Discrete Gauge Anomalies Revisited

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Anomalies in chiral gauge theories

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- A U(1) chiral gauge theory is anomalous if the anomaly cancellation condition

$$\text{Purely gauge}: \sum_{\text{left}} q_L^3 - \sum_{\text{right}} q_R^3 = 0$$

$$\text{Mixed gauge and grav}: \sum_{\text{left}} q_L - \sum_{\text{right}} q_R = 0$$

is not satisfied. Here $\{q_L\}$ and $\{q_R\}$ are U(1) charges of Weyl fermions.

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mention two caveats. First, there are discrete symmetries—those associated with global anomalies—that cannot be consistently gauged. Identification of such anomalies is a difficult but well developed art,[14] into which we shall not enter here. Second, it is not quite true that the identifications we envisage in field space are

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a difficult but well developed art!

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➢ In some cases, we might be able to write down such a theory as

$$\int \sum_i \overline{\psi}_i (i\partial\!\!\!/ + q_i A\!\!\!/)\psi_i + \underline{\frac{in}{2\pi}\int B\wedge dA + \frac{ipn}{4\pi}\int B\wedge B}$$

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➢ There are two kinds of $\mathbb{Z}_n$ chiral gauge theories, depending on the symm of ferm, which can be $\underbrace{\mathrm{Spin}(4) \times \mathbb{Z}_n}_{\text{"untwisted"}}$ or $\underbrace{(\mathrm{Spin}(4) \times \mathbb{Z}_{2m})/\mathbb{Z}_2}_{\text{"twisted"}}$

Previous works

- There have been several attempts to tackle this problem, such as the works by Ibanez-Ross (91), Banks-Dine (91), Csaki-Murayama (97), Araki *et al.* (08), etc.

- Let's review some of these works

Ibanez-Ross

Their argument (only for untwisted $\mathbb{Z}_n$ symm):

U(1) anomaly cancel. cond.

Z_n anomaly cancel. cond. $=$ $+$

charge constraints on massive states through SSB of U(1)

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The result (a *necessary* cond.):

$$\sum_i q_i^3 = pn + r\frac{n^3}{8}, \quad p, r \in \mathbb{Z};\ p \in 3\mathbb{Z} \text{ if } n \in 3\mathbb{Z},$$

$$\sum_i q_i = p'n + r'\frac{n}{2}, \quad p', r' \in \mathbb{Z}.$$

Contribution from Dirac and Majorana masses, respectively

Banks-Dine

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Comments on Ibanez-Ross:

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➢ It can be argued by considering the violation of the low energy Z_n symm in the presence of a **grav instanton** which is a **spin manifold**

- The nonlinear (cubic) constraint might be too restrictive and might not be required for consistency of the low energy theory

➢ It is not solely from the low energy considerations and would depend on assumptions about UV embedding theories

Csaki-Murayama

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- For Type I anomalies, the matching conditions have to be always satisfied *regardless of* the details of the massive bound state spectrum.

- The Type II anomalies have to be also matched *except* if there are **fractionally charged** massive bound states in the theory.

Our approach

- Here we revisit this problem from a more modern perspective based on the concept of symmetry protected topological (SPT) phases (from condensed matter physics) and also from a refined definition of *global anomalies* by Witten (2016)

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- Our approach is based on *geometrical* and *topological* considerations

➢ We compute the 't Hooft anomaly of $\mathbb{Z}_n$ (global) symm, deduced by the consistency of formulating the theory on a generic manifold w/ a **untwisted/twisted** spin structure and a background $\mathbb{Z}_n$ field

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$$(n^2 + 3n + 2) \sum_i s_i^3 = 0 \mod 6n$$

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$(\mathrm{Spin}(4)\times\mathbb{Z}_{2m})/\mathbb{Z}_2$ $\quad \tilde{s}_i \in 2\mathbb{Z}+1$

$$(2m^2+m+1)\sum_i \tilde{s}_i^3 - (m+3)\sum_i \tilde{s}_i = 0 \mod 48m$$

$$\sum_i \tilde{s}_i = 0 \mod 2m$$

Conclusion

- We propose a new formula for evaluating the anomalies (and the corresponding cancel. cond.) of an underlying chiral gauge theory.

- While agreeing with previous works by Ibanez and Ross and by Csaki and Murayama using anomaly matching argument, our result provides, from a **purely low-energy perspective**, a more complete aspect of discrete symmetry anomalies, respecting the viewpoint in the work of Banks and Dine.

Thank You!