Dualities and Dynamics in 2+1 Dimensions

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Strings 2018

25–29 June

Progress on IR dynamics of certain gauge theories in 2+1 dimensions

- with/without Chern-Simons interactions
- with scalar and/or fermionic matter, mostly in fundamental rep.

Relativistic QFT in the continuum

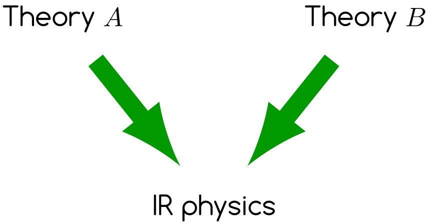
* Utilized in many condensed matter problems:

- quantum phase transitions of spin liquids and quantum magnets
- high-T_c superconductors
- edge modes of topological insulators
- half-filled Landau level

* *Engineer* QED with CS interactions and fermions on graphene films

[Lee, Wang, Zalatel, Vishwanath, He 18]

Progress driven by (conjectural) IR dualities (in HE sense)



Particularly powerful for IR fixed points

* Important role played by ('t Hooft) anomalies for global symmetries

Particle / vortex duality [Peskin 78; Dasgupta, Halperin 81]

$O(2)$ vector model	gauged $O(2)$ vector model
$\mathcal{L} = |\partial\phi|^2 + |\phi|^4 + m^2|\phi|^2$	$\mathcal{L} = |F_{\mu\nu}|^2 + |D_a\widetilde{\phi}|^2 + |\widetilde{\phi}|^4 - m^2|\widetilde{\phi}|^2$
	$U(1)_0$ with $\widetilde{\phi}$

Simple, but with lots of beautiful physics:

gapped & broken phases	$\leftrightarrow$	Higgsed & free-photon phases
perturbative excitations ($m^2 > 0$)	$\leftrightarrow$	(finite energy) vortices
fundamental field ϕ	$\leftrightarrow$	monopole operator $\mathfrak{M}$
$U(1)$ symmetry	$\leftrightarrow$	$U(1)$ magnetic symmetry
$O(2)$ Wilson-Fisher fixed point	$\leftrightarrow$	IR CFT

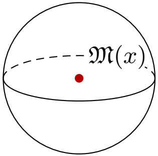
Lattice Monte Carlo: [Nguyen, Sudbø 99; Kajantie, Laine, Neuhaus, Rajantie, Rummukainen 04]

Monopole operators

Local disorder operators: defined by boundary conditions in path-integral

['t Hooft 78; Borokhov, Kapustin, Wu 02]

CFT: radial quantization [Chester, Dyer, Iliesiu, Mezei, Pufu, Radicevic, Sachdev, ...]



- Charged under magnetic symmetry: $\widehat{\pi_1(G_{\text{gauge}})}$

E.g.: $U(1)_{\text{gauge}}$ $\Rightarrow$ Magnetic (topological) symmetry $U(1)_M$

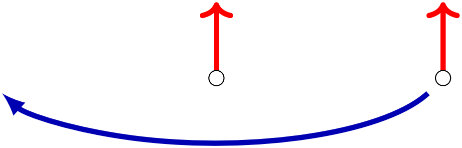
$$J_\mu = \epsilon_{\mu\nu\rho}F^{\nu\rho}$$

$SO(N \geq 3)$ $\Rightarrow$ $\mathbb{Z}_2$

- Semiclassical monopoles can have electric charges

Dressing by matter field zero-modes $\Rightarrow$ spin and global quantum numbers

Flux attachment



Attaching one unit of magnetic flux to particles, their statistics changes

[Wilczek 82; Polyakov 88; Zhang, Hansson, Kivelson 88; Jain 89; Shaji, Shankar, Sivakumar 90; Paul, Shankar, Sivakumar 91; Fradkin, Lopez 91; Chen, Fisher, Wu 93; Fradkin, Schaposnik 94; ...]

Realized through Chern-Simons interactions

$$\mathcal{L} = \frac{1}{4\pi} ada + \left| D_a \phi \right|^2$$

- Semiclassical "bare" monopole $\mathfrak{M}_{\text{bare}}$ is not gauge invariant
- Gauge-invariant monopole $\mathfrak{M} = $ "$\mathfrak{M}_{\text{bare}}\phi$" has spin $\frac{1}{2}$

Particle/vortex dualities with fermions

$U(1)_1$ with ϕ | free Dirac ψ

$$\mathcal{L} = \frac{1}{4\pi} ada + |D_a\phi|^2 + |\phi|^4 \quad \longleftrightarrow \quad \mathcal{L} = \bar{\psi}\partial\!\!\!/\psi$$

$U(1)_{\frac{1}{2}}$ with ψ | $O(2)$ vector model

$$\mathcal{L} = \frac{1}{4\pi} ada + \bar{\psi} D\!\!\!\!/_a \psi \quad \longleftrightarrow \quad \mathcal{L} = |\partial\phi|^2 + |\phi|^4$$

Also fermion/fermion duality: relevant for half-filled Landau level

[Barkeshli, McGreevy 12; Son 15; Wang, Senthil 15; Metlitski, Vishwanath 15; Mross, Alicea, Motrunich 15; Mulligan, Raghu, Fisher 16; Karch, Tong 16; Seiberg, Senthil, Wang, Witten 16; Murugan, Nastase 16]

Vector models at large N, k

Singlet sector of critical $O(N)$ model	$\xleftarrow{\tilde{\lambda} = \frac{k}{N}\ ,\ \ O(k)_{-N}}$ $\xrightarrow[O(N)_k\ ,\ \ \lambda = \frac{N}{k}]{}$	Singlet sector of k free fermions

- High-spin symmetry (large N): conserved currents $J^{(s)}$ with $s = 2, 4, 6, \ldots$

$$J^{(s)} = \varphi^i \partial^s \varphi^i + \ldots \qquad\qquad J^{(s)} = \psi^a \gamma \partial^{s-1} \psi^a + \ldots$$

  All primary operators are products of $J^{(s)}$'s and $J^{(0)} = \varphi^i \varphi^i$ or $\psi^a \psi^a$

- Parity-breaking deformation: couple to Chern-Simons gauge theory

- Duality: identify the two families with $\tilde{\lambda} = 1/\lambda$

[Giombi, Minwalla, Prakash, Trivedi, Wadia, Yin 11; Aharony, Gur-Ari, Yacoby 12]

- Spectrum of primaries is independent of λ (at large N)

  [Giombi, Minwalla, Prakash, Trivedi, Wadia, Yin 11; Aharony, Gur-Ari, Yacoby 12]

- Correlation functions (of single-trace ops): 3 conformal structures

$$\langle J^{(s_1)} J^{(s_2)} J^{(s_3)} \rangle = \alpha_{s_1 s_2 s_3} T_{\text{bos}} + \beta_{s_1 s_2 s_3} T_{\text{fer}} + \gamma_{s_1 s_2 s_3} T_{\text{odd}}$$

  α, β, γ fixed by high-spin symmetry, in terms of two parameters

  [Maldacena, Zhiboedov 11; 12]

$$c_1, c_2(N, k)$$

  Fix by direct computation. [Aharony, Gur-Ari, Yacoby 12; Gur-Ari, Yacoby 12]

- Thermal partition functions (*also away from fixed point*)

  [Giombi, Minwalla, Prakash, Trivedi, Wadia, Yin 11; Aharony, Giombi, Gur-Ari, Maldacena, Yacoby 12; Jain, Minwalla, Sharma, Takimi, Wadia, Yokoyama 13; Choudhury, Dey, Halder, Jain, Janagal, Minwalla, Prabhakar 18]

- Large N: blind to many details

3D dualities among vector models

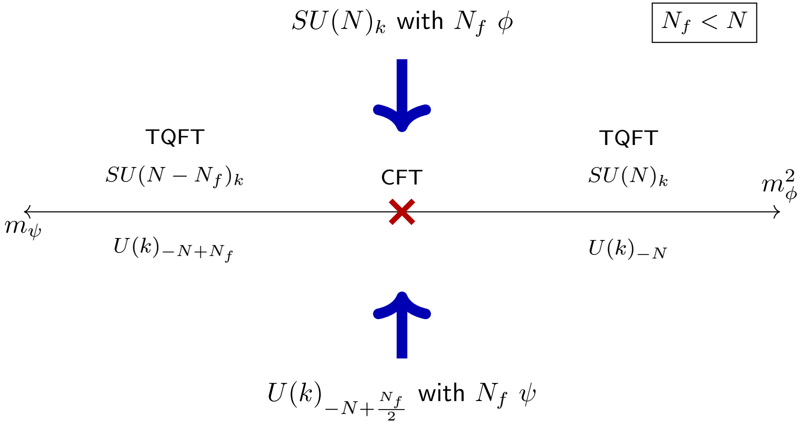
IR dualities between Chern-Simons gauge theories with matter in fundamental rep

[Aharony 15; Hsin, Seiberg 16; Aharony, FB, Hsin, Seiberg 16]

scalars ϕ with $|\phi|^4$ interactions		fermions ψ	
$SU(N)_k$ with N_f ϕ	$\longleftrightarrow$	$U(k)_{-N+\frac{N_f}{2}}$	with N_f ψ
$U(N)_k$ with N_f ϕ	$\longleftrightarrow$	$SU(k)_{-N+\frac{N_f}{2}}$	with N_f ψ
$U(N)_{k,k\pm N}$ with N_f ϕ	$\longleftrightarrow$	$U(k)_{-N+\frac{N_f}{2},-N\mp k+\frac{N_f}{2}}$	with N_f ψ
$USp(2N)_k$ with N_f ϕ	$\longleftrightarrow$	$USp(2k)_{-N+\frac{N_f}{2}}$	with N_f ψ
$SO(N)_k$ with N_f $\phi_{\mathbb{R}}$	$\longleftrightarrow$	$SO(k)_{-N+\frac{N_f}{2}}$	with N_f $\psi_{\mathbb{R}}$

valid for N_f less than a bound ($\lesssim N$)

Conjecturally, these theories have only *one* phase transition



Gapped phases: match via *level-rank duality*

Conjecture: 2nd order phase transition

Bosonization of free fermions

$U(N)_1$ with N_f ϕ

↓

N_f free Dirac ψ

$(N_f \leq N)$

[Aharony 15; Seiberg, Senthil, Wang, Witten 16; Karch, Tong 16; Hsin, Seiberg 16]

$SO(N \geq 3)_1$ with N_f $\phi_{\mathbb{R}}$

↓

N_f free Majorana $\psi_{\mathbb{R}}$

$(N_f \leq N - 2)$

[Metlitski, Vishwanath, Xu 16; Aharony, FB, Hsin, Seiberg 16]

Fermionization of Wilson-Fisher scalars

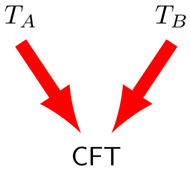
$SO(k \geq 3)_{-\frac{1}{2}}$ with $\psi_{\mathbb{R}}$ $\longleftrightarrow$ $O(1)$ (Ising) vector model

$U(k)_{-\frac{1}{2}}$ with ψ $\longleftrightarrow$ $O(2)$ vector model

IR quantum-enhanced symmetries

Symmetries can enhance at IR fixed points

* Sometimes dualities make this manifest



- T_B can have larger symmetry than T_A
- The symmetry groups of T_A and T_B might not commute

$$[G_A, G_B] \neq 0 \quad \Rightarrow \quad G_{\mathsf{IR}} \supset G_A, G_B$$

Caveat: assume IR fixed point with no symmetry breaking

∗ E.g.: QED with one fermion

$U(1)_{\frac{3}{2}}$ with 1 ψ UV: $G = O(2)$

$SU(2)_{-1}$ with 1 ψ UV: $G = SO(3)$

CFT with $SO(3)$ symmetry

Basic monopole operators $\mathfrak{M}^{\pm}$: spin 1

IR conserved currents $O(2) \to SO(3)$

[Aharony, FB, Hsin, Seiberg 17]

∗ E.g.: QED $U(1)_0$ with 2 ψ

self-duality

$Pin(2)^-$

$\cup$

UV: $\dfrac{Pin(2)^- \times SU(2)}{\mathbb{Z}_2}$

IR: $O(4)$

$\mathcal{M}^{\pm 2}$ become extra IR currents

[Xu, You 15; Karch, Tong 16; Hsin, Seiberg 16; FB, Hsin, Seiberg 17; Cordova, Hsin, Seiberg 18]

∗ Examples with emergent time-reversal symmetry

E.g.: $U(N)_1$ with 1 ϕ $\longleftrightarrow$ free Dirac ψ

A few generalizations

- More gauge groups from gauging global symmetries (choice of counter-terms)

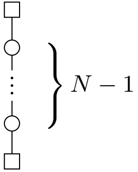
  E.g.: gauge groups O, $Spin$, $Pin^{\pm}$, ... [Cordova, Hsin, Seiberg 17]

Produce intricate nets, testing the conjectured dualities

- Quiver gauge theories (inspired by 3D SUSY mirror symmetry)

  [Karch, Robinson, Tong 16; Jensen, Karch 17]

E.g.: $U(1)_{-\frac{N}{2}}$ with N ψ $\longleftrightarrow$



$U(1)^{N-1}$ with N "bifundamental" ϕ

CS matrix $\kappa_{ab} = SU(N)$ Cartan matrix

(hard to precisely identify scalar potential)

- Vector models with scalars *and* fermions

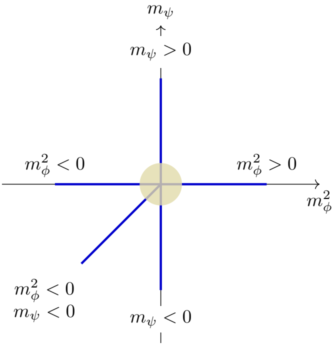
[Jain, Minwalla, Yokoyama 13; Gur-Ari, Yacoby 15; FB 17; Jensen 17]

$$SU(N)_{k-\frac{N_f}{2}} \text{ with } N_s\ \phi,\ N_f\ \psi \quad \longleftrightarrow \quad U(k)_{-N+\frac{N_s}{2}} \text{ with } N_f\ \phi,\ N_s\ \psi$$

$$\vdots$$

valid for N_s, N_f less than a bound.

Two-dimensional phase diagram under symmetry-preserving deformations $m_\phi^2|\phi|^2$ and $m_\psi\overline{\psi}\psi$



∗ Semiclassically: match gapped phases and gapless lines

Origin: multicritical fixed point? more intricate topology?

- Multi-critical point from extra tuning of vector models (at large N):

$U(N)_k$ with N_f ϕ tune $\lvert\phi\rvert^2$ and $\lvert\phi\rvert^4$ to zero	$\longleftrightarrow$	Legendre transform of $U(k)_{-N}$ with N_f ψ w.r.t. $\overline{\psi}\psi$ (Gross-Neveu critical point)

[Aharony, Jain, Minwalla to appear; see Minwalla's talk last year]

How to test such conjectures?

- Large N, k computations (already described)
- Consistency under relevant deformations
  (reduce to level-rank duality of TQFTs or other dualities)
- Deformation from SUSY dualities [Jain, Minwalla, Yokoyama 13; Gur-Ari, Yacoby 15]
  [Kachru, Mulligan, Torroba, Wang 16]
- 't Hooft anomaly matching (more later) [FB, Hsin, Seiberg 17]
- Embedding in String Theory
  [Jensen, Karch 17; Armoni, Niarchos 17; Argurio, Bertolini, Bigazzi, Cotrone, Niro 18]
- Lattice Monte Carlo computations? (not many results for parity-breaking theories)
  [Hands, Kogut, Scorzato, Strouthos; Karthik, Narayanan]

  $d = 4 - \epsilon$ expansion? [Di Pietro, Komargodski, Shamir, Stamou 15]

Anomalies

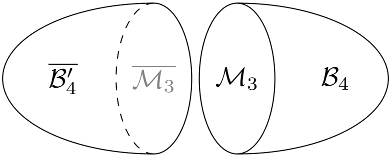
Theory with internal symmetry G can be coupled to background G gauge fields

$\rightarrow$ observables E.g.: $Z[A_{\text{background}}]$

Might be impossible (by local counterterms) to make Z
a well-defined or gauge-invariant function of G-bundles

$\rightarrow$ 't Hooft anomaly

- Extend G-bundles to 4-dim bulk: $Z[A_{\text{background}}]$ depends on extension



Quantify 't Hooft anomaly by dependence on extension

$\rightarrow$ classical TQFT (characteristic class) in 4-dim (anomaly inflow)

∗ 't Hooft anomalies are independent of RG flow

- Example in 3D (no anomalous currents) [FB, Hsin, Seiberg 17]

$SU(2)_k$ with N_f ϕ : $USp(2N_f)/\mathbb{Z}_2$ faithfully-acting symmetry

Gauge + Global Chern-Simons counter-terms: $\dfrac{SU(2)_k \times USp(2N_f)_L}{\mathbb{Z}_2}$

Quantization: $k \in \mathbb{Z}\,, \quad L \in \mathbb{Z}\,, \quad k + N_f L \in 2\mathbb{Z}$

k odd, N_f even: 't Hooft anomaly

$$S_{\text{anom}} = \pi \int \frac{\mathcal{P}(w_2)}{2}\,, \qquad e^{iS_{\text{anom}}} = \pm 1$$

- Anomalies can be computed and matched across dualities

[FB, Hsin, Seiberg 17; Komargodski, Seiberg 17;Cordova, Hsin, Seiberg 17]

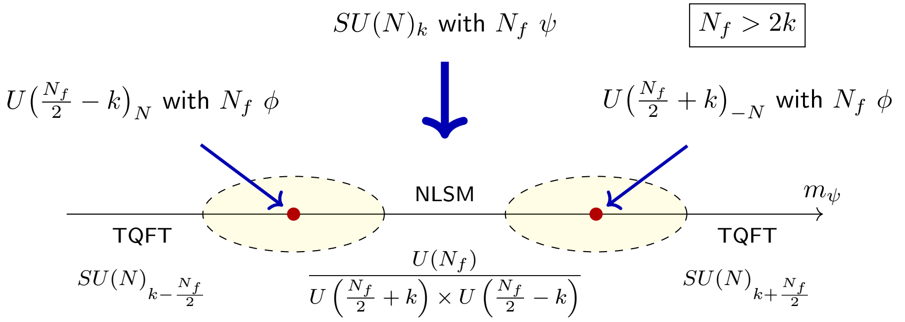
The dualities suggest

other interesting physical phenomena

Quantum phases: spontaneous symmetry breaking

Dualities suggest quantum phases with SSB in 3D QCD for: $2k < N_f < N_f^*$

[Komargodski, Seiberg 17]



- Two phase transitions with different dual descriptions
- Anomalies match. Reproduced by a WZ term in NLSM
- For $k = 0$, compatible with [Vafa, Witten 84]
- Cannot be true for arbitrarily large N_f [Appelquist, Nash 90; Grover 12; Sharon 18]
  Some numerical evidence (in $SU(2)_0$ with N_f ψ) [Karthik, Narayanan 18]

3D transitions from 4D domain walls

4D $SU(N)$ QCD with N_f flavors ($< N_f^{\text{(conf.)}}$)

[Gaiotto, Kapustin, Komargodski, Seiberg 17]
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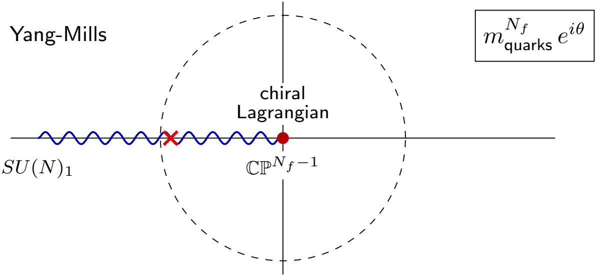


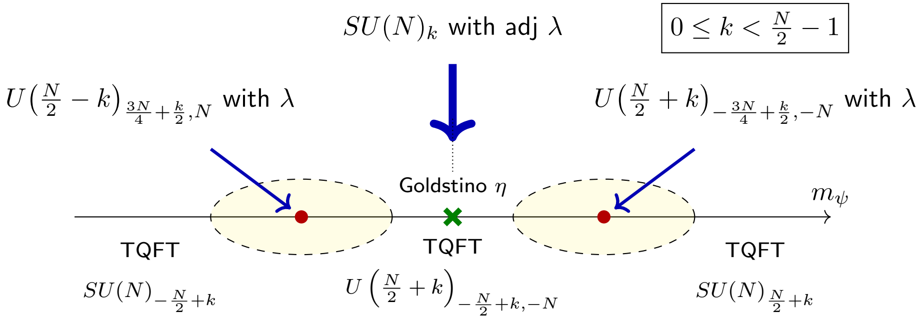
- 4D bulk 1st order transition for $\theta = \pi$ (broken CP) $\rightarrow$ 3D domain wall
- Large $|m_{\text{quark}}|$: $SU(N)_1$ TQFT (e.g. from [Acharya, Vafa 01])
- Small $|m_{\text{quark}}|$: $\mathbb{CP}^{N_f-1}$ NLSM
- 3D phase transition for some value m^*_{quark}

  described by 3D vector model: $SU(N)_{1-\frac{N_f}{2}}$ with N_f ψ

Spontaneous Supersymmetry Breaking

Dualities of gauge theories with adjoint fermions shed light on SUSY breaking

[Witten 99; Gomis, Komargodski, Seiberg 17]



- SUSY breaking point: massless Goldstino η + TQFT
- Three phase transitions, each with a dual description

Concluding remarks

- Lessons from SUSY QFTs $\Rightarrow$ Predictions about non-SUSY QFTs
- Explore a variety of directions

  E.g.: boundary conditions [Gaiotto; Aitken, Baumgartner, Karch, Robinson]

  more general defects
- Interactions with other non-perturbative methods. E.g. conformal bootstrap
- Experimentally testable?