SCFTs in 6d and IR symmetry enhancement in 4d

Shlomo S. Razamat

Technion

Kim, SSR, Vafa, Zafrir – 1709.02496, 1802.00620, and 1806.06720
SSR, Sela, Zafrir – 1711.02789

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Lagrangians/non Lagrangian models

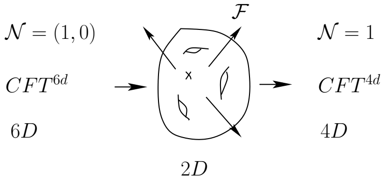
- This talk is about $4d$ QFTS with $\mathcal{N} = 1$ supersymmetry

- We believe that certain models exist for which one cannot write a Lagrangian with all the (super)symmetries manifest

- Many $\mathcal{N} = 2$ examples; Argyres-Douglas models, Minahan-Nemeschansky models, T_N models

- Insisting on manifestly $\mathcal{N} = 2$ supersymmetric Lagrangians no description can be found which reproduces some of the properties of these models

Compactifications from six dimensions – no Lagrangians



- Evidence for existence of non-Lagrangian models studying compactifications of $6d$ conformal field theories to $4d$
- Choices involved – $6d$ model, surface, flux for G^{6d}
- General choices lead to models with no known Lagrangian
- There is a tention between having a Lagrangian description and having all the symmetries of the models manifest

How should we think about such models?

Two options:

Give up Lagrangians:

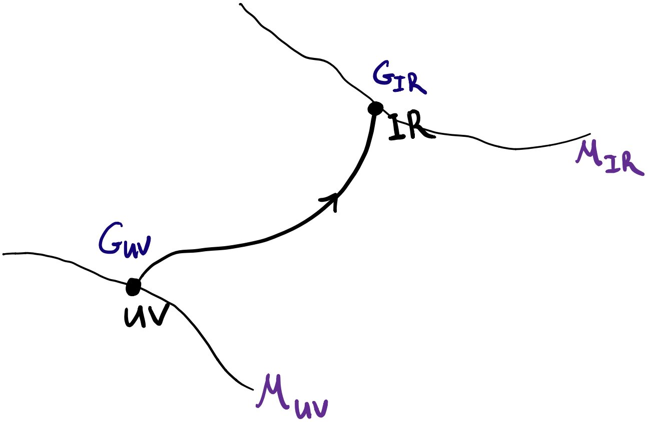
One can attempt to develop new tools to avoid the need of defining Lagrangians (see Bootstrap)

Alternatively:

...

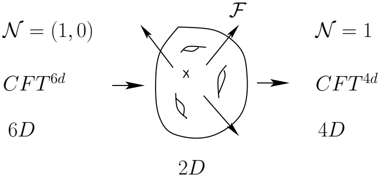
Give up symmetry!!

UV symmetry and IR symmetry



- Global (super)symmetries can be enhanced in IR, G_{IR} might be not equal to G_{UV}

Compactifications from six dimensions – Lagrangians



- What will be an organizing principle?
- A strong hint comes by studying same type of compactifications
- Many compactifications do have Lagrangian descriptions
- In many cases the UV symmetry is enhanced in the IR
- We will discuss how it comes about in the case of a torus with flux

Flux and symmetry

- The $6d$ $(1,0)$ theories have some symmetry G^{6d}
- Upon compactification one can turn on flux for abelian subgroups of this symmetry preserving $\mathcal{N}=1$ supersymmetry in $4d$
- Flux is specified by $r = Rank\, G^{6d}$ integers

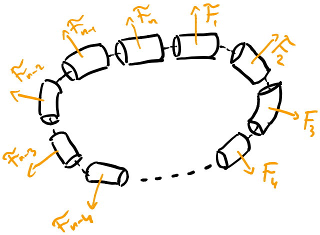
$$\mathcal{F} = (F_1\,,\, F_2 \cdots\,,F_{r-1}\,,F_r)$$

- The $4d$ symmetry G^{4d} is the subgroup of G^{6d} which commutes with the flux
- For example if G^{6d} is $SU(r+1)$,

$$\mathcal{F} = (r-1, -1, -1, \cdots, -1)$$

preserves $SU(r) \times U(1)$

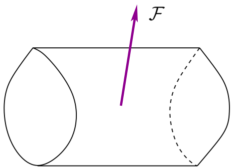
Torus built from tubes



We can think of the theories obtained by compactification on tori as combined from simple building blocks

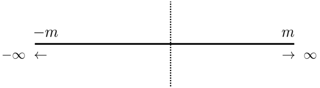
- The blocks correspond to tubes with some value of flux $\mathcal{F}_j$
- Each tube is a simple$4d$ theory, the IR symmetry visible in UV
- However, combining tubes the flux $\sum_{j=1}^{n} \mathcal{F}_j$ might be indicating enhanced symmetry, bigger than the symmetry of the blocks
- Example, $U(1)^2 \to U(1) \times SU(2)$,
  $(4, -1, -3) + (4, -3, -1) \to (8, -4, -4)$

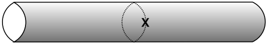
What are the tube models?



- To try and understand what are the tube models one can compactify the $6d$ models first on a circle to $5d$
- Many $6d$ theories have effective description as gauge theory in $5d$ which then can be used to understand the $4d$ models
- Example: ADE conformal matter (N M5 branes on ADE singularity), $5d$ description as quivers in the shape of affine ADE Dynkin diagram

Five dimensions, domain walls, and flux





- Upon compactification to $5d$ we have a choice of holonomy which translates to a choice of mass parameters
- Different holonomies might lead to different effective theories in $5d$ (Hayashi, Kim, Lee, Taki, Yagi 15)
- The $5d$ manifestation of the flux is in terms of domain walls interpolating between different values of the mass parameters, or different five dimensional descriptions
- The four dimensional theories can be constructed by understanding the theories on the domain walls (see N. Paquette's talk)
- Punctures: make the cylinder finite, choose bc for $5d$ fields

The basic reduction toolbox

- How do we know theories in four dimensions correspond to some compactification?
- Anomaly,

$$\int_{\Sigma_{g,0}} I_8^{T^{6d}}(\mathcal{F}) = I_6^{T^{4d}(T^{6d},\Sigma_{g,0},\mathcal{F})}$$

(Benini, Tachikawa, Wecht 9)

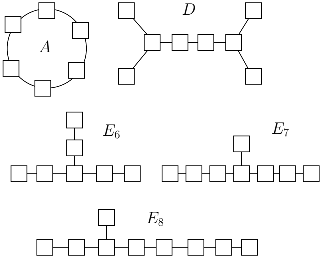
- Indices,

$$1 + \sum_{relevants} n_i(qp)^{R_i/2} + (Marg - Currents)qp + \dots$$

(Beem, Gadde 12)

- If at order qp you see adjoint of some group it has to be the symmetry of the theory (unless there is an accidental $U(1)$)
- Consistency checks

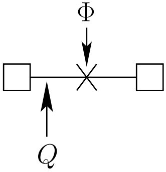
Examples: ADE conformal matter



- We will give examples of $G = ADE$ conformal matter

- Five dimensional description is in terms of G affine quiver

Flip is the trick

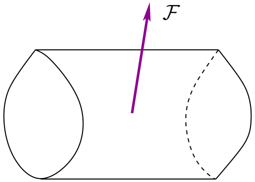


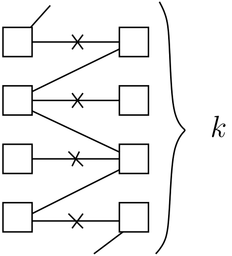
$$W = \Phi\, \epsilon^{i_1 \cdots i_N} \epsilon_{j_1 \cdots j_N} Q^{j_1}_{i_1} \cdots Q^{j_N}_{i_N}$$

- The theories are usual quiver theories
- In addition to gauge charged matter one also has gauge singlets
- The gauge singlet fields flip some of the gauge invariant baryons
- The flips are very important, without these the anomalies do not match with $6d$ and in some cases the symmetries do not enhance

- Generally these are just free fields

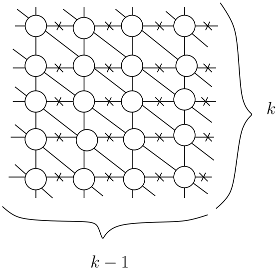
Example of A





- Typical building block is a WZ model
- The UV symmetry is two copies of $SU(N)^k$ associated to the boundary and $U(1)^{2k-1}$ which is the Cartan subgroup of $G^{6d} = SU(k) \times SU(k) \times U(1)$
- The pattern of the bifundamental fields is related to flux $\mathcal{F}$

Example of A



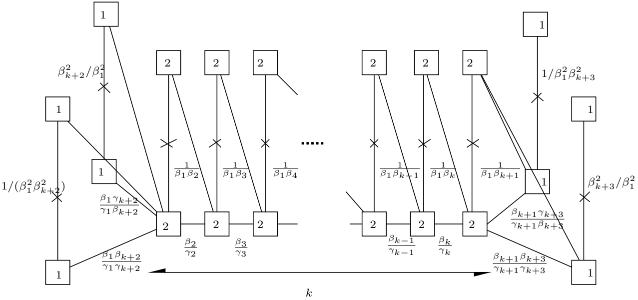
- Gluing $k-1$ blocks (triangulation of torus) one obtains theory corresponding to flux

$$\mathcal{F} = (k, -1, -1, \cdots - 1, 0, 0, ...)$$

- Symmetry enhanced from $U(1)^{2k-1}$ to $SU(k) \times SU(k-1) \times U(1)^2$

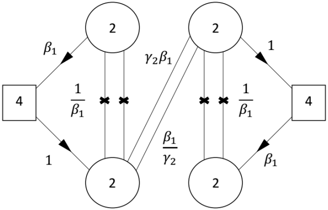
- Anomalies and indices work

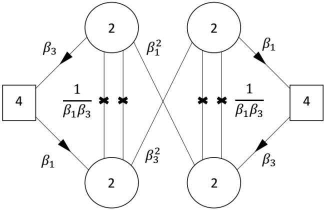
Example of D



- The building block becomes more complicated
- Two copies of affine Dynkin diagram connected by bifundamental fields

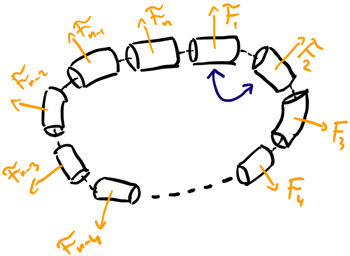
Example of D_5 minimal

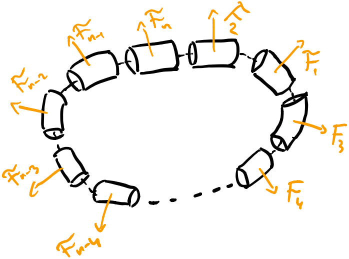




- On the left flux is such that $SU(4)^2 \times U(1)^2$ enhances to $SO(15) \times U(1)$
- On the right flux is such that $SU(4)^2 \times U(1)^2$ enhances to $SO(12) \times SU(2) \times U(1)$
- Anomalies and indices are consistent with this

Dualities, olden



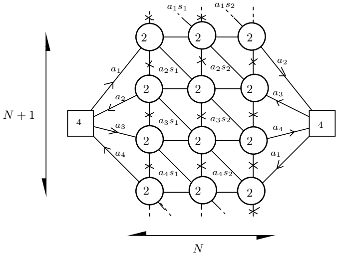


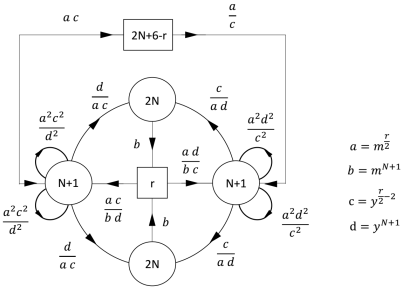
- Changing the order in which we combine the blocks one obtains different looking quiver theories

- Six dimensional interpretation tells all these have to be the same CFTs, dual to each other

- These reduce to Seiberg (Intriligator-Pouliot, etc) dualities

Dualities, new



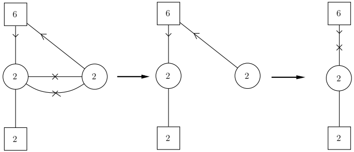


- Different $5d$ descriptions of a $6d$ model can give different blocks but equivalent $4d$ models for closed surfaces

- Minimal D_{N+3} conformal matter has three description, $SU(2)^N$, $USp(2N)$, $SU(N+1)$

- The models above are dual $(r = N+1)$ with different manifest symmetry which enhances to $SO(2N+10) \times SU(N+1) \times U(1)$

Complicated to simple

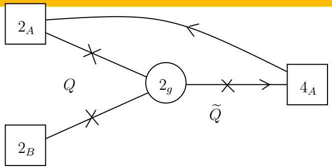


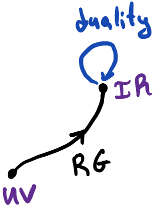
- Theories obtained by compactification can be simplified by deformations, still exhibit enhanced symmetry
- Example, reductions with D_4 minimal conformal matter deformed
- The $6d$ logic implies should have $E_6 \times U(1)$ symmetry
- The theory is $SU(2)$ SQCD with four flavors and a superpotential

$$1 + \overline{\mathbf{27}} h^{-1} (qp)^{\frac{4}{9}} + h^3 (qp)^{\frac{2}{3}} + \ldots + (-\mathbf{78} - 1) qp + \ldots .$$

- Assumptions of no accidental abelian symmetry, CFT, proof symmetry is $E_6 \times U(1)$

Symmetry and duality



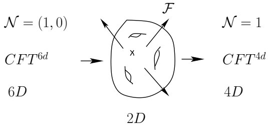


- The symmetry is related to Seiberg duality
- The duality mixes $SU(2)$ with $SU(6)$ enhancing it to E_6
- The conformal manifold is a point
- Generalizations: Self-dualities of $Spin(4+n)$ gauge theories with n vectors and spinors with 32 components (Csaki, Schmaltz, Skiba, Terning 97, Karch 97) lead to models with enhanced symmetry, for example symmetry rotating the spinors is commutant of $SU(2)$ in E_{9-n}.

Summary

- Can construct a lot of examples of theories with IR symmetry being much bigger than UV symmetry through compactifications
- The construction is (almost) algorithmic
- Simple compactifications lead to involved models
- Simple enhancements often related to deformations of compactifications
- Symmetry emerges but is not completely accidental

Open questions



- The key is to identify a set of building blocks
- Understand more systematically the domain walls
- Higher genus known for A_0 $N = 2$ and $N = 3$, A_1 and $N = 2$, D_4 and $N = 1$, pure glue $SU(3)$ and $SO(8)$ 6d SCFTS (SSR, Zafrir 18)
- Develop the general dictionary between six and four dimensions
- Three dimensions

Is everything Lagrangian?

- Derived Lagrangians for many models with flavor symmetry enhancing
- Have Lagrangian for Argyres-Douglas models (Maruyoshi, Song 2016)
- Have Lagrangian constructions for E_6 (and many others, E_7, ...)
- In such examples also supersymmetry enhances
- Can we write a Lagrangian for any model?

Thank You