A thrilling time to study black holes

• LIGO has ushered in a new era of data collection.

• Black hole physics is now an experimental science!

• The Event Horizon Telescope will soon deliver the first up-close pictures of two black holes: the one at the center of our Galaxy (Sagittarius A*) and the other in M87.

• This also poses a pressing challenge to theorists! Question: What will the images look like?

Difficult to predict…
Astrophysics is hard…

• Black holes are typically surrounded by an accretion disk and a plasma magnetosphere, and may also have a corona, jets, etc.

• Each of these ingredients obeys messy physics.

• The traditional approach is to run computer simulations.

• Detailed simulations are time-consuming and costly.

• They can be incomplete or altogether fail in a special corner of parameter space: high spin black holes.
...but symmetries are powerful

- A maximally rotating black hole saturates the Kerr bound: $|J| \leq M^2$

- Near extremality, the spacetime geometry near the event horizon develops an infinitely deep throat.

- In this regime, numerical studies are confronted with a separation of scales: resolving the near-horizon physics requires a large grid with an increasingly fine resolution.

- This also implies the existence of a perturbative expansion in a small parameter: analytics and numerics are complimentary.

- Here, the small parameter is the Hawking temperature, which vanishes at extremality.
Application of an old tool (conformal symmetry) to a new arena (black hole astrophysics).

• At this critical temperature, the region near the horizon develops local scale invariance, or conformal symmetry:
  
  • The global conformal group is geometrically realized in this “gravitating box” by the emergence of isometries that enhance time translation symmetry to $\text{SL}(2,\mathbb{R})$.

  • The infinite-dimensional local extension arises from an asymptotic symmetry group analysis (like in AdS$_3$) and relates black holes of different small temperatures.

• This observation suggests that physics near rapidly spinning black holes should be constrained by conformal symmetry in much the same way as many near-critical condensed matter systems are governed by conformal field theories.

• Using this insight, we can greatly reduce the number of parameters, leading to striking displays of universal behavior.
A prediction for the polarization from M87 from conformal symmetry

• Guided by this symmetry, we analytically compute the polarization profile of near-horizon emissions from a high spin black hole.

• This result is directly relevant to ongoing observations at the EHT of the black hole at the center of M87.

• Our prediction relies only on three assumptions:

  1. The black hole is rapidly spinning: |J| ∼ M².

  2. The EHT can observe near-horizon emissions.

  3. The polarization profile at the source is invariant under the enhanced isometries of the near-horizon geometry.
Symmetry assumption

• It is natural to expect that a spacetime imparts its symmetries to the physics that it drives. *For instance, accretion onto a black hole is driven by gravity and as a result, slowly-accreting matter disks tend to relax to a stationary and axisymmetric state.*

• We therefore assume that the source polarization is invariant under the local symmetries, *i.e.*, the source polarization vector field $f^\mu$ commutes with the four Killing vector fields (stationarity, axisymmetry, dilation invariance & special conformal invariance) that generate the isometries of the near-horizon geometry.

• Because the polarization $f^\mu$ must also obey parallel transport along outgoing geodesics, this is an overconstrained problem.

• Surprisingly, a solution nonetheless exists and moreover, it is unique!
Polarimetric images

• A backlit black hole absorbs all incident light up to some impact parameter and therefore casts a shadow upon an observer’s screen.

• Because gravity bends light, this silhouette depends on the polar angle $\theta_o$ of the distant observer.

• All the near-horizon emissions turn out to be confined to the shadow.

• I will now show you the following three cases:

  1. “Face-on” observer on the rotation axis ($\theta_o = 0^\circ$).

  2. “Edge-on” observer in the equatorial plane ($\theta_o = 90^\circ$).

  3. The intermediate case $\theta_o \sim 15^\circ$ of relevance to M87.
Face-on: The Polarization Whorl

$\beta_o$

$\alpha_o$
Edge-on: Double Whorl
\[ F_0 \propto A(R_o, \theta_o, \beta, g) = \frac{\sqrt{3} M g^3}{\bar{g} |\beta| R_0^2 \sin \theta_o} \left( \frac{q^3}{1 + q \partial_q \log G_\theta} \right) \]

\[ \bar{g} = \sqrt{(4q^2 - 15)g^2 + 2\sqrt{3}g + 3}, \]

\[ q = \sqrt{3 - \beta^2 + \cos^2 \theta_o - 4 \cot^2 \theta_o}, \]

\[ G_\theta = \frac{\pi}{\sqrt{Q_+}} _2 F_1 \left( \frac{1}{2}, \frac{1}{2}, 1, \frac{Q_-}{Q_+} \right), \]

\[ Q_\pm = 3 - \frac{q^2}{2} \pm \sqrt{12 - (2q)^2 + \left( \frac{q^2}{2} \right)^2}. \]
Down the barrel: M87
1) The near-horizon geometry of a high spin black hole is a gravitating box, like AdS. Unlike AdS, it is realized in nature, and the EHT could detect its signature!

2) The maximally spinning black hole is an example of critical point in astronomy. Emergent (infinite-dimensional) conformal symmetry.

The analogue of the divergent correlation length is the throat’s depth.

3) The critical near-horizon behavior is universal. The leading behavior near extremality is fixed by conformal symmetry: power law behavior, with critical exponent fixed by the rank of the field.
Thank you

And stay tuned for the images!
Application of an old tool (conformal symmetry) to a new arena (black hole astrophysics).

- Fields in a (Near-)Extreme Kerr background (NHEK) necessarily develop self-similarity in the near-horizon limit:
  \[ x^\mu \rightarrow \lambda x^\mu \quad F \rightarrow \lambda^h F \]

- Stationary, axisymmetric fields have a surprisingly simple limiting behavior in NHEK (here, \( H_+ = \partial_t \) and \( h \) is the conformal weight):
  \[ L_{H_+} F = 0 \quad L_{H_0} F = h F \quad L_{\partial_\phi} F = 0 \]
  i.e., they form highest-weight representations of SL(2,R).

- Hence, complicated dynamics near rapidly spinning black holes reduces to a tractable representation theory problem in NHEK!
**Powerful tool** with **practical applications to near-horizon physics**

\[ x^\mu \rightarrow \lambda x^\mu \quad F \rightarrow \lambda^h F \quad L_{H_0}F = h F \]

- The power \( h \) is the **conformal weight**. It labels representations and also dictates their relative importance to physics near the critical point.

- \( F \sim r^h \) so the **smaller weights contribute more**.

- If \( F \) is a tensor field of rank \( p \), then \( h \geq -p \).

- The **leading behavior** is obtained by isolating the piece with the smallest allowed weight. Subleading terms are **power-suppressed** by their less relevant weight.
Scaling symmetry, self-similarity and power laws with critical exponents

- This discussion exactly parallels the classification of IR fixed points in terms of operator scaling dimensions, familiar from the study of critical phenomena in condensed matter systems or quantum field theory.

- In the astrophysical context, the infrared limit arises geometrically from the scaling to a region of infinite redshift near the horizon.

- Extremal black holes therefore provide examples of critical conformal fixed points in astronomy, with the conformal weights playing the role of critical exponents.