Bootstrapping $4d \mathcal{N} = 2$ theories

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together with C. Beem, M. Cornaglioatto, P. Liendo, W. Peelaers, L. Rastelli, V. Schomerus, B. van Rees
What is the 3d Ising model of 4d (S)CFTs?

One $\mathbb{Z}_2$—even, one $\mathbb{Z}_2$—odd relevant scalar operator
3d Ising Model

[Poland Simmons-Duffin Kos, Simmons-Duffin, Poland Simmons-Duffin Kos Vichi]

What other theories are within reach?

What is the 3d Ising model of 4d (S)CFTs?

One $\mathbb{Z}_2$—even, one $\mathbb{Z}_2$—odd relevant scalar operator
Outline

1 The Superconformal Bootstrap Program

2 $(A_1, A_2)$ Argyres-Douglas Theory

3 Landscape of $4d \mathcal{N} = 2$ SCFTs

4 Summary & Outlook
1. The Superconformal Bootstrap Program

2. $(A_1, A_2)$ Argyres-Douglas Theory

3. Landscape of $4d \mathcal{N} = 2$ SCFTs

4. Summary & Outlook
What is the space of consistent $4d$ SCFTs?
The Superconformal Bootstrap Program

What is the space of consistent $4d$ SCFTs?

→ Maximally supersymmetric theories: $\mathcal{N} = 4$ SYM (?)
What is the space of consistent $4d$ SCFTs?

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- $\mathcal{N} = 2$ theories: growing list of theories [see Argyres’ talk]
What is the space of consistent 4d SCFTs?

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→ $\mathcal{N} = 3$ theories  [García-Etxebarria Regalado]
→ $\mathcal{N} = 2$ theories: growing list of theories  [see Argyres’ talk]
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Can we bootstrap specific theories?
The Superconformal Bootstrap Program

What is the space of consistent 4d SCFTs?

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→ $\mathcal{N} = 2$ theories: growing list of theories  [see Argyres’ talk]

Can we bootstrap specific theories?

→ “Simplest” $\mathcal{N} = 2$ Argyres-Douglas theory?
Conformal field theory defined by [see Simmons-Duffin’s talk]

Set of local operators and all their correlation functions

Operator Product Expansion

\[ O_1(x) O_2(0) = \sum_{k} f_{O_1 O_2 O_k} x^{\Delta_k - \Delta_1 - \Delta_2} (O_k(0) + \ldots) \]
Conformal field theory defined by [see Simmons-Duffin’s talk]
Set of local operators and all their correlation functions

Operator Product Expansion

\[ \mathcal{O}_1(x)\mathcal{O}_2(0) = \sum_{k_{\text{prim.}}} f_{\mathcal{O}_1\mathcal{O}_2\mathcal{O}_k} x^{\Delta_k - \Delta_1 - \Delta_2} (\mathcal{O}_k(0) + \ldots) \]
Conformal field theory defined by \( \{ \mathcal{O}_{\Delta, \ell, \ldots}(x) \} \) and \( \{ f_{\mathcal{O}_i \mathcal{O}_j \mathcal{O}_k} \} \)

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Operator Product Expansion

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Subject to

- Unitarity
- Associativity of the operator product algebra
Conformal field theory defined by \( \{ \mathcal{O}_{\Delta,\ell} \} \) and \( \{ f_{\mathcal{O}_i\mathcal{O}_j\mathcal{O}_k} \} \)

Operator Product Expansion

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\mathcal{O}_1(x)\mathcal{O}_2(0) = \sum_{k \text{prim.}} f_{\mathcal{O}_1\mathcal{O}_2\mathcal{O}_k} x^{\Delta_k - \Delta_1 - \Delta_2} (\mathcal{O}_k(0) + \ldots)
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Subject to

▶ Unitarity
▶ Crossing equations for all four-point functions
Conformal field theory defined by \[ \{ \mathcal{O}_{\Delta, \ell, \ldots}(x) \} \text{ and } \{ f_{\mathcal{O}_i \mathcal{O}_j \mathcal{O}_k} \} \] [see Simmons-Duffin’s talk]
Conformal field theory defined by [see Simmons-Duffin’s talk]
\{\mathcal{O}_{\Delta,\ell,...}(x)\} \text{ and } \{f_{i,j,k}\}

The Superconformal Bootstrap

- Conformal families $\leadsto$ Superconformal families
Conformal field theory defined by [see Simmons-Duffin’s talk]
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- Conformal families \(\leadsto\) Superconformal families
- Finite re-organization of an infinite amount of data
Conformal field theory defined by \[ \{ \mathcal{O}_{\Delta,\ell,...}(x) \} \] and \( \{ f_{\mathcal{O}_i \mathcal{O}_j \mathcal{O}_k} \} \)

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Q: Is there a solvable truncation of the crossing equations?
The Superconformal Bootstrap

Conformal field theory defined by \[ \{ \mathcal{O}_{\Delta,\ell,...}(x) \} \text{ and } \{ f_{\mathcal{O}_i \mathcal{O}_j \mathcal{O}_k} \} \]

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- Conformal families \( \rightsquigarrow \) Superconformal families
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Q: Is there a solvable truncation of the crossing equations?

\[ \rightarrow \text{ Yes, for } 4d \mathcal{N} \geq 2 \quad [\text{Beem ML Liendo Peelaers Rastelli van Rees}] \]

(and also \( 6d \mathcal{N} = (2, 0) \) and \( 2d \mathcal{N} = (0, 4) \) \[\text{[Beem Rastelli van Rees]}\])
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(and also \(6d \mathcal{N} = (2, 0)\) and \(2d \mathcal{N} = (0, 4) \quad [\text{Beem Rastelli van Rees}]\))

\[\rightarrow\text{ Subsector } \mathcal{N} \geq 2 \text{ SCFTs captured by } 2d \text{ chiral algebra} \]
A solvable subsector

$4d \mathcal{N} = 2$ SCFTs $\rightarrow 2d$ chiral algebra
A solvable subsector

4d $\mathcal{N} = 2$ SCFTs $\rightarrow$ 2d chiral algebra

- $SU(2)_R$ current $\mapsto$ 2d stress tensor $T(z)$
A solvable subsector

\[ 4d \mathcal{N} = 2 \text{ SCFTs} \rightarrow 2d \text{ chiral algebra} \]

- \( SU(2)_R \) current \( \rightarrow \) 2d stress tensor \( T(z) \)

\( \mathcal{O}_{2d} \sum \mathcal{O}_{2d} f^2 \mathcal{O}_{2d} \rightarrow f^2 \mathcal{O}_{4d} \geq \mathcal{O}_{4d} \) unitarity

\[ \Rightarrow \] New unitarity bounds

assumptions: interacting theory, unique stress tensor
A solvable subsector

$4d$ $\mathcal{N} \geq 2$ $\text{SCFTs} \rightarrow 2d$ chiral algebra

- Super-stress tensor multiplet$_{4d}$ $\leftrightarrow$ (Super-)stress tensor$_{2d}$
A solvable subsector

$4d \, \mathcal{N} \geq 2$ SCFTs $\rightarrow 2d$ chiral algebra

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A trivial statement in $2d$

$\rightarrow$ (super-)stress tensor four-point function fixed in terms of $c_{2d}$
A solvable subsector

$4d \mathcal{N} \geq 2$ SCFTs $\rightarrow 2d$ chiral algebra

- Super-stress tensor multiplet$_{4d} \mapsto (\text{Super-})\text{stress tensor}_{2d}$

A trivial statement in $2d$

$\rightarrow (\text{super-})\text{stress tensor four-point function fixed in terms of} \quad c_{2d} \quad \left( \langle TT \rangle \propto c \right)$
A solvable subsector

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- Super-stress tensor multiplet$_{4d}$ $\leftrightarrow$ (Super-)stress tensor$_{2d}$

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$\rightarrow$ (super-)stress tensor four-point function fixed in terms of $c_{2d} = -12c_{4d}$ ($\langle TT \rangle \propto c$)
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$\rightarrow 2d$ Superblock decomposition:

$$\sum_{\mathcal{O}_{2d}} f_{\mathcal{O}_{2d}}^2 \mathcal{O}_{2d}$$
A solvable subsector

$4d \mathcal{N} \geq 2$ SCFTs $\rightarrow$ $2d$ chiral algebra

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$$\sum_{\mathcal{O}_{2d}} f_{\mathcal{O}_{2d}}^2 \quad \begin{array}{c}
\mathcal{O}_{2d} \\
\mathcal{O}_{2d}
\end{array}$$

$\rightarrow$ $f_{\mathcal{O}_{2d}}^2$
A solvable subsector

$4d\ \mathcal{N} \geq 2\ \text{SCFTs} \rightarrow 2d\ \text{chiral algebra}$

- Super-stress tensor multiplet$_{4d} \leftrightarrow (\text{Super-})\text{stress tensor}_{2d}$

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\[ \sum_{\mathcal{O}_{2d}} f_{\mathcal{O}_{2d}}^2 \]

$\rightarrow f_{\mathcal{O}_{2d}}^2 \sim f_{\mathcal{O}_{4d}}^2$

assumptions: interacting theory, unique stress tensor
A solvable subsector

4d $\mathcal{N} \geq 2$ SCFTs $\rightarrow$ 2d chiral algebra

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$\rightarrow$ (super-)stress tensor four-point function fixed in terms of $c_{2d} = -12c_{4d}$ \quad (\langle TT \rangle \propto c)$

$\rightarrow$ 2d Superblock decomposition:

$\sum_{\mathcal{O}_{2d}} f_{\mathcal{O}_{2d}}^2 \mathcal{O}_{2d} \mathcal{O}_{2d}$

$\rightarrow$ $f_{\mathcal{O}_{2d}}^2 \sim f_{\mathcal{O}_{4d}}^2 \geq 0$

4d unitarity

assumptions: interacting theory, unique stress tensor
A solvable subsector

4d $\mathcal{N} \geq 2$ SCFTs $\rightarrow$ 2d chiral algebra

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$$\sum_{\mathcal{O}_{2d}} f_{\mathcal{O}_{2d}}^2 \quad \mathcal{O}_{2d}$$

$\rightarrow$ $f_{\mathcal{O}_{2d}}^2 \sim f_{\mathcal{O}_{4d}}^2 \quad \geq 0 \Rightarrow$ New unitarity bounds

4d unitarity assumptions: interacting theory, unique stress tensor
Landscape of $4d \mathcal{N} \geq 2$ SCFTs

From $2d$ (super-)stress tensor four-point function
(assumptions: interacting theory, unique stress tensor)

$\rightarrow 4d \mathcal{N} = 4$ SCFTs $c = a \geq \frac{3}{4}$ [Beem Rastelli van Rees]
Landscape of $4d \, \mathcal{N} \geq 2$ SCFTs

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(assumptions: interacting theory, unique stress tensor)

$\rightarrow \ 4d \, \mathcal{N} = 4$ SCFTs $c = a \geq \frac{3}{4} \quad \text{[Beem Rastelli van Rees]}$

$\rightarrow \ 4d \, \mathcal{N} \geq 3$ SCFTs $c = a > \frac{13}{24} \quad \text{[Cornagliotto ML Schomerus]}$

from interpreting $\mathcal{O}_{2d}$ as a $4d$ operator

\[ su(2) \quad \mathcal{N}=4 \text{ SYM} \]
Landscape of $4d \mathcal{N} \geq 2$ SCFTs

From $2d$ (super-)stress tensor four-point function
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from interpreting $O_{2d}$ as a $4d$ operator

\[
\begin{align*}
\text{su}(2) & \quad \mathcal{N}=4 \text{ SYM} \\
\text{'smallest'} & \quad \mathcal{N}=3
\end{align*}
\]
Landscape of $4d \mathcal{N} \geq 2$ SCFTs

From $2d$ (super-)stress tensor four-point function

(assumptions: interacting theory, unique stress tensor)

$\rightarrow 4d \mathcal{N} = 4$ SCFTs $c = a \geq \frac{3}{4}$ [Beem Rastelli van Rees]
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from interpreting $\mathcal{O}_{2d}$ as a $4d$ operator

$\rightarrow 4d \mathcal{N} \geq 2$ SCFTs $c \geq \frac{11}{30}$ [Liendo Ramirez Seo]
Landscape of $4d\ N \geq 2$ SCFTs

From $2d$ (super-)stress tensor four-point function
(assumptions: interacting theory, unique stress tensor)

→ $4d\ N = 4$ SCFTs $c = a \geq \frac{3}{4}$ [Beem Rastelli van Rees]
→ $4d\ N \geq 3$ SCFTs $c = a > \frac{13}{24}$ [Cornagliootto ML Schomerus]
from interpreting $O_{2d}$ as a $4d$ operator
→ $4d\ N \geq 2$ SCFTs $c \geq \frac{11}{30}$ [Liendo Ramirez Seo]

↔ Saturated by the $(A_1, A_2)$ Argyres-Douglas theory
1 The Superconformal Bootstrap Program

2 \((A_1, A_2)\) Agyres-Douglas Theory

3 Landscape of 4d \(\mathcal{N} = 2\) SCFTs

4 Summary & Outlook
The “simplest” Argyres-Douglas theory

→ Originally obtained on the Coulomb branch of a 4d $\mathcal{N} = 2$ susy gauge theory with gauge group $SU(3)$
The “simplest” Argyres-Douglas theory

→ Originally obtained on the Coulomb branch of a $4d$ $\mathcal{N} = 2$ susy gauge theory with gauge group $SU(3)$

→ $\mathcal{N} = 1$ Lagrangian description  [see Song’s talk]
The “simplest” Argyres-Douglas theory

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→ Strongly coupled isolated SCFT – no marginal deformations
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→ Chiral algebra[$(A_1, A_2)$] = Lee-Yang minimal model [Beem Rastelli]
The “simplest” Argyres-Douglas theory

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→ Chiral algebra\([A_1, A_2]\) = Lee-Yang minimal model

[Beem Rastelli]

Our tools beyond protected subsector

► Numerical bootstrap

[Rattazzi Rychkov Tonni Vichi]
The “simplest” Argyres-Douglas theory

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→ Chiral algebra[$(A_1, A_2)$] $= \text{Lee-Yang minimal model}$ [Beem Rastelli]

Our tools beyond protected subsector

- Numerical bootstrap
  [Rattazzi Rychkov Tonni Vichi]
- Lightcone bootstrap
  [Fitzpatrick Kaplan Poland Simmons-Duffin, Komargodski Zhiboedov]
The “simplest” Argyres-Douglas theory

\(\rightarrow\) Originally obtained on the Coulomb branch of a 4d \(\mathcal{N} = 2\) susy gauge theory with gauge group \(SU(3)\)

\(\rightarrow\) \(\mathcal{N} = 1\) Lagrangian description \([\text{see Song's talk}]\)

\(\rightarrow\) Strongly coupled isolated SCFT – no marginal deformations

\(\rightarrow\) Just another SCFT

\(\rightarrow\) Chiral algebra\([\mathcal{A}_1, \mathcal{A}_2]\) = Lee-Yang minimal model \([\text{Beem Rastelli}]\)

Our tools beyond protected subsector

\(\triangleright\) Numerical bootstrap
\([\text{Rattazzi Rychkov Tonni Vichi}]\)

\(\triangleright\) Lightcone bootstrap
\([\text{Fitzpatrick Kaplan Poland Simmons-Duffin, Komargodski Zhiboedov}]\)

\(\leftrightarrow\) Lorentzian inversion formula of \([\text{Caron-Huot}]\)
The “simplest” Argyres-Douglas theory

How can we approach it?
The “simplest” Argyres-Douglas theory

How can we approach it?

- Known: $4d \\mathcal{N} = 2$ chiral operator $\phi$

\[ \Delta_\phi = \frac{6}{5} \]

Two OPE channels:

$\phi\phi \sim \phi^2 + \cdots$

$\phi\bar{\phi} \sim \text{Identity} + \text{Super-stress tensor} + \cdots$

Conformal blocks $\mapsto$ superconformal blocks (only in $\phi\bar{\phi}$ channel)
The “simplest” Argyres-Douglas theory

How can we approach it?

- Known: $4d \, \mathcal{N} = 2$ chiral operator $\phi$  \hspace{1cm} $(\mathcal{Q}_\alpha^I \phi = 0)$

\begin{equation}
\Delta \phi = \frac{6}{5}
\end{equation}
The “simplest” Argyres-Douglas theory

How can we approach it?

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$$\Delta \phi = \frac{6}{5}$$

$U(1)_r$ charge $r = \Delta \phi$
The “simplest” Argyres-Douglas theory

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$U(1)_r$ charge $r = \Delta_\phi$

► Study $\langle \phi(x_1)\phi(x_2)\bar{\phi}(x_3)\bar{\phi}(x_4) \rangle$
The “simplest” Argyres-Douglas theory

How can we approach it?

- Known: $4d \mathcal{N} = 2$ chiral operator $\phi$ \hfill $(Q^I_\alpha \phi = 0)$
  \[ \Delta \phi = \frac{6}{5} \]

  $U(1)_r$ charge $r = \Delta \phi$

- Study $\langle \phi(x_1) \phi(x_2) \bar{\phi}(x_3) \bar{\phi}(x_4) \rangle$
  conjugate of $\phi$
The “simplest” Argyres-Douglas theory

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The “simplest” Argyres-Douglas theory

How can we approach it?

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The “simplest” Argyres-Douglas theory

How can we approach it?

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\[
\Delta_\phi = \frac{6}{5}
\]

$U(1)_r$ charge $r = \Delta_\phi$

- Study $\langle \phi(x_1) \phi(x_2) \phi(x_3) \phi(x_4) \rangle$

\[\text{conjugate of } \phi\]

- Two OPE channels:

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- Conformal blocks $\leadsto$ superconformal blocks
The “simplest” Argyres-Douglas theory

How can we approach it?

► Known: 4d $\mathcal{N} = 2$ chiral operator $\phi$ ($Q_\alpha^I \phi = 0$)

$$\Delta_\phi = \frac{6}{5}$$

$U(1)_r$ charge $r = \Delta_\phi$

► Study $\langle \phi(x_1) \phi(x_2) \overline{\phi}(x_3) \overline{\phi}(x_4) \rangle$

conjugate of $\phi$

► Two OPE channels:

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$\leftrightarrow \phi \overline{\phi} \sim \text{Identity} + \text{Super-stress tensor} + \cdots$

► Conformal blocks $\rightsquigarrow$ superconformal blocks

(only in $\phi \overline{\phi}$ channel) [Fitzpatrick Kaplan Khandker Li Poland Simmons-Duffin]
Minimum allowed central charge

Does $\langle \phi \phi \phi \phi \rangle$ know about $c \geq \frac{11}{30}$?
Does $\langle \phi \phi \bar{\phi} \phi \rangle$ know about $c \geq \frac{11}{30}$?
Minimum allowed central charge

Does $\langle \phi \phi \bar{\phi} \bar{\phi} \rangle$ know about $c \geq \frac{11}{30}$?

[Cornaglioni ML Liendo]
Does $\langle \phi \phi \bar{\phi} \bar{\phi} \rangle$ know about $c \geq \frac{11}{30}$?

[Cornaglotti ML Liendo]
Bounding OPE coefficients

\[ \phi \phi \sim f_\phi^2 \phi^2 + \cdots \]

unknown

\[ \Delta = 2 \Delta_\phi \]
Bounding OPE coefficients

\[ \phi \phi \sim f^2_{\phi^2} \phi^2 + \cdots \]

unknown

\[ \Delta = 2\Delta_{\phi} \]

Excluded

Excluded

[Cornagliotto ML Liendo]
Bounding OPE coefficients

\[ \phi \phi \sim f_{\phi^2}^2 \phi^2 + \cdots \]

unknown
\[ \Delta = 2\Delta_{\phi} \]

Excluded

Unique solution at \( c_{\text{min}} \)

[Cornaglott ML Liendo]
\( \phi \phi \sim f_{\phi^2}^2 \phi^2 \Delta = 2\Delta \phi \)

\((A_1, A_2)\) lives here \( \sim 1.2\% \)
Lorentzian inversion formula

\[ \phi \phi \sim f_\phi^2 \phi^2 + f_{C_\ell}^2 C_{\ell>0} + \cdots \]

\[ \Delta = 2\Delta_\phi \quad \Delta = 2\Delta_\phi + \ell \]
Lorentzian inversion formula

\[ \phi \phi \sim f_{\phi^2}^2 \phi^2 + f_{C_{\ell}}^2 C_{\ell>0} + \cdots \]

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[Cornagliotto ML Liendo]
Lorentzian inversion formula

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Inverting the \( \phi\phi \) OPE

→ Same as bosonic inversion, valid for \( \ell > 1 \)
Inverting the $\phi \phi$ OPE

→ Same as bosonic inversion, valid for $\ell > 1$
→ Feed in low twist in $t/u$-channel: $\bar{\phi} \phi$ OPE
Lorentzian inversion formula

\[ \phi \phi \sim f_{\phi^2}^2 \phi^2 + f_{C_\ell}^2 C_{\ell>0} + \cdots \]

\[ \Delta = 2\Delta_\phi \quad \Delta = 2\Delta_\phi + \ell \]

Inverting the \( \phi \phi \) OPE

\[ \rightarrow \text{Same as bosonic inversion, valid for } \ell > 1 \]
\[ \rightarrow \text{Feed in low twist in } t/u\text{-channel: } \bar{\phi} \phi \text{ OPE} \]
\[ \leftrightarrow \text{Only input: } \bar{\phi} \phi \sim 1 + \text{Stress tensor multiplet} \]
Lorentzian inversion formula

\[ \phi \phi \sim f_{\phi}^2 \phi^2 + f_{C,\ell}^2 C_{\ell>0} + \cdots \]

\[ \Delta = 2\Delta_{\phi} \]
\[ \Delta = 2\Delta_{\phi} + \ell \]

Inverting the \( \phi \phi \) OPE

→ Same as bosonic inversion, valid for \( \ell > 1 \)

→ Feed in low twist in \( t/u \)-channel: \( \bar{\phi} \phi \) OPE
  
  ← Only input: \( \bar{\phi} \phi \sim 1 + \text{Stress tensor multiplet} \)

→ Get \( s \)-channel (\( \phi \phi \)) large spin

[Cornaglio ML Liendo]
Lorentzian inversion formula

\[ \phi \phi \sim f^2_{\phi^2} \phi^2 + f^2_{C_\ell} C_{\ell>0} + \cdots \]

\[ \Delta = 2\Delta_\phi \quad \Delta = 2\Delta_\phi + \ell \]

\[ \Rightarrow \text{Rigorous bounds for } (A_1, A_2) \]

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[Cornaglito ML Liendo]
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\[ \sim \text{analytic approximation} \]

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→ Get s-channel ($\phi\phi$) large spin

[Cornagliozzo ML Liendo]
1. The Superconformal Bootstrap Program

2. $(A_1, A_2)$ Argyres-Douglas Theory

3. Landscape of $4d \mathcal{N} = 2$ SCFTs

4. Summary & Outlook
Landscape of $4d \mathcal{N} \geq 2$ SCFTs

Projection of space of SCFTs to an axis

$\rightarrow 4d \mathcal{N} = 4$ SCFTs $c = a \geq \frac{3}{4}$ [Beem Rastelli van Rees]

$\rightarrow 4d \mathcal{N} \geq 3$ SCFTs $c = a > \frac{13}{24}$ [Cornaglioitto ML Schomerus]

$\rightarrow 4d \mathcal{N} \geq 2$ SCFTs $c \geq \frac{11}{30}$ [Liendo Ramirez Seo]
Landscape of $4d \, \mathcal{N} \geq 2$ SCFTs

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Finer view of the space of theories:

⇒ Organize theories by flavor symmetry
Landscape of $4d \mathcal{N} \geq 2$ SCFTs

Projection of space of SCFTs to an axis

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Finer view of the space of theories:

$\Rightarrow$ Organize theories by flavor symmetry

$\langle TT \rangle \propto c$, $\langle JJ \rangle \propto k$
4d $\mathcal{N} = 2$ SCFT with $su(2)$ flavor symmetry

- 4d Flavor current supermultiplet

![Graph showing the relationship between $1/k_{4d}$ and $c_{4d}$ for different SCFTs with $su(2)$ flavor symmetry.](image-url)

Only for $su(2)$, $su(3)$, $so(8)$, $g^2$, $f_{4d}$, $e_{6d}$, $e_{8d}$. 15/21
4d $\mathcal{N} = 2$ SCFT with $su(2)$ flavor symmetry

- 4d Flavor current supermultiplet $\mapsto \langle JJJJ \rangle_{2d}$
4d $\mathcal{N} = 2$ SCFT with $su(2)$ flavor symmetry

- 4d Flavor current supermultiplet $\leftrightarrow \langle JJJJ \rangle_{2d} \sim f_{4d}^2 \geq 0$

[Beem ML Liendo Peelaers Rastelli van Rees]
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- 4d Flavor current supermultiplet $\leftrightarrow \langle JJJJ \rangle_{2d} \sim \sum f_{4d}^2 \geq 0$

[Beem ML Liendo Peelaers Rastelli van Rees]
$4d \mathcal{N} = 2$ SCFT with $su(2)$ flavor symmetry

- $4d$ Flavor current supermultiplet $\leftrightarrow \langle JJJJ \rangle_{2d} \sim \sum f_{4d}^2 \geq 0$

- $\langle TTTT \rangle$ & $\langle JJTT \rangle \sim$ distinguishes more operators

Analytically ruled out

assumptions:
unique stress tensor, interacting SCFT

[Beem ML Liendo Peelaers Rastelli van Rees, ML Liendo]
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[Beem ML Liendo Peelaers Rastelli van Rees, ML Liendo]
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- $\langle TTTT \rangle$ & $\langle JJTT \rangle$ $\sim$ distinguishes more operators

Analytically ruled out

Argyres-Douglas SCFT

Only for $su(2)$, $su(3)$, $so(8)$, $g_2$, $f_4$, $e_6$, $e_7$, $e_8$

[Beem ML Liendo Peelaers Rastelli van Rees, ML Liendo]
1 The Superconformal Bootstrap Program

2 \((A_1, A_2)\) Argyres-Douglas Theory

3 Landscape of \(4d \, \mathcal{N} = 2\) SCFTs

4 Summary & Outlook
Constrained the “simplest” Argyres-Douglas theory
Constrained the “simplest” Argyres-Douglas theory

Zoom in to other strongly coupled $\mathcal{N} = 2$ SCFTs?
(at corners of $su(2)$, $su(3)$, $e_6$, $e_7$, $e_8$ exclusion curves)
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**Superblocks for Super-stress tensor multiplets**
Summary & Outlook

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→ Bounds on $(c, k)$ did not come from superprimary of stress tensor – compute whole superblock?
→ Two-dimensional long blocks [Cornagliotto ML Schomerus]
  needed for $c > \frac{13}{24}$ for $\mathcal{N} = 3$ SCFTs
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→ Bounds on $(c, k)$ did not come from superprimary of stress tensor – compute whole superblock?
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  needed for $c > \frac{13}{24}$ for $\mathcal{N} = 3$ SCFTs
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What is the “smallest” $\mathcal{N} = 3$ SCFT?
5. Lorentzian inversion formula for $(A_1, A_2)$

6. Constraining the space of $4d \mathcal{N} = 2$ SCFTs
Lorentzian inversion formula: Superconformal case

**Invert** $\phi\phi$ OPE

→ Same as bosonic inversion, valid for $\ell > 1$
→ Feed in $\bar{\phi}\phi \sim 1 + $ Stress tensor multiplet $+ \ldots$
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**Invert $\bar{\phi}\phi$ OPE**

→ Supersymmetric inversion: valid for $\ell \geq 0$
Lorentzian inversion formula: Superconformal case

Invert $\phi\phi$ OPE

$\Rightarrow$ Same as bosonic inversion, valid for $\ell > 1$

$\Rightarrow$ Feed in $\bar{\phi}\phi \sim 1 + $ Stress tensor multiplet + . . .

Invert $\bar{\phi}\phi$ OPE

$\Rightarrow$ Supersymmetric inversion: valid for $\ell \geq 0$

$\Rightarrow$ Feed in low twist in $t$-channel ($\bar{\phi}\phi$)
Lorentzian inversion formula:
Superconformal case

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**Invert \( \bar{\phi} \phi \) OPE**

- Supersymmetric inversion: valid for \( \ell \geq 0 \)
- Feed in low twist in \( t \)-channel (\( \bar{\phi} \phi \))
  \[ \leftrightarrow \bar{\phi} \phi \sim 1 + \text{Stress tensor multiplet} + \ldots \]
- and in \( u \)-channel (\( \phi \phi \))
  \[ \leftrightarrow \phi \phi \sim \phi^2 + \ldots \]
\[ \phi^2 \sim f_{\phi^2}^2 \phi^2 + f_{C_\ell}^2 C_{\ell>0} + \cdots \]

\[ \Delta = 2\Delta_\phi \quad \Delta = 2\Delta_\phi + \ell \]

[Cornagliotto ML Liendo]
Bounding OPE coefficients

\[ \phi \phi \sim f_{\phi^2}^2 \phi^2 + f_{\delta\ell}^2 C_{\ell>0} + \cdots \]

\[ \Delta = 2\Delta \phi \]

\[ \Delta = 2\Delta \phi + \ell \]

\[ c_{f/\delta\ell} = 4 \]

[Coraglioitto ML Liendo]
A Lorentzian inversion formula

Inverting the $\phi\bar{\phi}$ OPE

→ Supersymmetric inversion: valid for $\ell \geq 0$
→ Only input: $\bar{\phi}\phi \sim 1 +$ Stress tensor multiplet
A Lorentzian inversion formula

Inverting the $\phi \bar{\phi}$ OPE

→ Supersymmetric inversion: valid for $\ell \geq 0$
→ Only input: $\bar{\phi} \phi \sim 1 + $ Stress tensor multiplet
5 Lorentzian inversion formula for \((A_1, A_2)\)

6 Constraining the space of \(4d \mathcal{N} = 2\) SCFTs
Constraining the space of $4d$ $\mathcal{N} = 2$ SCFTs

$su(2)$ flavor symmetry

Analytically ruled out

Numerically ruled out

[Beem, ML, Liendo, Peelaers, Rastelli, van Rees; ML, Liendo]

[Beem, ML, Liendo, Rastelli, van Rees]
Constraining the space of $4d \ N = 2$ SCFTs

$e_6$ flavor symmetry

Numerically ruled out

Ruled out

[Beem, ML, Liendo, Peelaers, Rastelli, van Rees; ML, Liendo]

[Beem, ML, Liendo, Rastelli, van Rees]
Constraining the space of $4d$ $\mathcal{N} = 2$ SCFTs

$su(4)$ flavor symmetry

Analytically ruled out

$[\text{Beem, ML, Liendo, Peelaers, Rastelli, van Rees}; \text{ML, Liendo}]$
Constraining the space of $4d \mathcal{N} = 2$ SCFTs

$su(2)$ flavor symmetry

Analytically ruled out

[Beem, ML, Liendo, Peelaers, Rastelli, van Rees; ML, Liendo]