Discrete Gauge Anomalies Revisited

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Anomalies in chiral gauge theories

- Cancellation of gauge anomalies – in a chiral theory such as the *standard model* – is a fundamental constraint on a consistent quantum field theory.
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• Cancellation of gauge anomalies – in a chiral theory such as the standard model – is a fundamental constraint on a consistent quantum field theory.

• A U(1) chiral gauge theory is anomalous if the anomaly cancellation condition

\[
\text{Purely gauge : } \sum_{\text{left}} q_L^3 - \sum_{\text{right}} q_R^3 = 0
\]

\[
\text{Mixed gauge and grav : } \sum_{\text{left}} q_L - \sum_{\text{right}} q_R = 0
\]

is not satisfied. Here \(q_L\) and \(q_R\) are U(1) charges of Weyl fermions.
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Discrete Gauge Symmetry in Continuum Theories

Lawrence M. Krauss\(^{(a)}\) and Frank Wilczek\(^{(b)}\)

mention two caveats. First, there are discrete symmetries—those associated with global anomalies—that cannot be consistently gauged. Identification of such anomalies is a difficult but well developed art,\(^{14}\) into which we shall not enter here. Second, it is not quite true that the identifications we envisage in field space are
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  ➢ In some cases, we might be able to write down such a theory as

$$\int \sum_i \bar{\psi}_i (i\partial + q_i A) \psi_i + \frac{in}{2\pi} \int B \wedge dA + \frac{ipn}{4\pi} \int B \wedge B$$

[Kapustin-Seiberg 14]
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[Kapustin-Seiberg 14]

- There are two kinds of $\mathbb{Z}_n$ chiral gauge theories, depending on the symm of ferm, which can be $\text{Spin}(4) \times \mathbb{Z}_n$ or $(\text{Spin}(4) \times \mathbb{Z}_{2m})/\mathbb{Z}_2$

  “untwisted”

  “twisted”
Previous works

• There have been several attempts to tackle this problem, such as the works by Ibanez-Ross (91), Banks-Dine (91), Csaki-Murayama (97), Araki et al. (08), etc.

• Let’s review some of these works
Ibanez-Ross

Their argument (only for untwisted $\mathbb{Z}_n$ symm):

\[ Z_n \text{ anomaly cancel. cond.} = U(1) \text{ anomaly cancel. cond.} + \text{charge constraints on massive states through SSB of U(1)} \]
Ibanez-Ross

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charge constraints on massive states through SSB of $U(1)$

The result (a necessary cond.):

\[ \sum_i q_i^3 = pn + r \frac{n^3}{8}, \quad p, r \in \mathbb{Z}; \ p \in 3\mathbb{Z} \text{ if } n \in 3\mathbb{Z}, \]

\[ \sum_i q_i = p'n + r' \frac{n}{2}, \quad p', r' \in \mathbb{Z}. \]

Contribution from Dirac and Majorana masses, respectively
Banks-Dine

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Comments on Ibanez-Ross:

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- It can be argued by considering the violation of the low energy $Z_n$ symm in the presence of a grav instanton which is a spin manifold

- The nonlinear (cubic) constraint might be too restrictive and might not be required for consistency of the low energy theory

- It is not solely from the low energy considerations and would depend on assumptions about UV embedding theories
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Argument by ’t Hooft anomaly matching. Two types of discrete anomalies are involved:

- For **Type I anomalies**, the matching conditions have to be always satisfied *regardless of* the details of the massive bound state spectrum.

- The **Type II anomalies** have to be also matched *except* if there are *fractionally charged* massive bound states in the theory.
Our approach

• Here we revisit this problem from a more modern perspective based on the concept of symmetry protected topological (SPT) phases (from condensed matter physics) and also from a refined definition of global anomalies by Witten (2016)
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• Our approach is based on geometrical and topological considerations

➢ We compute the ’t Hooft anomaly of $\mathbb{Z}_n$ (global) symm, deduced by the consistency of formulating the theory on a generic manifold w/ a untwisted/twisted spin structure and a background $\mathbb{Z}_n$ field
Main result

- The anomaly cancel. cond. *(necessary & sufficient)* of untwisted/twisted $\mathbb{Z}_n$ symm:
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\[ \text{Spin}(4) \times \mathbb{Z}_n \quad s_i \in \mathbb{Z} \]

\[ (n^2 + 3n + 2) \sum_i s_i^3 = 0 \mod 6n \]

\[ 2 \sum_i s_i = 0 \mod n \]
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\[
\text{(Spin}(4) \times \mathbb{Z}_{2m})/\mathbb{Z}_2 \quad \tilde{s}_i \in 2\mathbb{Z} + 1
\]

\[
(2m^2 + m + 1) \sum_i \tilde{s}_i^3 - (m + 3) \sum_i \tilde{s}_i = 0 \mod 48m
\]

\[
\sum_i \tilde{s}_i = 0 \mod 2m
\]
Conclusion

• We propose a new formula for evaluating the anomalies (and the corresponding cancel. cond.) of an underlying chiral gauge theory.

• While agreeing with previous works by Ibanez and Ross and by Csaki and Murayama using anomaly matching argument, our result provides, from a purely low-energy perspective, a more complete aspect of discrete symmetry anomalies, respecting the viewpoint in the work of Banks and Dine.
Thank You!