Light-ray operators in conformal field theory

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Motivation

Conformal partial wave expansion for a scalar four-point function

\[ \langle \phi_1 \phi_2 \phi_3 \phi_4 \rangle = \sum_J \int \frac{d\Delta}{2\pi i} C(\Delta, J) \ G_{\Delta, J}(x_i) \]

\[ C(\Delta, J) \sim \frac{-f_{12k} f_{34k}}{\Delta - \Delta_k} \]

Lorentzian inversion formula [Caron-Huot '17]

\[ C(\Delta, J) = \frac{\kappa \Delta^+ + J}{4} \int d^2 \mu(z, \bar{z}) \ G_{J+d-1, \Delta-d+1}(z, \bar{z}) \langle 0 | [\phi_4, \phi_1][\phi_2, \phi_3] | 0 \rangle \]

- Defines an analytic continuation of \( C(\Delta, J) \) in spin \( J \)
- \( J \notin \mathbb{Z}_{\geq 0} \) is physically meaningful, e.g. Regge behavior
Questions

- Do continuous-spin operators even make sense?
- Can the analytic continuation be lifted to operator level?
- Can we generalize the Lorentzian inversion formula to four-pt functions of operators with spin? (What's up with the funny block?)
- Can conformal Regge theory be phrased in terms of continuous-spin operators?
- Integer-spin operators on leading twist Regge trajectory have ANEC-like positivity properties. Does this also hold for continuous-spin operators?
Answers

- Do continuous-spin operators even make sense?
  - Yes: $\mathcal{O}(x, z)$ is a homogeneous function of null $z$ of degree $J$.
  - Lemma: if $J \notin \mathbb{Z}_{\geq 0}$ then $\mathcal{O}(x, z)|0\rangle = 0$.
  - Corollary: $\mathcal{O}(x, z)$ are necessarily non-local.
Answers

- Do continuous-spin operators even make sense?
  - Yes: $\mathcal{O}(x, z)$ is a homogeneous function of null $z$ of degree $J$.
  - Lemma: if $J \notin \mathbb{Z}_{\geq 0}$ then $\mathcal{O}(x, z)|0\rangle = 0$.
  - Corollary: $\mathcal{O}(x, z)$ are necessarily non-local.

- Can the analytic continuation be lifted to operator level?
  - No for local operators, $\mathcal{O}(x, z)|0\rangle \neq 0$.
  - Yes for light transforms of local operators, $L[\mathcal{O}](x, z)|0\rangle = 0$.
    - Light transform is a conformally-invariant integral transform [Knapp-Stein ‘71], $L \in \mathcal{D}_8$.
    - $L[\mathcal{O}]$ is a primary of dimension $1 - J$, spin $1 - \Delta$.
    - $L[T]$ is the ANEC operator.

- Continuous-spin operators can be constructed as residues

$$\mathcal{O}_J(x, z) = \text{res}_\Delta \int d^d x_1 d^d x_2 K_{\Delta, J}(x, z; x_1, x_2) \mathcal{O}_1(x_1) \mathcal{O}_2(x_2)$$

They come from light ray or at least light cone.

- By construction they reduce to light transforms

$$\mathcal{O}_J = L[\mathcal{O}_J], \quad J \in \mathbb{Z}_{\geq 0}. \quad (1)$$
Summary

- Analytic continuation in spin at operator level
- Natural generalization of Lorentzian inversion formula

\[ C(\Delta, J) = \frac{-1}{2\pi i} \int [dx_i] \langle 0 | [O_4, O_1][O_2, O_3]|0 \rangle \]
\[ \times \frac{\langle O_1 O_2 L[O] \rangle^{-1} \langle O_3 O_4 L[O] \rangle^{-1}}{\langle L[O]L[O] \rangle^{-1}} \]

- Interpretation of conformal Regge theory as an expansion in light-ray operators

\[ \langle O_1 O_2 O_3 O_4 \rangle \sim \int dJ \int d\Delta \frac{C(\Delta, J) \langle O_1 O_2 L[O] \rangle \langle O_3 O_4 L[O] \rangle}{2\pi i 1 - e^{-2\pi i J} \langle L[O]L[O] \rangle}, \]

- Positivity constraints for the entire leading Regge trajectory (including ANEC at \( J = 2 \))

\[ \langle \Psi | O_J | \Psi \rangle \geq 0, \quad (J \geq J_0). \]