Bootstrapping $4d$ $\mathcal{N} = 2$ theories

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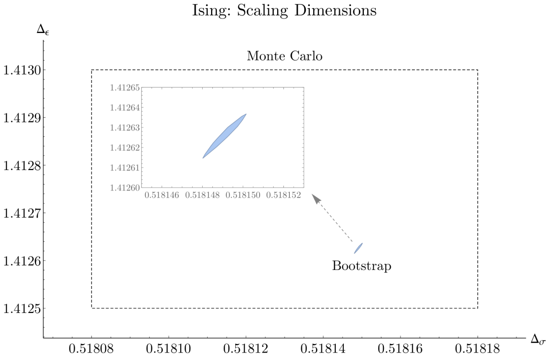
together with C. Beem, M. Cornagliotto, P. Liendo, W. Peelaers, L. Rastelli, V. Schomerus, B. van Rees

$3d$ Ising Model

[Poland Simmons-Duffin Kos, Simmons-Duffin, Poland Simmons-Duffin Kos Vichi]



One $\mathbb{Z}_2-$even, one $\mathbb{Z}_2-$odd relevant scalar operator

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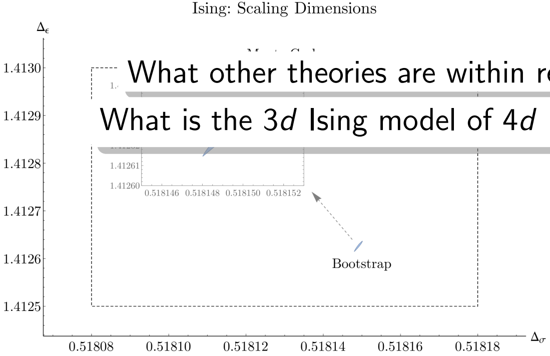

What other theories are within reach?

What is the 3*d* Ising model of 4*d* (S)CFTs?

One $\mathbb{Z}_2-$even, one $\mathbb{Z}_2-$odd relevant scalar operator

Outline

1. **The Superconformal Bootstrap Program**

2. (A_1, A_2) **Argyres-Douglas Theory**

3. **Landscape of** $4d$ $\mathcal{N} = 2$ **SCFTs**

4. **Summary & Outlook**

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$\rightarrow$ "Simplest" $\mathcal{N} = 2$ Argyres-Douglas theory?

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Conformal field theory defined by **[see Simmons-Duffin's talk]**

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$$\mathcal{O}_1(x)\mathcal{O}_2(0) = \sum_{k\,\mathrm{prim.}} f_{\mathcal{O}_1\mathcal{O}_2\mathcal{O}_k} x^{\Delta_k-\Delta_1-\Delta_2}\left(\mathcal{O}_k(0)+\ldots\right)$$

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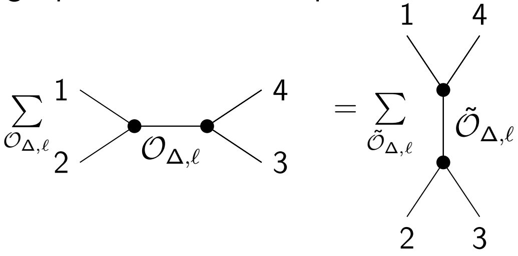

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(and also $6d$ $\mathcal{N} = (2,0)$ and $2d$ $\mathcal{N} = (0,4)$ [Beem Rastelli van Rees])

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$4d$ $\mathcal{N} = 2$ **SCFTs** $\rightarrow$ $2d$ **chiral algebra**

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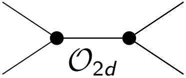
$\sum_{\mathcal{O}_{2d}} f^2_{\mathcal{O}_{2d}}$ 

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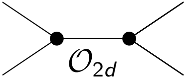
$\sum_{\mathcal{O}_{2d}} f^2_{\mathcal{O}_{2d}}$ 

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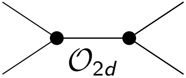

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assumptions: interacting theory, unique stress tensor

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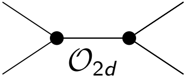
$\sum_{\mathcal{O}_{2d}} f^2_{\mathcal{O}_{2d}}$ 

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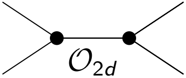

$\to$ $f^2_{\mathcal{O}_{2d}} \rightsquigarrow f^2_{\mathcal{O}_{4d}} \underbrace{\geqslant}_{4d \text{ unitarity}} 0 \Rightarrow$ **New unitarity bounds**

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Landscape of $4d\ \mathcal{N} \geqslant 2$ SCFTs

From $2d$ (super-)stress tensor four-point function

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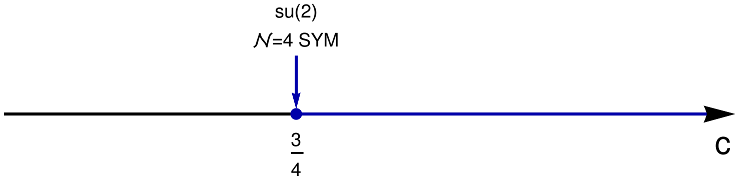
$\rightarrow$ $4d\ \mathcal{N} = 4$ SCFTs $c = a \geqslant \frac{3}{4}$ [Beem Rastelli van Rees]



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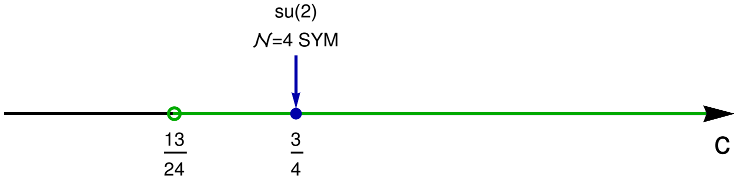

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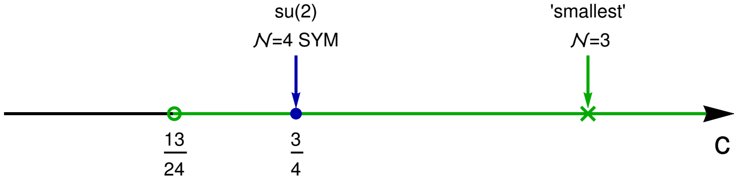

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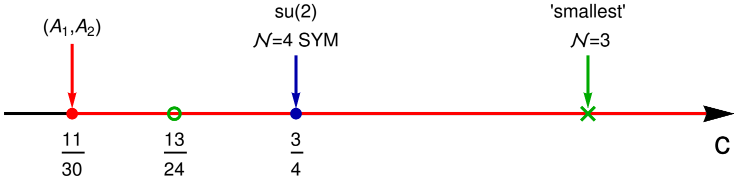

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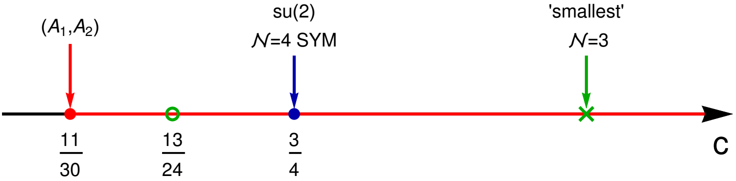
$\hookrightarrow$ Saturated by the (A_1, A_2) Argyres-Douglas theory



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How can we approach it?

- Known: $4d$ $\mathcal{N}=2$ chiral operator ϕ $\quad(\mathcal{Q}^I_\alpha \phi = 0)$

  $$\boxed{\Delta_\phi = \frac{6}{5}} \qquad U(1)_r \text{ charge } r = \Delta_\phi$$

- Study $\langle \phi(x_1)\phi(x_2)\underbrace{\bar{\phi}(x_3)}_{\text{conjugate of } \phi}\bar{\phi}(x_4)\rangle$

- Two OPE channels:
  - $\hookrightarrow$ $\phi\phi \sim \phi^2 + \cdots$
  - $\hookrightarrow$ $\phi\bar{\phi} \sim \mathrm{Identity} + \text{Super-stress tensor} + \cdots$

- Conformal blocks $\rightsquigarrow$ superconformal blocks

(only in $\phi\bar{\phi}$ channel) [Fitzpatrick Kaplan Khandker Li Poland Simmons-Duffin]

Minimum allowed central charge

Does $\langle \phi\phi\bar{\phi}\bar{\phi} \rangle$ know about $c \geqslant \frac{11}{30}$?

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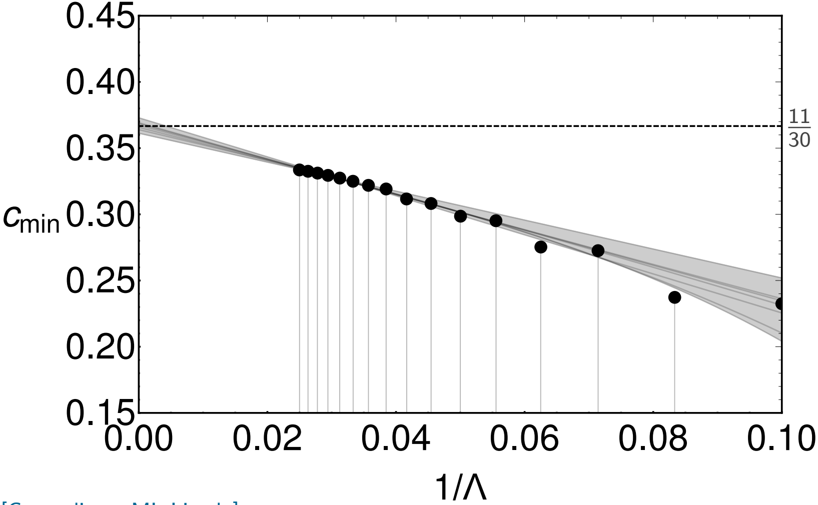

[Cornagliotto ML Liendo]

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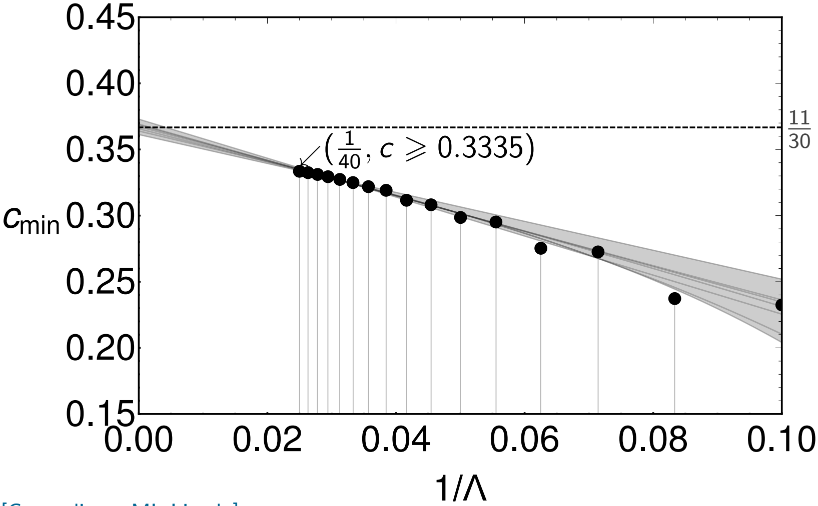

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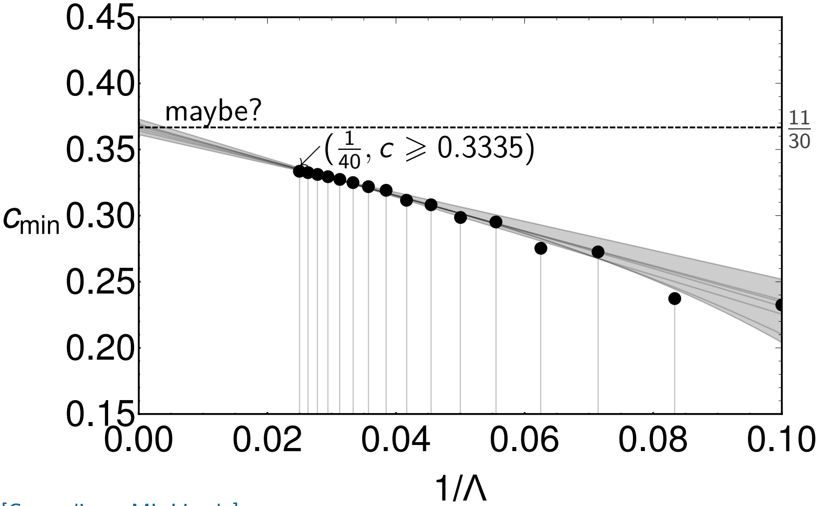

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Bounding OPE coefficients

$$\phi\phi \sim \underbrace{f_{\phi^2}^2}_{\textbf{unknown}} \underbrace{\phi^2}_{\Delta=2\Delta_\phi} + \cdots$$

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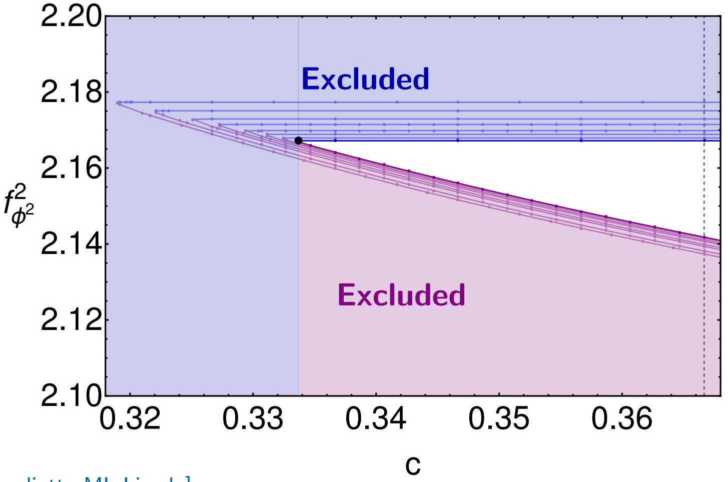

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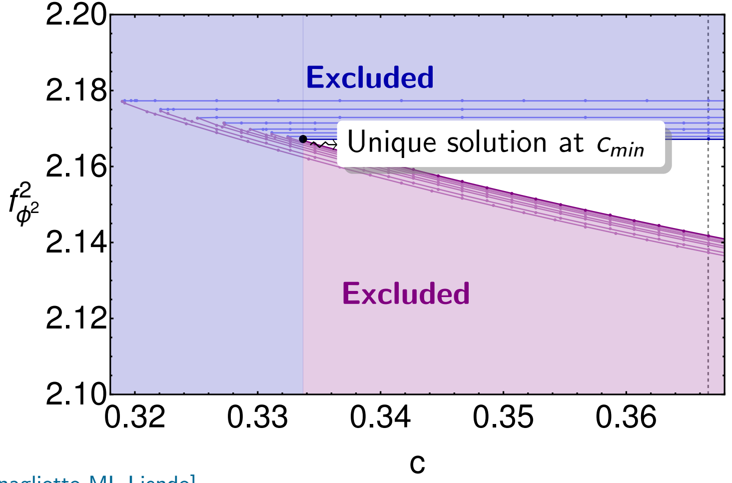

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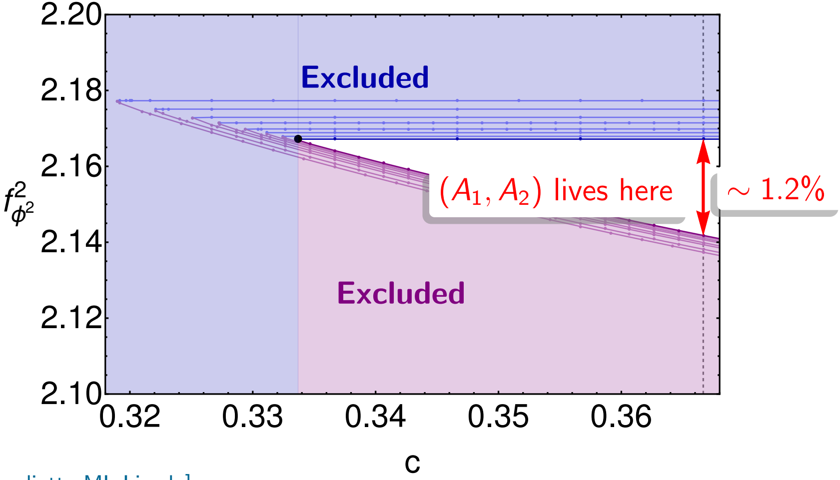

[Cornagliotto ML Liendo]

Lorentzian inversion formula

$$\phi\phi \sim f^2_{\phi^2} \underbrace{\phi^2}_{\Delta=2\Delta_\phi} + f^2_{\mathcal{C}_\ell} \underbrace{\mathcal{C}_{\ell>0}}_{\Delta=2\Delta_\phi+\ell} + \cdots$$

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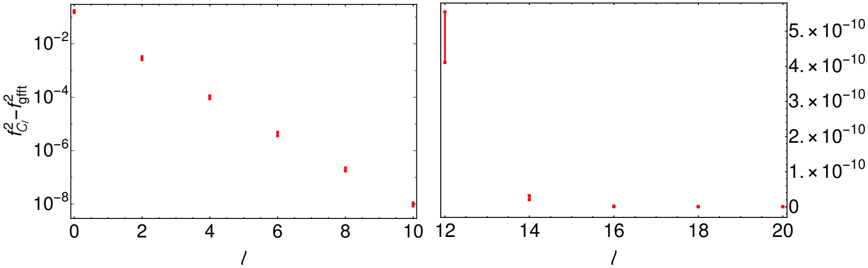

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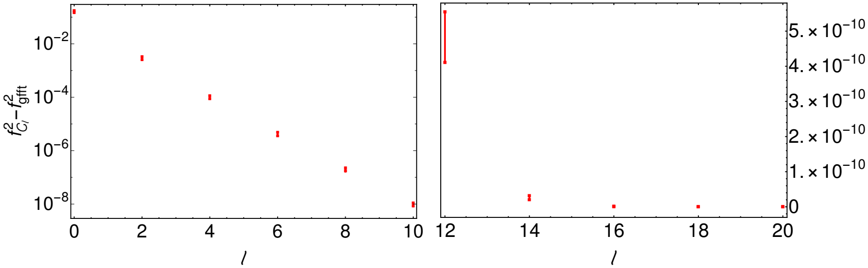

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Inverting the $\phi\phi$ OPE

$\rightarrow$ Same as bosonic inversion, valid for $\ell > 1$

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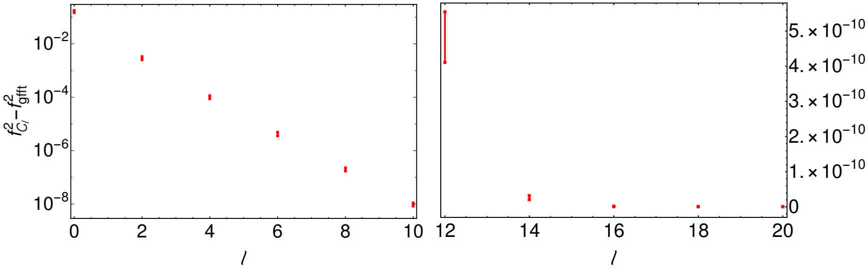

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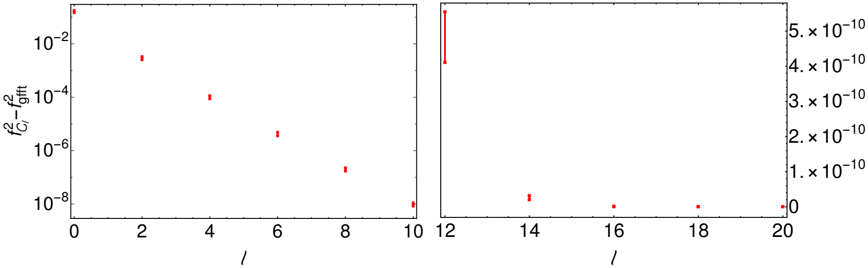

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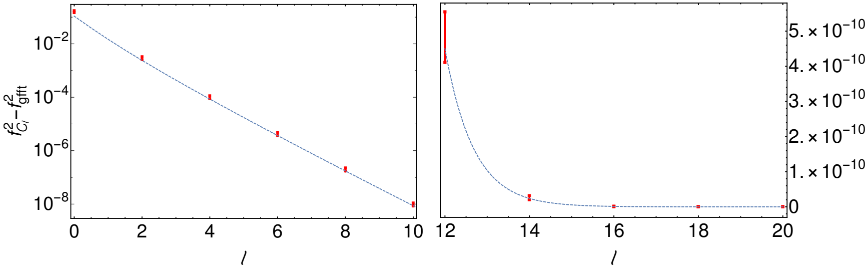

[Cornagliotto ML Liendo]

Inverting the $\phi\phi$ OPE

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- $\rightarrow$ Feed in low twist in t/u-channel: $\bar{\phi}\phi$ OPE
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- $\rightarrow$ Get s-channel ($\phi\phi$) large spin

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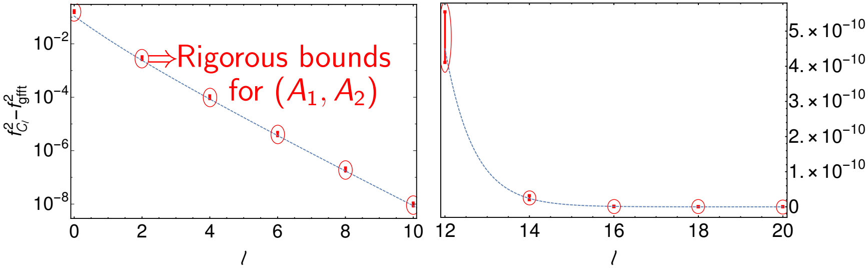

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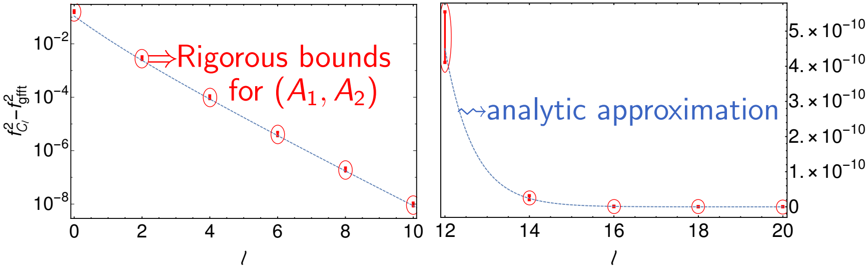

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Outline

1. The Superconformal Bootstrap Program
2. (A_1, A_2) Argyres-Douglas Theory
3. **Landscape of $4d$ $\mathcal{N} = 2$ SCFTs**
4. Summary & Outlook

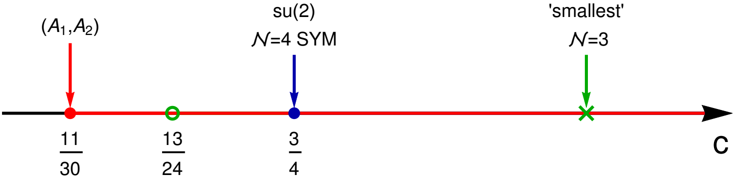
Landscape of $4d$ $\mathcal{N} \geqslant 2$ SCFTs

Projection of space of SCFTs to an axis

→ $4d$ $\mathcal{N} = 4$ SCFTs $c = a \geqslant \frac{3}{4}$ [Beem Rastelli van Rees]

→ $4d$ $\mathcal{N} \geqslant 3$ SCFTs $c = a > \frac{13}{24}$ [Cornagliotto ML Schomerus]

→ $4d$ $\mathcal{N} \geqslant 2$ SCFTs $c \geqslant \frac{11}{30}$ [Liendo Ramirez Seo]



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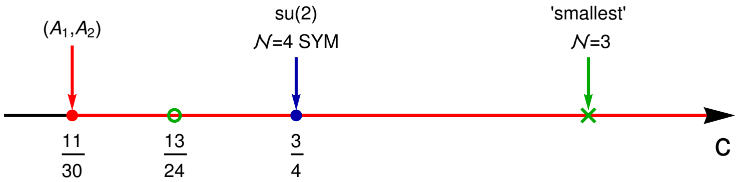

Finer view of the space of theories:

⇒ Organize theories by flavor symmetry

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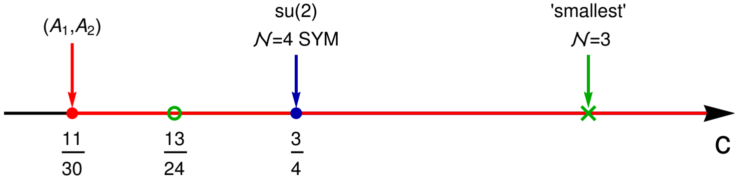


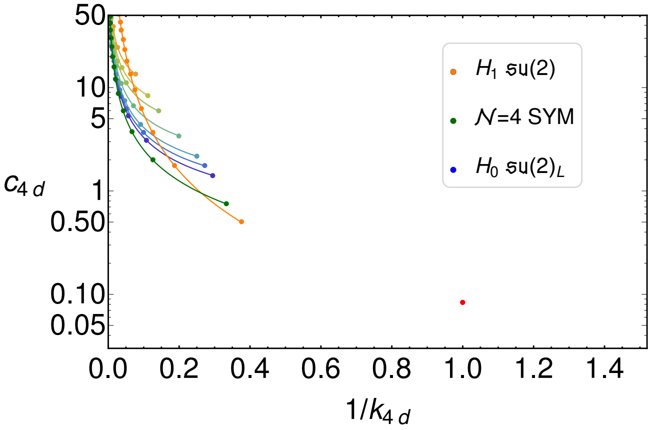
Finer view of the space of theories:

⇒ Organize theories by flavor symmetry

$\langle TT \rangle \propto c, \qquad \langle JJ \rangle \propto k$

$4d$ $\mathcal{N} = 2$ SCFT with $su(2)$ flavor symmetry

- $4d$ Flavor current supermultiplet



$4d$ $\mathcal{N}=2$ SCFT with $su(2)$ flavor symmetry

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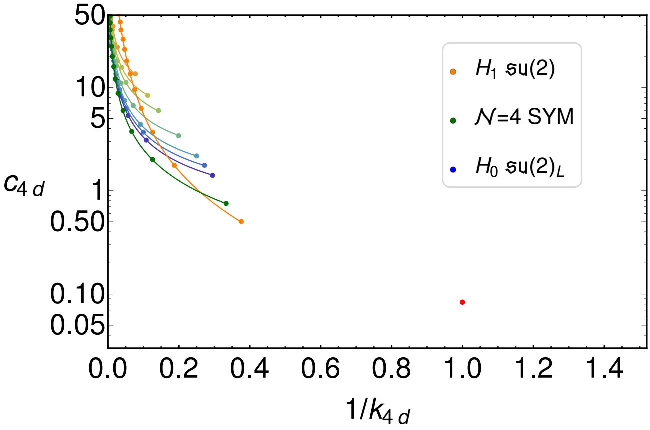

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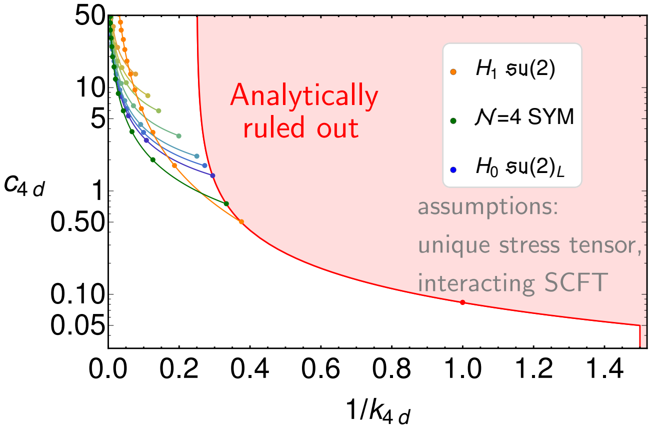

[Beem ML Liendo Peelaers Rastelli van Rees]

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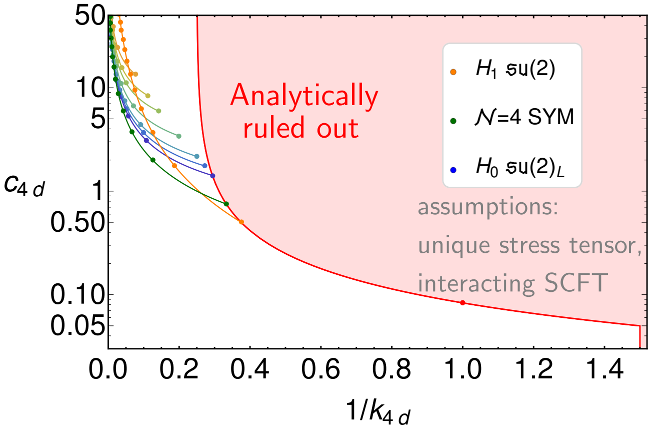

[Beem ML Liendo Peelaers Rastelli van Rees]

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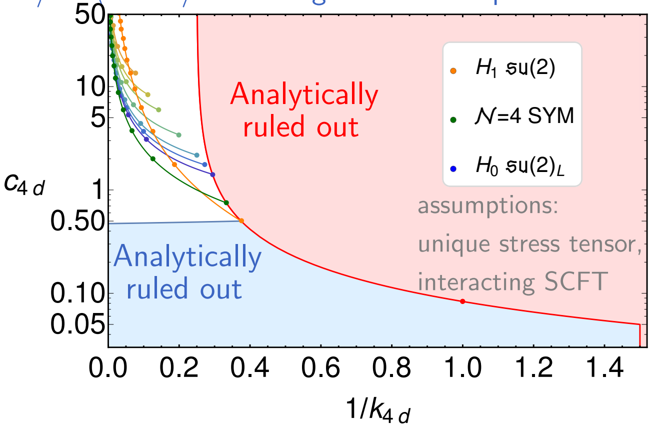

[Beem ML Liendo Peelaers Rastelli van Rees, ML Liendo]

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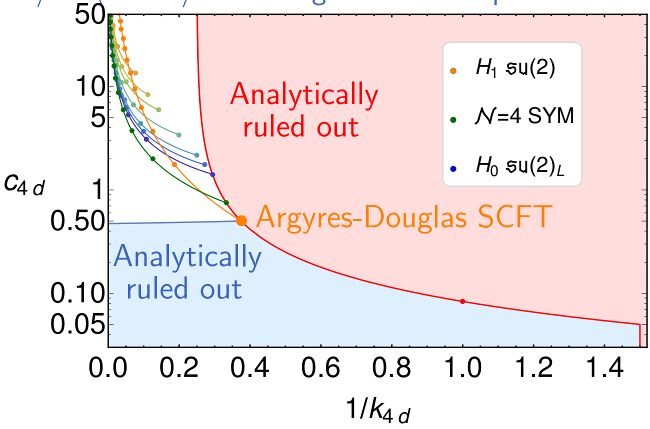

[Beem ML Liendo Peelaers Rastelli van Rees, ML Liendo]

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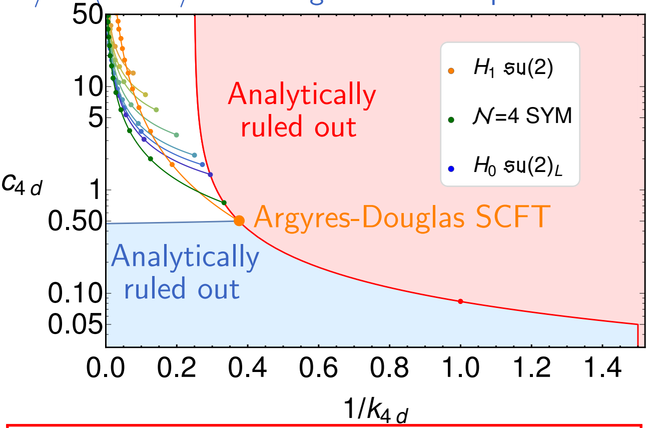

Only for $su(2)$, $su(3)$, $so(8)$, g_2, f_4, e_6, e_7, e_8

[Beem ML Liendo Peelaers Rastelli van Rees, ML Liendo]

Outline

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(at corners of $su(2)$, $su(3)$, e_6, e_7, e_8 exclusion curves)

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→ Weight-shifting operators? [Karateev Kravchuk Simmons-Duffin]

What is the "smallest" $\mathcal{N} = 3$ SCFT?

Thank you!

Backup slides

Outline

5. **Lorentzian inversion formula for** (A_1, A_2)

6. **Constraining the space of** $4d$ $\mathcal{N} = 2$ **SCFTs**

Lorentzian inversion formula: Superconformal case

Invert $\phi\phi$ OPE

- $\rightarrow$ Same as bosonic inversion, valid for $\ell > 1$
- $\rightarrow$ Feed in $\bar{\phi}\phi \sim 1 +$ Stress tensor multiplet $+ \ldots$

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- $\rightarrow$ Feed in low twist in t-channel $(\bar{\phi}\phi)$

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Invert $\bar{\phi}\phi$ OPE

- $\rightarrow$ Supersymmetric inversion: valid for $\ell \geqslant 0$
- $\rightarrow$ Feed in low twist in *t*-channel ($\bar{\phi}\phi$)
  - $\hookrightarrow$ $\bar{\phi}\phi \sim 1 +$ Stress tensor multiplet $+ \ldots$
- $\rightarrow$ and in *u*-channel ($\phi\phi$)
  - $\hookrightarrow$ $\phi\phi \sim \phi^2 + \ldots$

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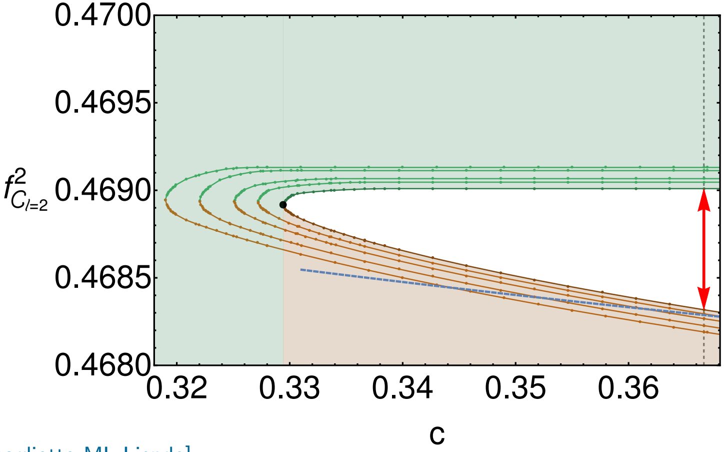

[Cornagliotto ML Liendo]

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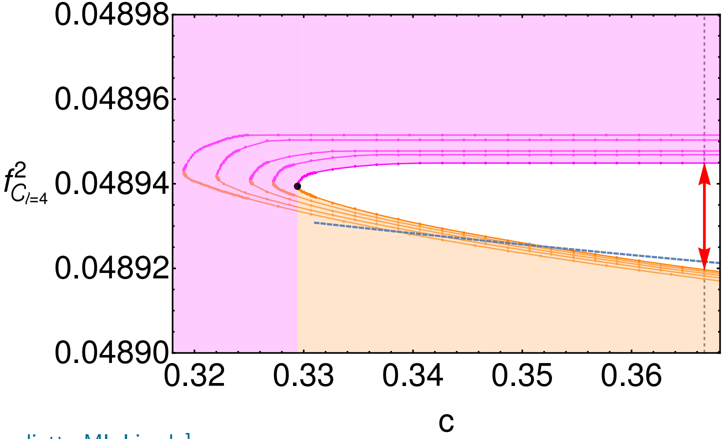

[Cornagliotto ML Liendo]

A Lorentzian inversion formula

Inverting the $\phi\bar{\phi}$ OPE

- $\rightarrow$ Supersymmetric inversion: valid for $\ell \geqslant 0$
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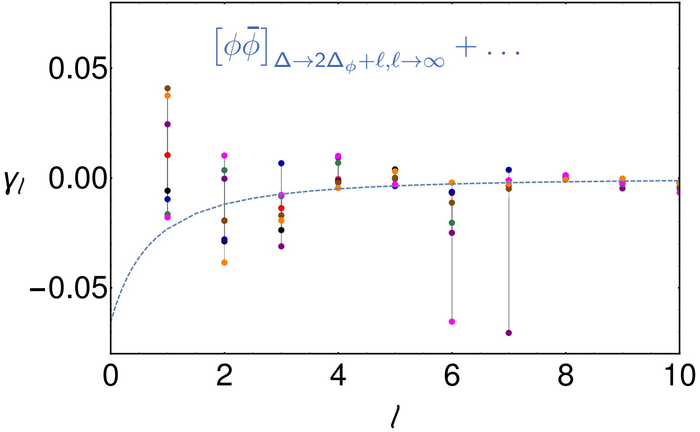


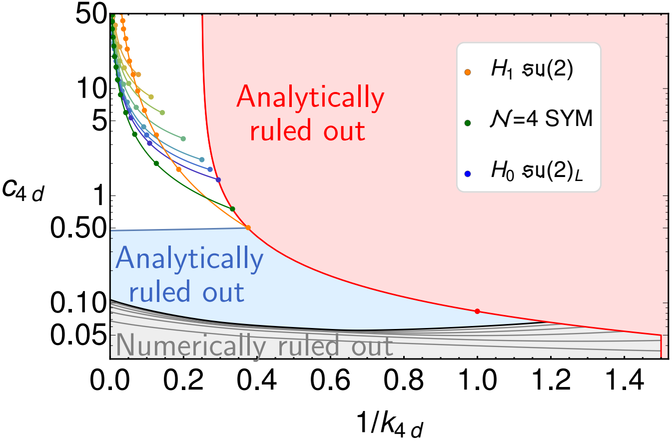
Outline

5 Lorentzian inversion formula for (A_1, A_2)

6 **Constraining the space of $4d$ $\mathcal{N} = 2$ SCFTs**

Constraining the space of $4d$ $\mathcal{N} = 2$ SCFTs

$su(2)$ flavor symmetry

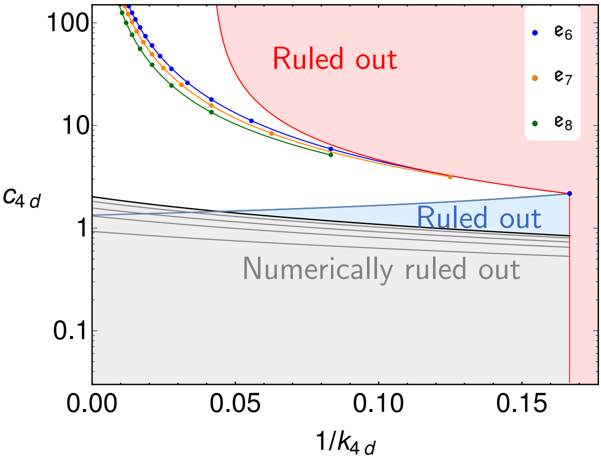


[Beem, ML, Liendo, Peelaers, Rastelli, van Rees; ML, Liendo]

[Beem, ML, Liendo, Rastelli, van Rees]

Constraining the space of $4d$ $\mathcal{N} = 2$ SCFTs

e_6 flavor symmetry

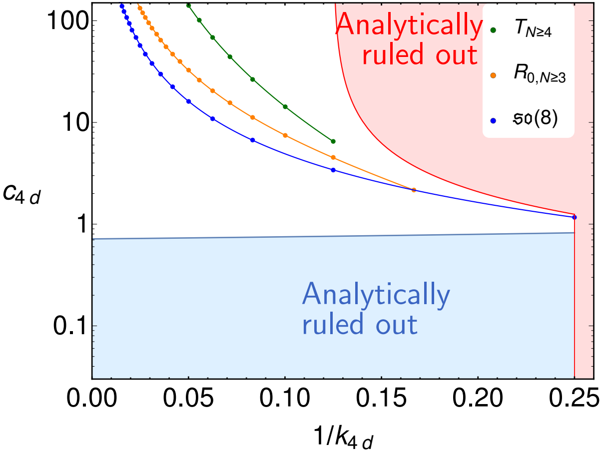


[Beem, ML, Liendo, Peelaers, Rastelli, van Rees; ML, Liendo]

[Beem, ML, Liendo, Rastelli, van Rees]

Constraining the space of $4d$ $\mathcal{N}=2$ SCFTs

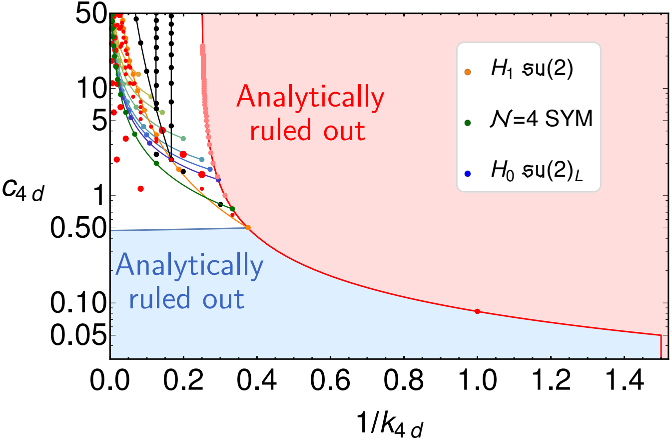
$su(4)$ flavor symmetry



[Beem, ML, Liendo, Peelaers, Rastelli, van Rees; ML, Liendo]

Constraining the space of $4d$ $\mathcal{N} = 2$ SCFTs

$su(2)$ flavor symmetry



[Beem, ML, Liendo, Peelaers, Rastelli, van Rees; ML, Liendo]