SCFTs, Compact CY 3-folds, and Topological Strings

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(to appear) in collaboration with: Hirotaka Hayashi, Hee-Cheol Kim, Kantaro Ohmori, and Cumrun Vafa

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Since 6d/5d theories can be engineered by compactifying F/M theory on singular 3-folds X, the basic insight is that the relationship between SCFTs and SUGRA can be interpreted as a relationship between 3-folds:

$$
\begin{array}{ccc}
\text{SCFT} & \hookrightarrow & \text{SUGRA} \\
\updownarrow & & \updownarrow \\
X_{\text{non-compact}} & \hookrightarrow & X_{\text{compact}}
\end{array}
$$

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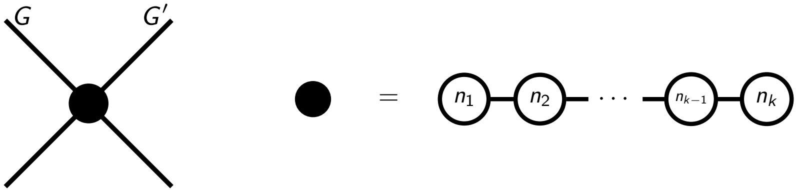
6d SCFTs called (G, G') conformal matter theories (with global symmetry subgroup $G \times G'$) have an orbifold realization in terms of **non-compact** 3-folds $T^2 \times \mathbb{C}^2/\mathbb{Z}_m \times \mathbb{Z}_n$:



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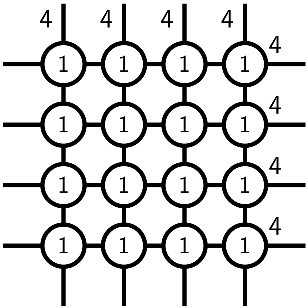
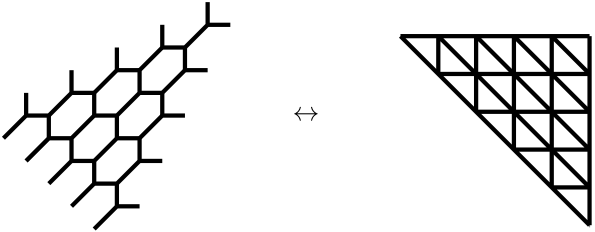
Example: $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ contains 16 copies of the (D_4, D_4) conformal matter theory. The non-compact curves $\mathbb{C}$ carrying D_4 global symmetry are compactified into $\mathbb{P}^1$'s with self-intersection -4:



Next, let's consider 5d theories. 5d SCFTs are associated non-elliptic 3-folds, such as toric 3-folds. The 5d T_5 theory, associated to the **non-compact** 3-fold $\mathbb{C}^3/\mathbb{Z}_5 \times \mathbb{Z}_5$, can be represented (in a particular Coulomb phase) as



Note T_5 theory has global symmetry $SU(5)^3$.

Toric singularities naturally appear in mirror Fermat hypersurfaces

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An example we study is the mirror quintic 3-fold $\mathbb{P}^4[5]/\mathbb{Z}_5^3$, whose singularities consist of 10 lines of $SU(5)$ singularities meeting triple-wise in 10 singular points T_{ijk} with normal geometry $\mathbb{C}^3/\mathbb{Z}_5^2$.

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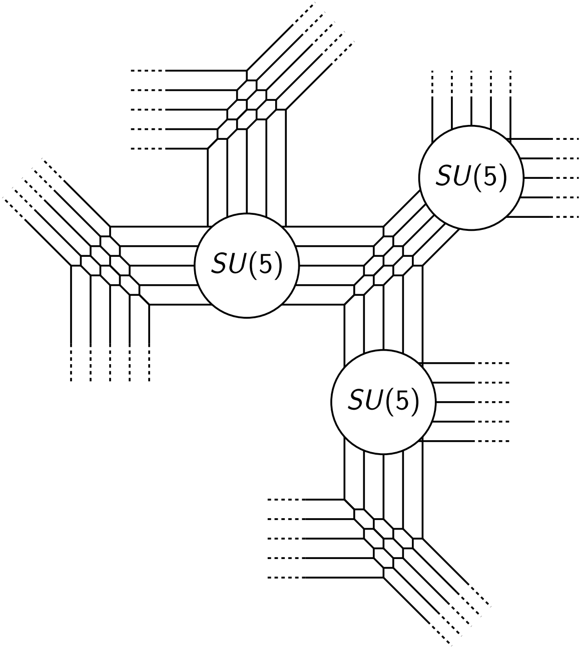
Locally, these points are T_5 theories, but their arrangement in the mirror quintic means their global $SU(5)$ symmetries are **gauged**:


SU(5)
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1. **Holography**. We use our description of $T^6/\mathbb{Z}_2^2$ to propose a 2d $\mathcal{N} = (0,4)$ quiver holographically dual to type IIB on $\mathrm{AdS}_3 \times S^3 \times T^4/\mathbb{Z}_2^2$.
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2. **Topological string partition function**. Observing that

$$Z_{\text{top}}(\tau, t, \lambda) = Z_{\text{BPS}}^{\text{5d}} = Z_{\text{BH}} = Z_0(\tau, \lambda) \sum_C Z_C(\tau, \lambda) e^{-t\cdot C}$$

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3. We make some progress towards generalizing the topological vertex to $SU(5)$ gaugings, which in principle permits computation of **topological string amplitudes** for the **mirror quintic**.

Thank you!