Topological Holography and Chiral Algebras

Work in progress with Kevin Costello

General Motivations

- Topological twist of sugra/superstrings as holographic dual of twisted SQFT (Costello)

- Topological Holography: a consistent subset of protected correlation functions and dual dynamics

- It may be rigorously provable and yet rather rich, including loop effects in supergravity or string theory.

Possible payoffs

- Fully solvable examples of holography
- Insights on perturbative and non-perturbative aspects of supergravity and superstrings in Ramond backgrounds
- Non-perturbative definition of topological string theories
- Mathematics

The chiral subsector

- 2d chiral algebra hidden in 4d N=2 gauge theories (Beem, Lemos, Liendo, Peelaers, Rastelli, van Rees)
- Cohomology of “Q+S” supercharge
- Gauged beta-gamma system: $S_{\rm ch} = \int_{\mathbb{C}} \langle Z, \bar{D}_A Z \rangle$
- Alternative constructions?

The N=4 chiral algebra

- Adjoint beta-gamma: $S_{\rm ch}^{\mathcal{N}=4} = \int_{\mathbb{C}} \mathrm{Tr} X \bar{D}_A Y$
- Hidden supersymmetry: N=4 super-Virasoro

$$\mathrm{sTr}[X^n Y^m] \qquad \mathrm{sTr}[X^n \partial X Y^m + \cdots]$$

$$\mathrm{sTr}[X^n Y^m \partial c + \cdots] \qquad \mathrm{sTr}[X^n Y^m b + \cdots]$$

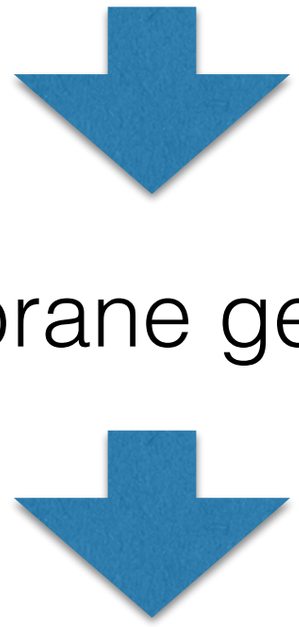
- Action is dimensional reduction of Holomorphic Chern-Simons (hCS). Same as B-model D-branes!

Chiral algebra from Holography

- Objective: twist sugra/superstrings on $AdS_5 \times S^5$
- Lowest KK modes on AdS_5 localize to Chern-Simons on AdS_3 (Bonetti, Rastelli)
- Direct twist is challenging.
- Shortcut: twist flat space and compute topological back reaction

AdS/CFT from open/closed duality

N D3 branes in flat space



Black brane geometry

Near horizon/decoupling limit
(Maldacena)

A quick refresh on B-model

- Topological sector of IIB superstrings on $\mathbb{R}^4 \times \mathrm{CY}_3$ (Antoniadis, Gava, Narain, Taylor)

- Closed B-model: Kodaira-Spencer theory of complex structure deformations (Bershadsky, Cecotti, Ooguri, Vafa)

- Topological twist of D-branes on $\mathbb{R}^2 \times \Sigma$ (Ooguri-Vafa)

- Open B-model: (dimensionally reduced) hCS (Witten)

Back to the (B)asics, continued

- Topological twist of D3 branes on $\mathbb{R}^2 \times \mathbb{C}$
- B-model topological open string on $\mathbb{C}$
- Worldvolume action: U(N) $S_{\rm ch}^{\mathcal{N}=4} = \int_{\mathbb{C}} {\rm Tr} X \bar{D}_A Y$
- N=4 Chiral algebra!

Back-reaction and near-horizon limit

N topological branes on $\mathbb{C} \subset \mathbb{C}^3$

Backreaction: $\mathbb{C}^3 \to SL(2,\mathbb{C})$

Decoupling limit is automatic

Complex structure deformation

- Backreaction: $N \frac{\bar{u} d\bar{v} - \bar{v} d\bar{u}}{(|u|^2 + |v|^2)^2} \frac{\partial}{\partial z}$

- New holomorphic functions:

$$a = vz + N \frac{\bar{u}}{|u|^2 + |v|^2}$$

$$b = uz - N \frac{\bar{v}}{|u|^2 + |v|^2}$$

$$au - bv = N$$

- Deformed conifold, aka $SL(2, \mathbb{C})$ with $\int_{S^3} \Omega = N$

Some geometry

$$g = \begin{pmatrix} u & v \\ b & a \end{pmatrix} \qquad\qquad \det g = N$$

- Original SU(2) acts from the right
- Emergent SL(2,C) acts from the left!
- The space has topology $AdS_3 \times S^3$
- Projection to AdS3: $X = gg^\dagger$

KK reduction

- Projection to AdS_3 helps understand chiral algebra
- KK reduction of KS theory: infinite tower of topological higher spin fields in AdS_3
- “Good” boundary conditions give a chiral algebra. Easy to match quantum numbers.

Back to 6d

- Lift boundary conditions to KS theory on SL(2,C)
- Boundary is (a circle bundle over) $\mathbb{CP}^1_z \times \mathbb{CP}^1_x$
- Deform boundary conditions at a point in $\mathbb{CP}^1_z$ with spherical harmonics on $\mathbb{CP}^1_x$

Status report

- Tree level 2pt and 3pt functions can be matched to large N chiral algebra
- Systematic 1/N expansion?
- Giant gravitons as generating functions?

Some simple extensions

- "Standard" Dijkgraaf-Vafa is a further twist.
- 4d N=4 with flavor: breaks conformal invariance, but we can fix with ghosts.
- k flavors and k ghost flavors give u(k|k) hCS theory
- ``Orbifold'' 4d N=2 quivers and quotients of deformed conifold

Open questions

- Find a way to do all-loop calculations
- New interpretation of local Calabi-Yau. Which ones have boundary chiral algebras?
- New non-perturbative definition of B-model?
- Relation to integrability?
  (Aganagic, Dijkgraaf, Klemm, Marino, Vafa)