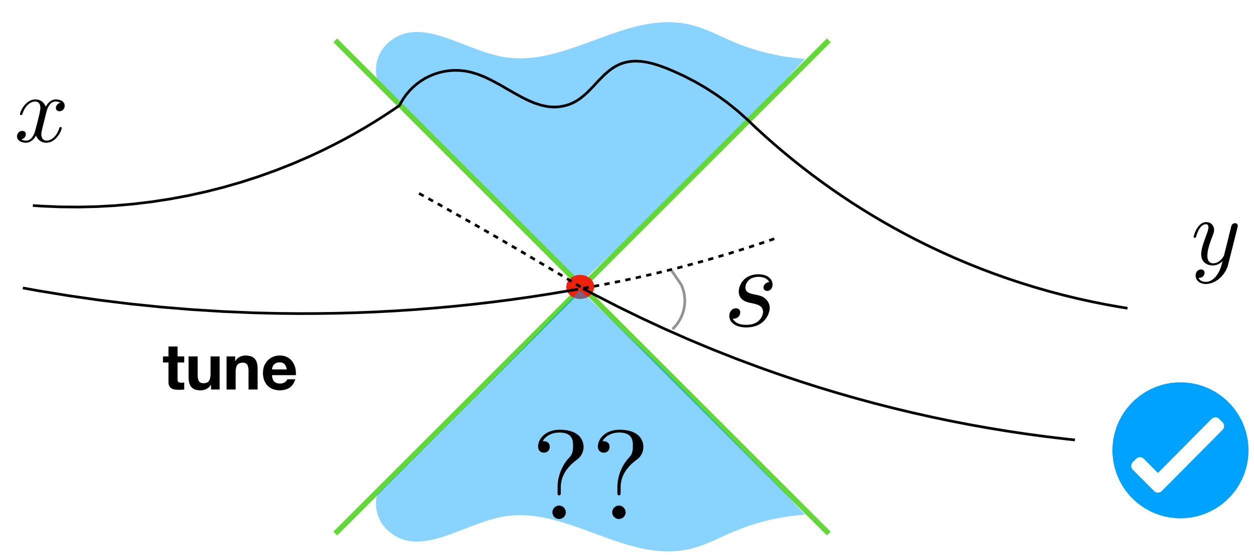


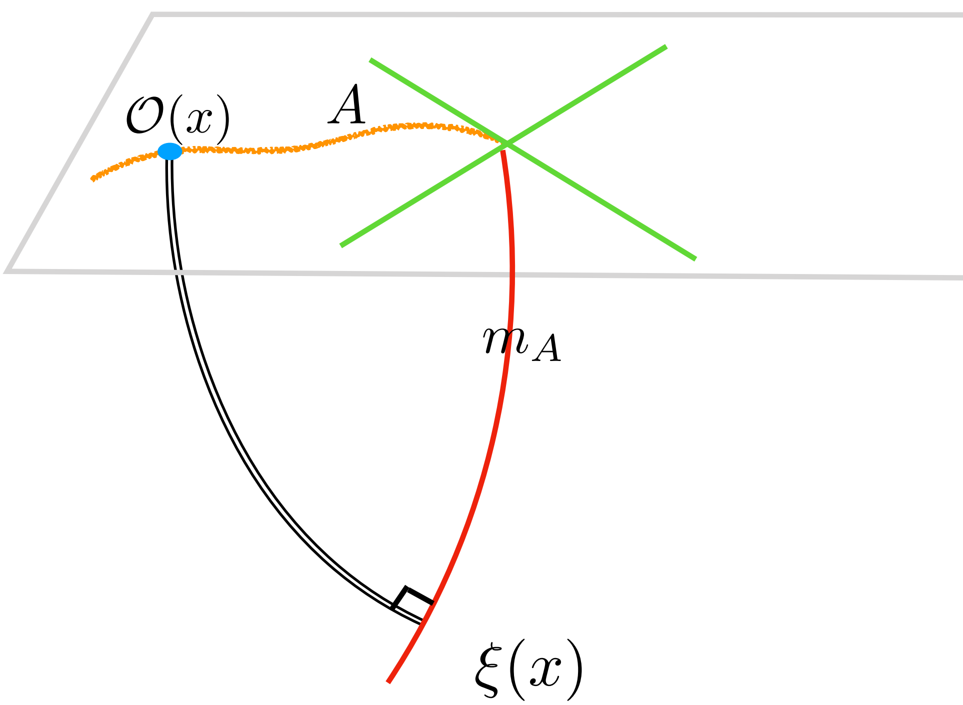




Also: replica trick argument

# Mirrors for mirror ops

- Complex boosts: Euclidean rotations - $\pi$ rotation: mirror!

$$\left\langle \mathcal{O}(x)\Delta^{1/2}\mathcal{O}(y)\right\rangle = \langle \mathcal{O}(x)\mathcal{O}_J(y)\rangle$$

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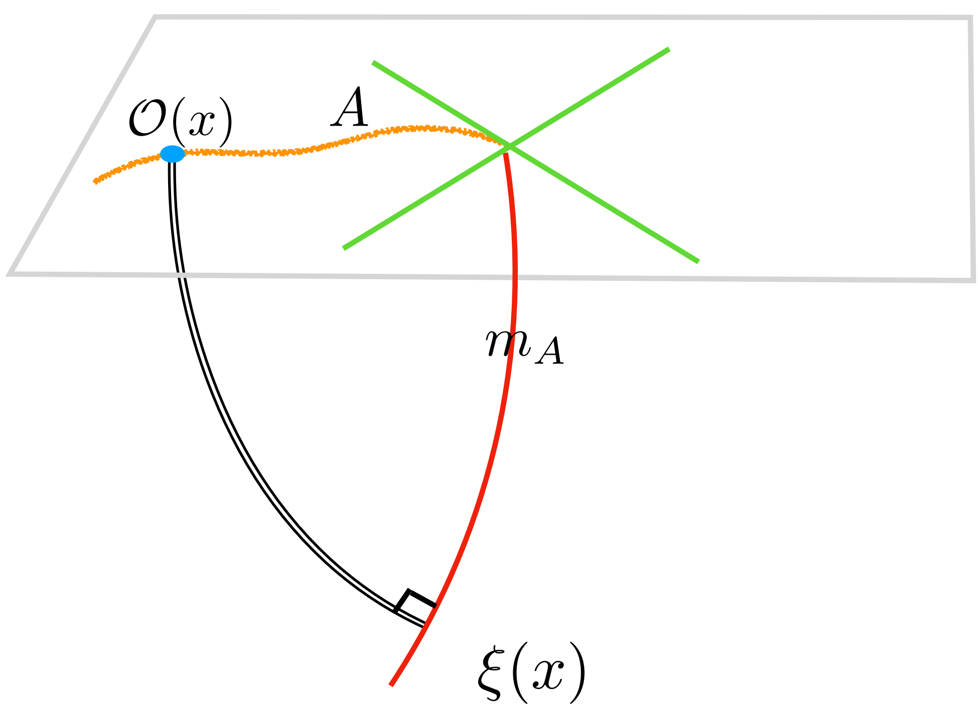


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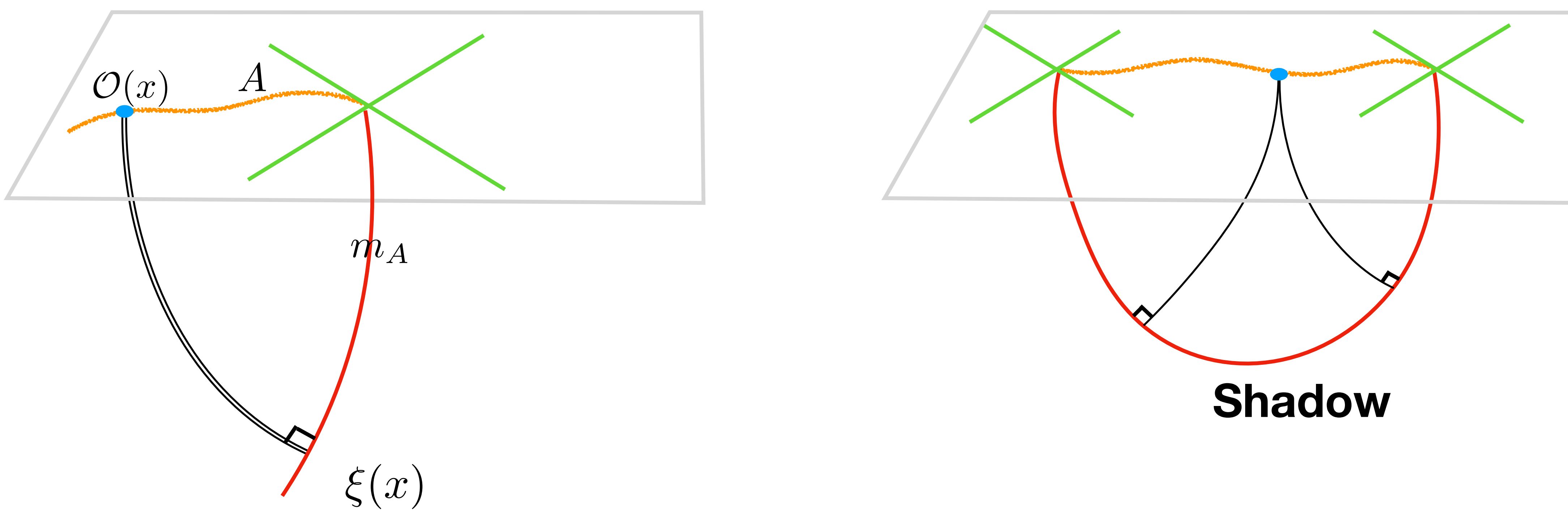


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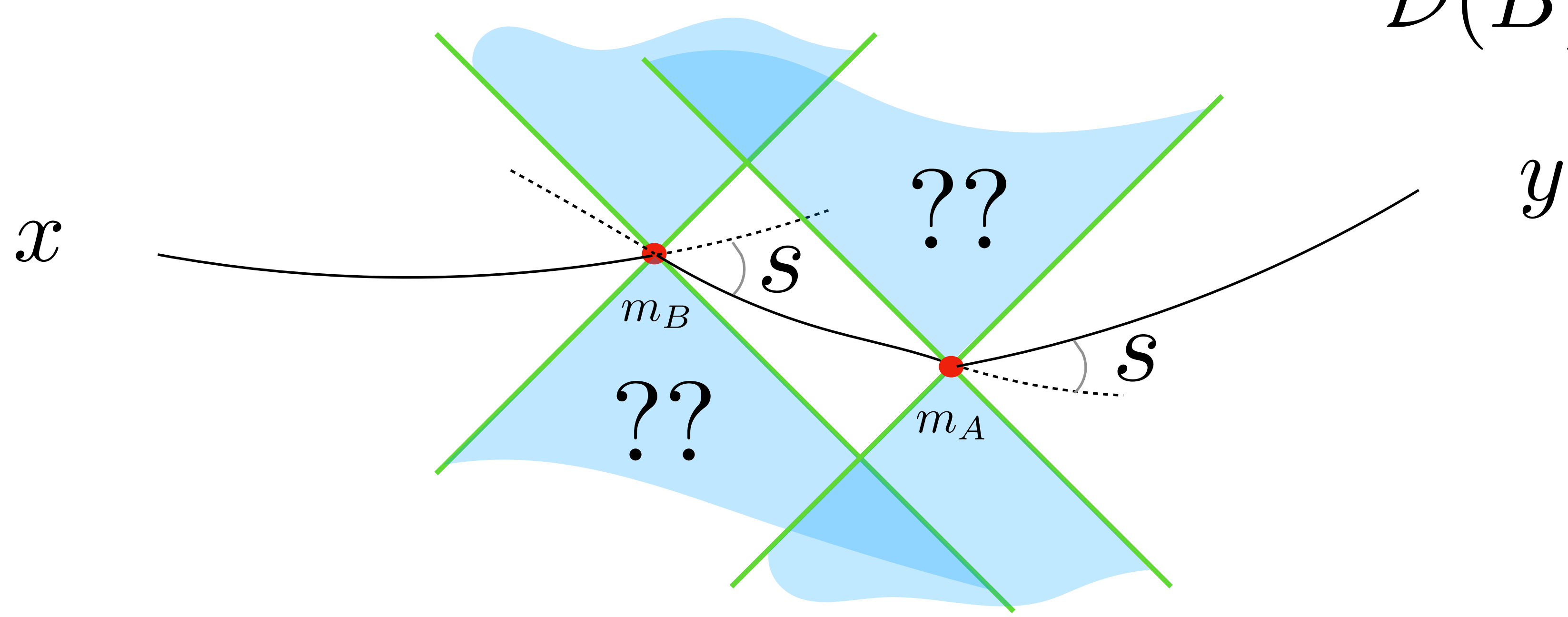
# Nested flows

- We can use these rules for more complicated correlators:

$$\mathcal{O}(s_{AB}) \equiv \rho_A^{is}\rho_B^{-is}\mathcal{O}\rho_B^{is}\rho_A^{-is}$$

$$\mathcal{D}(B) \subset \mathcal{D}(A)$$




- As long as we can thread the geodesic through - satisfying the boost conditions at each RT surface

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- We would like to combine these two ideas (mirrors and double flow) to compute:

mirror

$$\left\langle \mathcal{O}(x)\overbrace{\Delta_B^{is}\Delta_A^{-is+1/2}}^{\text{double flow}}\mathcal{O}(x)\right\rangle$$

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- Some intuition: double flow in the Rindler case gives action of two boosts = translation. More generally acting on the geodesic correlator we will be able to extract properties of this translation deep in the bulk at:

$$\xi(x) \in m_A$$

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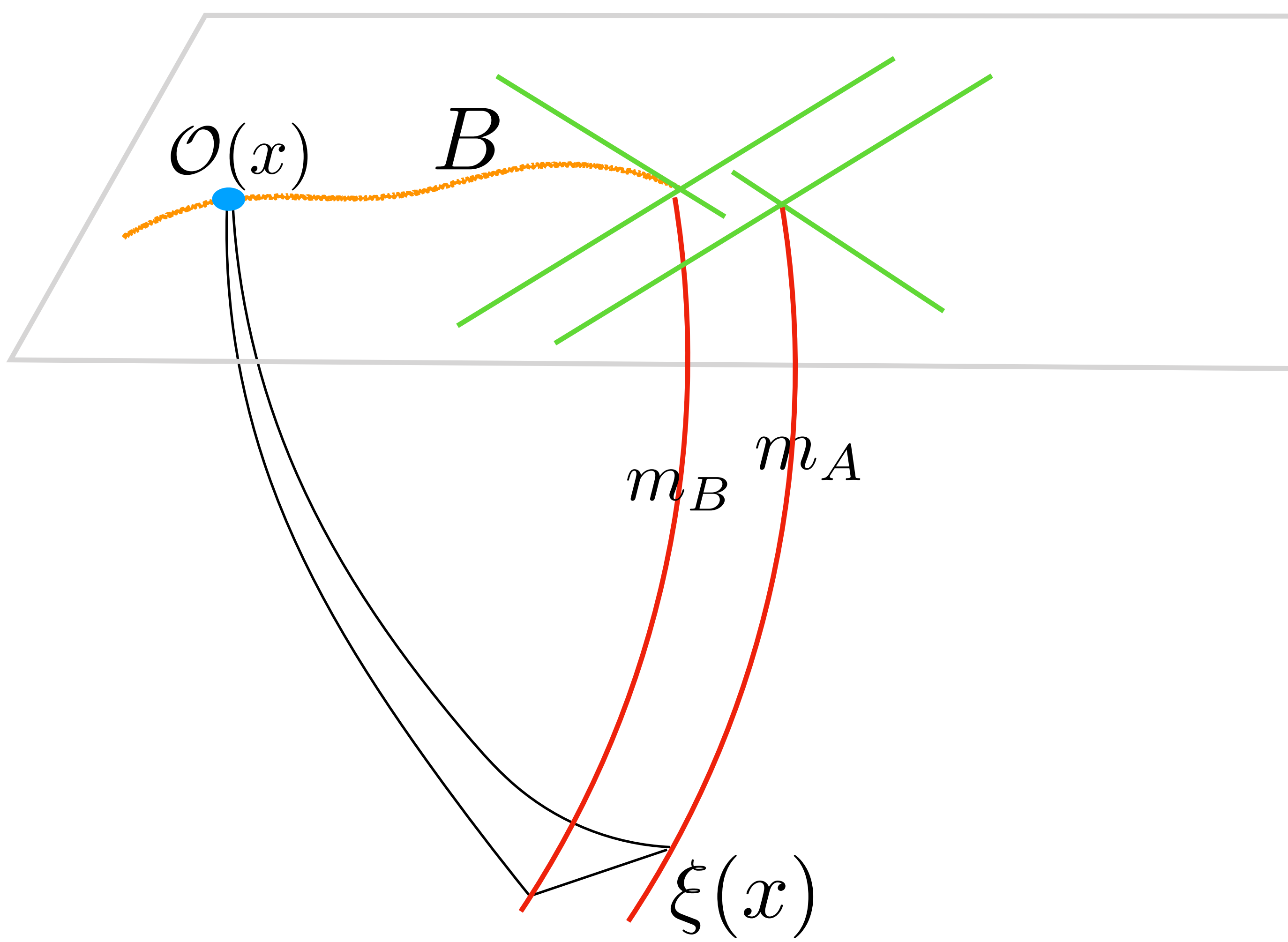
- Still hard to satisfy the boost condition while varying s



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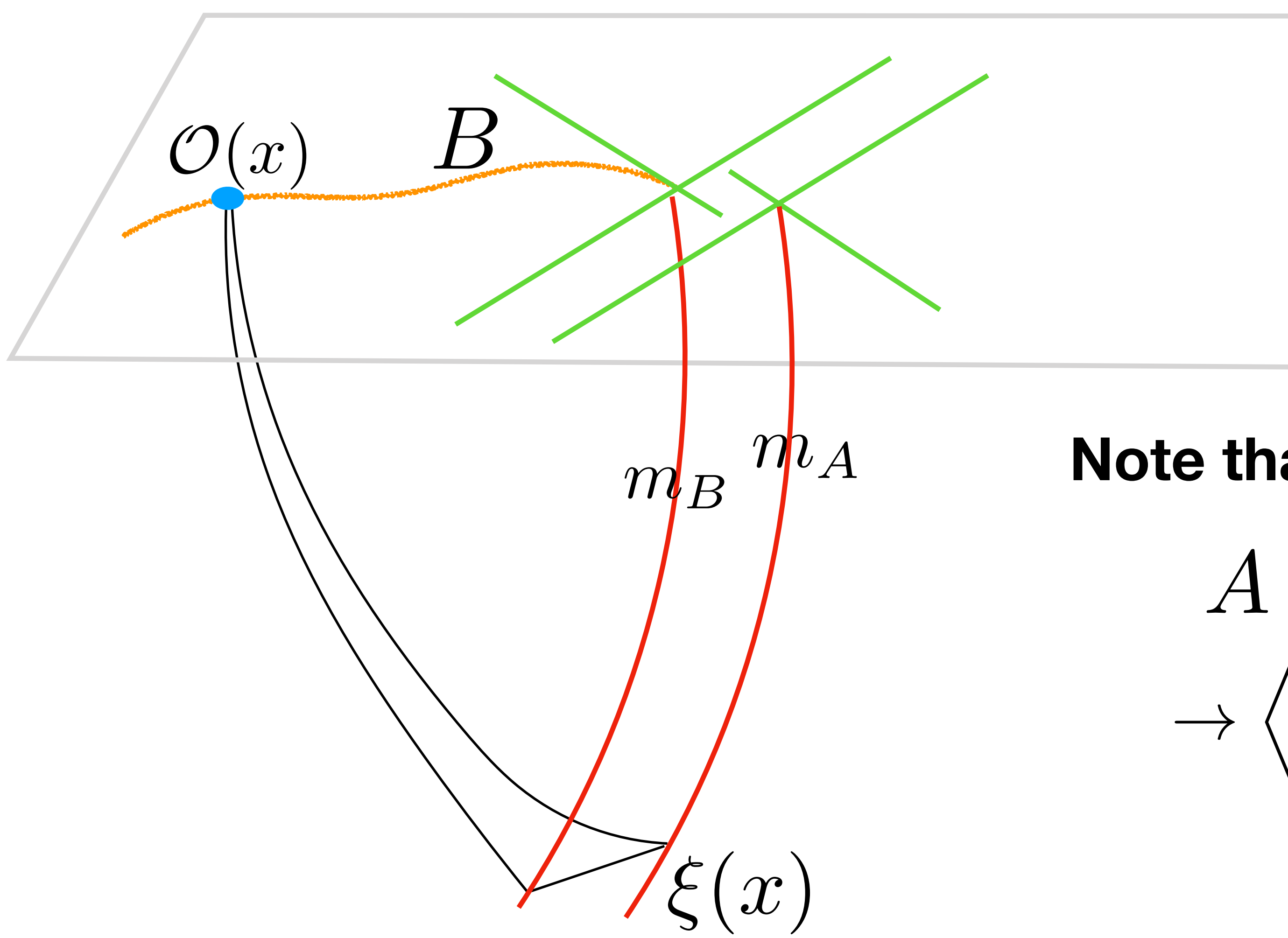


**Note that when:**

$$A = B$$

$$\to \left\langle \mathcal{O}(x)\Delta_A^{1/2}\mathcal{O}(x)\right\rangle$$

- For small deformations $A \to B$, reflected geodesics come close enough to use the rules to linear order in the deformation

# Nested boosts

- Consider: $$iM + 1 \equiv \frac{\left\langle \mathcal{O} \Delta_B^{is} \Delta_A^{-is+1/2} \mathcal{O} \right\rangle}{\sqrt{\left\langle \mathcal{O} \Delta_A^{1/2} \mathcal{O} \right\rangle \left\langle \mathcal{O} \Delta_B^{1/2} \mathcal{O} \right\rangle}}$$

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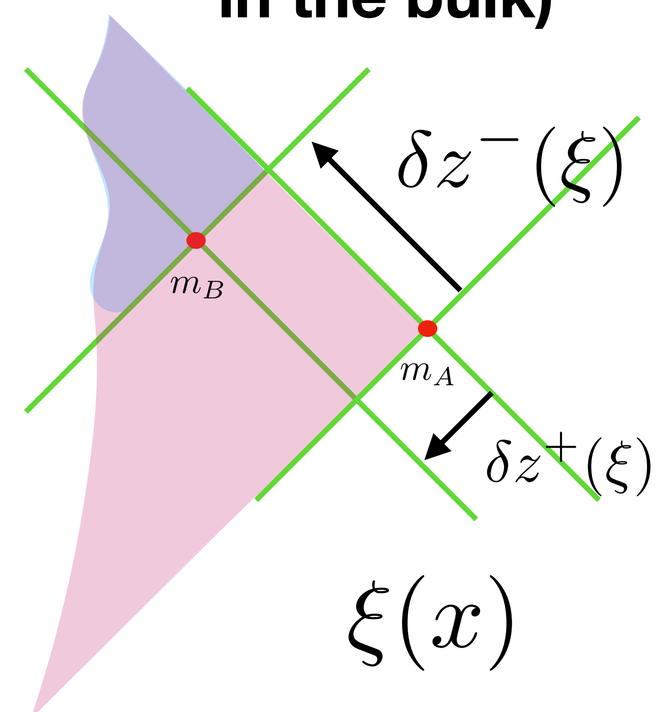
- Which we calculated using the rules:

$$\mathcal{M} = ie^{2\pi(s-s_\star)}\delta z^+(\xi) + ie^{-2\pi(s-s_\star)}\delta z^-(\xi)$$

**(somewhere deep in the bulk)**



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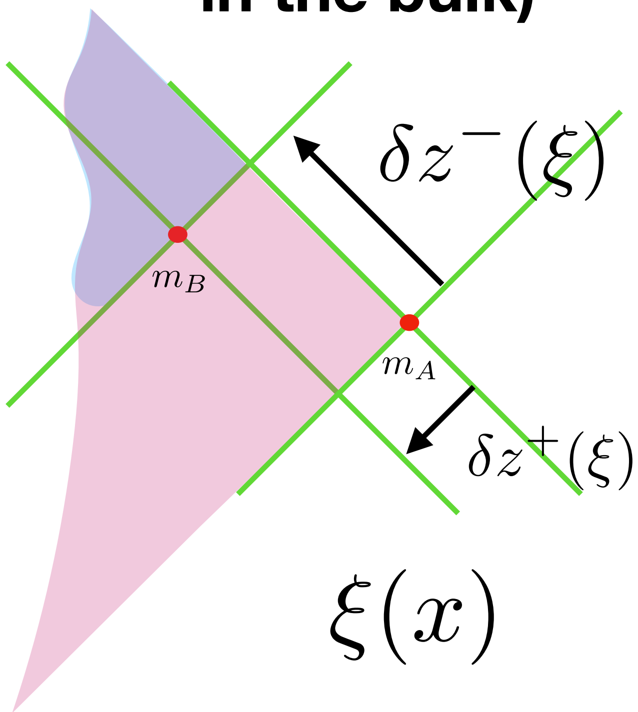


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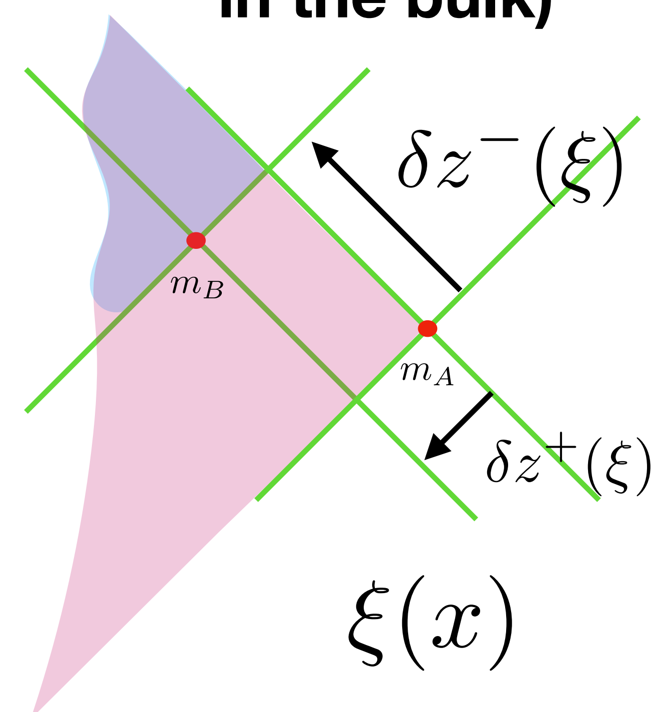


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**saturates chaos bound (T=1)**

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# EWN

- This is the entanglement wedge nesting property

$$\mathcal{D}(B) \subset \mathcal{D}(A)$$

$$\longrightarrow \quad \mathcal{E}_b \subset \mathcal{E}_a$$

**(spacelike movement of the RT surface)**

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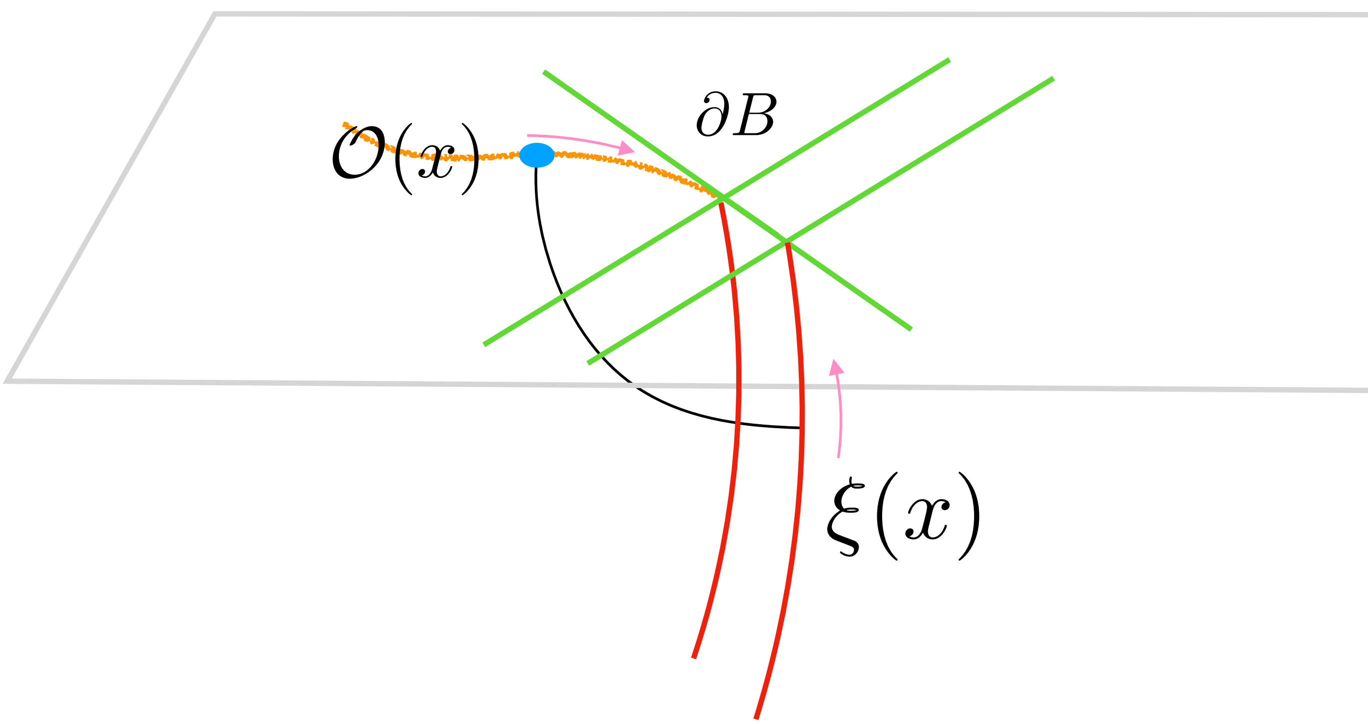
- Imposing this condition near the boundary of AdS gives the QNEC of the boundary CFT

**(Leichenhaur, Koeller)**

# QNEC near boundary




(Bousso, Fisher, Koeller, Leichenauer, Wall)

$$\langle T_{++} \rangle \geq S''_{EE}$$

**Metric fall off near bdry of AdS**

**Shape of RT surface near bdry**

- This connects to a general proof of the QNEC, which used such a modular flow correlator, but calculated it using other CFT methods

(Balakrishnan, TF, Khandker, Wang)

- Such methods fail unless $\mathcal{O}(x)$ is close to $\partial B$

- To get away from this, we needed to use holographic CFTs

# Other things …

- We have only started to use these new tools …

- Can get more information about the bulk:

$$h_{ij} \quad \textbf{can we get:} \quad K^{\alpha}_{ij}, T^{\text{bulk}}_{++}, \ldots ??$$

- What about the bulk NEC? Quantum Focusing Condition? Einstein's equations?

- Stringy corrections? Non saturation of the chaos bound?

ありがとう