Role of Complexified Supersymmetric Solutions

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Reference:
M.H., arXiv:1710.05010 [hep-th], to be published in PRL

Title is changed by PRL:
Supersymmetric solutions and Borel singularities for N=2 supersymmetric Chern–Simons theories

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In SUSY theories,

\[ \exists \text{ configurations} \]

which formally satisfy SUSY conditions: \( Q(\text{fields}) = 0 \)
but are not on original path integral contour.
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which formally satisfy SUSY conditions: \( Q(\text{fields}) = 0 \) but are not on original path integral contour.

Let us call them

“Complexified SUSY solutions”

What are their physical roles?
What are physical roles of complexified SUSY solutions?

Our proposal for a part of answers:
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Our proposal for a part of answers:

they provide important information on large order behavior of perturbative series in SUSY QFT
Preparation: Borel resummation

Borel transformation:

\[ \mathcal{O}(g) \simeq \sum_{\ell=0}^{\infty} c_{\ell} g^{\alpha+\ell} \quad \Rightarrow \quad \mathcal{B}\mathcal{O}(t) = \sum_{\ell=0}^{\infty} \frac{c_{\ell}}{\Gamma(\alpha + \ell)} t^{\alpha+\ell-1} \]
Preparation: Borel resummation

Borel transformation:

\[ O(g) \simeq \sum_{\ell=0}^{\infty} c_\ell g^{a+\ell} \]

\[ B O(t) = \sum_{\ell=0}^{\infty} \frac{c_\ell}{\Gamma(a + \ell)} t^{a+\ell-1} \]

Borel resummation (along R+):

\[ S_0 O(g) = \int_0^\infty dt \, e^{-\frac{t}{g}} \, B O(t) \]
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Borel transformation:

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\[ S_0 \mathcal{O}(g) = \int_0^\infty dt \ e^{-\frac{t}{g}} \mathcal{B}\mathcal{O}(t) \]

Locations of Borel singularities determine

\[ \begin{cases} \bullet \text{ Whether or not perturbative series are Borel summable} \\ \bullet \text{ factorial divergence of perturbative series:} \end{cases} \]

\[ \mathcal{B}\mathcal{O}(t) \sim \frac{1}{t-t_0} \quad \Rightarrow \quad c_\ell \sim \frac{\ell!}{t_0^\ell} \]
Proposal

Idea:

Conjecture:
Proposal

Idea:

Bosonic solution \iff Fermionic solution

Conjecture:
Proposal

Idea:

Bosonic solution $\leftrightarrow$ Pole of Borel trans.

Fermionic solution $\leftrightarrow$ Zero of Borel trans.

Conjecture:
Proposal

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Conjecture:

If there are $n_B$ bosonic & $n_F$ fermionic solutions with action $S=S_c/g$, then

(Borel trans.) $\supset$ $\prod_{\text{solutions}} \frac{1}{(t-S_c)^{n_B-n_F}}$ (in the same topological sector)
In poster:

We explicitly check

\[(\text{Borel trans.}) \supset \prod_{\text{solutions}} \frac{1}{(t-S_c)^{n_B-n_F}}\]

for SUSY observables in 3d N=2 SUSY Chern-Simons matter theories
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for SUSY observables in 3d N=2 SUSY Chern-Simons matter theories

- construct $\infty$ complexified SUSY sols. in general 3d N=2 SUSY theory on $S^3$ with Lagrangian

- comparison of their actions & Borel singularities for "1/k-expansion" (=perturbative series by inverse CS level)
In poster:

We explicitly check

\[(\text{Borel trans.) } \supset \prod_{\text{solutions}} \frac{1}{(t - S^c)^{n_B - n_F}}\]

for SUSY observables in 3d N=2 SUSY Chern-Simons matter theories

- construct infinite complexified SUSY sols. in general 3d N=2 SUSY theory on $S^3$ with Lagrangian
- comparison of their actions & Borel singularities for “1/k-expansion” (=perturbative series by inverse CS level)

Related progress not in poster:

- Resurgence
  - [Fujimori-M.H.-Kamata-Misumi-Sakai’18]

- SUSY breaking & complexified SUSY sols.  - [M.H., work in progress]

- Spectrum of complexified SUSY sols. & effective action  - [M.H., work in progress]

Thanks!