SCFTs in $6d$ and IR symmetry enhancement in $4d$

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Lagrangians/non Lagrangian models

- This talk is about 4d QFTS with $\mathcal{N} = 1$ supersymmetry

- We believe that certain models exist for which one cannot write a Lagrangian with all the (super)symmetries manifest

- Many $\mathcal{N} = 2$ examples; Argyres-Douglas models, Minahan-Nemeschansky models, $T_N$ models

- Insisting on manifestly $\mathcal{N} = 2$ supersymmetric Lagrangians no description can be found which reproduces some of the properties of these models
Evidence for existence of non-Lagrangian models studying compactifications of 6d conformal field theories to 4d

Choices involved – 6d model, surface, flux for \( G^{6d} \)

General choices lead to models with no known Lagrangian

There is a tension between having a Lagrangian description and having all the symmetries of the models manifest
How should we think about such models?

Two options:

Give up Lagrangians:

One can attempt to develop new tools to avoid the need of defining Lagrangians (see Bootstrap)

Alternatively:

...
Give up symmetry!!
Global (super)symmetries can be enhanced in IR, $G_{IR}$ might be not equal to $G_{UV}$
What will be an organizing principle?

A strong hint comes by studying same type of compactifications

Many compactifications do have Lagrangian descriptions

In many cases the UV symmetry is enhanced in the IR

We will discuss how it comes about in the case of a torus with flux
The 6d (1, 0) theories have some symmetry $G^{6d}$

Upon compactification one can turn on flux for abelian subgroups of this symmetry preserving $\mathcal{N} = 1$ supersymmetry in 4d

Flux is specified by $r = \text{Rank } G^{6d}$ integers

$$\mathcal{F} = (F_1, F_2, \cdots, F_{r-1}, F_r)$$

The 4d symmetry $G^{4d}$ is the subgroup of $G^{6d}$ which commutes with the flux

For example if $G^{6d}$ is $SU(r + 1)$,

$$\mathcal{F} = (r - 1, -1, -1, \cdots, -1)$$

preserves $SU(r) \times U(1)$
We can think of the theories obtained by compactification on tori as combined from simple building blocks.

- The blocks correspond to tubes with some value of flux $F_j$.
- Each tube is a simple 4d theory, the IR symmetry visible in UV.
- However, combining tubes the flux $\sum_{j=1}^{n} F_j$ might be indicating enhanced symmetry, bigger than the symmetry of the blocks.
- Example, $U(1)^2 \rightarrow U(1) \times SU(2)$, $(4, -1, -3) + (4, -3, -1) \rightarrow (8, -4, -4)$. 
What are the tube models?

To try and understand what are the tube models one can compactify the 6d models first on a circle to 5d.

Many 6d theories have effective description as gauge theory in 5d which then can be used to understand the 4d models.

Example: ADE conformal matter ($N$ M5 branes on ADE singularity), 5d description as quivers in the shape of affine ADE Dynkin diagram.
• Upon compactification to 5d we have a choice of holonomy which translates to a choice of mass parameters

• Different holonomies might lead to different effective theories in 5d ([Hayashi, Kim, Lee, Taki, Yagi 15])

• The 5d manifestation of the flux is in terms of domain walls interpolating between different values of the mass parameters, or different five dimensional descriptions

• The four dimensional theories can be constructed by understanding the theories on the domain walls ([see N. Paquette’s talk])

• Punctures: make the cylinder finite, choose bc for 5d fields
The basic reduction toolbox

- How do we know theories in four dimensions correspond to some compactification?

- Anomaly,

\[ \int_{\Sigma_{g,0}} I_8^{T^6d}(\mathcal{F}) = I_6^{T_4d}(T^6d,\Sigma_{g,0},\mathcal{F}) \]

(Benini, Tachikawa, Wecht 9)

- Indices,

\[ 1 + \sum_{relevant} n_i(qp)^{R_i/2} + (\text{Marg - Currents})qp + \ldots \]

(Beem, Gadde 12)

- If at order \( qp \) you see adjoint of some group it has to be the symmetry of the theory (unless there is an accidental \( U(1) \))

- Consistency checks
We will give examples of $G = ADE$ conformal matter

Five dimensional description is in terms of $G$ affine quiver
The theories are usual quiver theories.
In addition to gauge charged matter one also has gauge singlets.
The gauge singlet fields flip some of the gauge invariant baryons.
The flips are very important, without these the anomalies do not match with 6d and in some cases the symmetries do not enhance.

Generally these are just free fields.
Example of A

- Typical building block is a WZ model

- The UV symmetry is two copies of $SU(N)^k$ associated to the boundary and $U(1)^{2k-1}$ which is the Cartan subgroup of $G^{6d} = SU(k) \times SU(k) \times U(1)$

- The pattern of the bifundamental fields is related to flux $\mathcal{F}$
Example of $A$

- Gluing $k-1$ blocks (triangulation of torus) one obtains theory corresponding to flux

\[ \mathcal{F} = (k, -1, -1, \cdots - 1, 0, 0, \ldots) \]

- Symmetry enhanced from $U(1)^{2k-1}$ to $SU(k) \times SU(k-1) \times U(1)^2$

- Anomalies and indices work
The building block becomes more complicated

Two copies of affine Dynkin diagram connected by bifundamental fields
Example of $D_5$ minimal

- On the left flux is such that $SU(4)^2 \times U(1)^2$ enhances to $SO(15) \times U(1)$

- On the right flux is such that $SU(4)^2 \times U(1)^2$ enhances to $SO(12) \times SU(2) \times U(1)$

- Anomalies and indices are consistent with this
Changing the order in which we combine the blocks one obtains different looking quiver theories

Six dimensional interpretation tells all these have to be the same CFTs, dual to each other

These reduce to Seiberg (Intriligator-Pouliot, etc) dualities
• Different $5d$ descriptions of a $6d$ model can give different blocks but equivalent $4d$ models for closed surfaces

• Minimal $D_{N+3}$ conformal matter has three description, $SU(2)^N$, $USp(2N)$, $SU(N+1)$

• The models above are dual ($r = N + 1$) with different manifest symmetry which enhances to $SO(2N + 10) \times SU(N + 1) \times U(1)$
Theories obtained by compactification can be simplified by deformations, still exhibit enhanced symmetry.

Example, reductions with $D_4$ minimal conformal matter deformed.

The $6d$ logic implies should have $E_6 \times U(1)$ symmetry.

The theory is $SU(2)$ SQCD with four flavors and a superpotential

$$1 + 27h^{-1}(qp)^{\frac{4}{9}} + h^3(qp)^{\frac{2}{3}} + ... + (\textbf{-78} - 1)qp + ....$$

Assumptions of no accidental abelian symmetry, CFT, proof symmetry is $E_6 \times U(1)$.
The symmetry is related to Seiberg duality

The duality mixes $SU(2)$ with $SU(6)$ enhancing it to $E_6$

The conformal manifold is a point

**Generalizations:** Self-dualities of $Spin(4 + n)$ gauge theories with $n$ vectors and spinors with 32 components ([Csaki, Schmaltz, Skiba, Terning 97, Karch 97](#)) lead to models with enhanced symmetry, for example symmetry rotating the spinors is commutant of $SU(2)$ in $E_{9-n}$.
Can construct a lot of examples of theories with IR symmetry being much bigger than UV symmetry through compactifications.

The construction is (almost) algorithmic.

Simple compactifications lead to involved models.

Simple enhancements often related to deformations of compactifications.

Symmetry emerges but is not completely accidental.
The key is to identify a set of building blocks

Understand more systematically the domain walls

Higher genus known for $A_0 N = 2$ and $N = 3$, $A_1$ and $N = 2$, $D_4$ and $N = 1$, pure glue $SU(3)$ and $SO(8)$ 6d SCFTS (SSR, Zafrir 18)

Develop the general dictionary between six and four dimensions

Three dimensions
Is everything Lagrangian?

- Derived Lagrangians for many models with flavor symmetry enhancing

- Have Lagrangian for Argyres-Douglas models (Maruyoshi, Song 2016)

- Have Lagrangian constructions for $E_6$ (and many others, $E_7$, ...)

- In such examples also supersymmetry enhances

- Can we write a Lagrangian for any model?
Thank You