Light-ray operators in conformal field theory

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Motivation

Conformal partial wave expansion for a scalar four-point function

$$\langle \phi_1 \phi_2 \phi_3 \phi_4 \rangle = \sum_J \int \frac{d\Delta}{2\pi i} C(\Delta, J)\ G_{\Delta,J}(x_i)$$

$$C(\Delta, J) \sim \frac{-f_{12k} f_{34k}}{\Delta - \Delta_k}$$

normal conformal block

Lorentzian inversion formula [Caron-Huot '17]

$$C(\Delta, J) = \frac{\kappa_{\Delta+J}}{4} \int d^2\mu(z,\bar{z})\ G_{J+d-1,\Delta-d+1}(z,\bar{z})\ \langle 0|[\phi_4,\phi_1][\phi_2,\phi_3]|0\rangle$$

funny conformal block

- Defines an analytic continuation of $C(\Delta, J)$ in spin J
- $J \notin \mathbb{Z}_{\geq 0}$ is physically meaningful, e.g. Regge behavior

Questions

- Do continuous-spin operators even make sense?
- Can the analytic continuation be lifted to operator level?
- Can we generalize the Lorentzian inversion formula to four-pt functions of operators with spin? (What's up with the funny block?)
- Can conformal Regge theory be phrased in terms of continuous-spin operators?
- Integer-spin operators on leading twist Regge trajectory have ANEC-like positivity properties. Does this also hold for continuous-spin operators?

Answers

- Do continuous-spin operators even make sense?
  - Yes: $\mathbb{O}(x, z)$ is a homogeneous function of null z of degree J.
  - Lemma: if $J \notin \mathbb{Z}_{\geq 0}$ then $\mathbb{O}(x, z)|0\rangle = 0$.
  - Corollary: $\mathbb{O}(x, z)$ are necessarily non-local.

Answers

- Do continuous-spin operators even make sense?
  - Yes: $\mathbb{O}(x,z)$ is a homogeneous function of null z of degree J.
  - Lemma: if $J \notin \mathbb{Z}_{\geq 0}$ then $\mathbb{O}(x,z)|0\rangle = 0$.
  - Corollary: $\mathbb{O}(x,z)$ are necessarily non-local.
- Can the analytic continuation be lifted to operator level?
  - No for local operators, $\mathcal{O}(x,z)|0\rangle \neq 0$.
  - Yes for light transforms of local operators, $\mathbf{L}[\mathcal{O}](x,z)|0\rangle = 0$.
    - Light transform is a conformally-invariant integral transform [Knapp-Stein '71], $\mathbf{L} \in \mathrm{D}_8$.
    - $\mathbf{L}[\mathcal{O}]$ is a primary of dimension $1-J$, spin $1-\Delta$.
    - $\mathbf{L}[T]$ is the ANEC operator.
  - Continuous-spin operators can be constructed as residues

$$\mathbb{O}_J(x,z) = \mathrm{res}_\Delta \int d^d x_1 d^d x_2 K_{\Delta,J}(x,z;x_1,x_2)\mathcal{O}_1(x_1)\mathcal{O}_2(x_2)$$

    They come from light ray or at least light cone.
  - By construction they reduce to light transforms

$$\mathbb{O}_J = \mathbf{L}[\mathcal{O}_J], \qquad J \in \mathbb{Z}_{\geq 0}. \tag{1}$$

Summary

- Analytic continuation in spin at operator level
- Natural generalization of Lorentzian inversion formula

$$C(\Delta, J) = \frac{-1}{2\pi i} \int [dx_i] \langle 0 | [\mathcal{O}_4, \mathcal{O}_1][\mathcal{O}_2, \mathcal{O}_3] | 0 \rangle \times \frac{\langle \mathcal{O}_1 \mathcal{O}_2 \mathbf{L}[\mathcal{O}] \rangle^{-1} \langle \mathcal{O}_3 \mathcal{O}_4 \mathbf{L}[\mathcal{O}] \rangle^{-1}}{\langle \mathbf{L}[\mathcal{O}] \mathbf{L}[\mathcal{O}] \rangle^{-1}}$$

- Interpretation of conformal Regge theory as an expansion in light-ray operators

$$\langle \mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_3 \mathcal{O}_4 \rangle \sim \oint dJ \oint \frac{d\Delta}{2\pi i} \frac{C(\Delta, J)}{1 - e^{-2\pi i J}} \frac{\langle \mathcal{O}_1 \mathcal{O}_2 \mathbf{L}[\mathcal{O}] \rangle \langle \mathcal{O}_3 \mathcal{O}_4 \mathbf{L}[\mathcal{O}] \rangle}{\langle \mathbf{L}[\mathcal{O}] \mathbf{L}[\mathcal{O}] \rangle},$$

- Positivity constraints for the entire leading Regge trajectory (including ANEC at $J = 2$)

$$\langle \Psi | \mathbb{O}_J | \Psi \rangle \geq 0, \qquad (J \geq J_0).$$