A semiclassical ramp in SYK and in gravity

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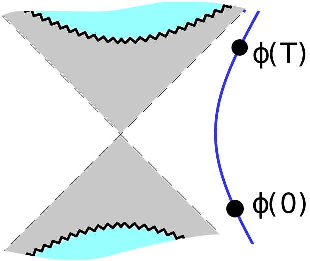
Based on work with Phil Saad and Steve Shenker



In a black hole geometry, correlators like $\langle\phi(T)\phi(0)\rangle$ seem to just decay with time T. However, AdS/CFT says

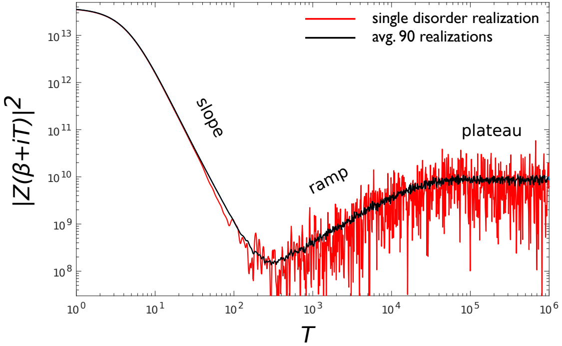
$$\langle\phi(T)\phi(0)\rangle_\beta = \frac{1}{Z(\beta)}\sum_{m,n} e^{-(\frac{\beta}{2}-iT)E_m} e^{-(\frac{\beta}{2}+iT)E_n} |\langle n|\phi|m\rangle|^2.$$

Which is NOT asymptotically decaying as a function of T [Maldacena, Barbon/Rabinovici, Dyson/Lindesay/Kleban/Goheer/Susskind].

A similar story applies to [Papadodimas/Raju]

$$|Z(\beta/2 - iT)|^2 = \sum_{m,n} e^{-(\frac{\beta}{2} - iT)E_m} e^{-(\frac{\beta}{2} + iT)E_n}, \qquad Z(x) \equiv \mathrm{Tr}[e^{-xH}]$$

This is essentially the "spectral form factor," heavily studied in the quantum chaos community [Berry, ...]. E.g. for SYK [CGHPSSSST]



- slope $\sim \frac{e^{2S}}{T^3}$ ✓
- ramp $\sim T$
- plateau $\sim e^S$.

Random matrix universality $\implies$ ramp and plateau should look somewhat similar for many systems, e.g. $\mathcal{N} = 4$ SYM.

Challenge: Compute this curve using the collective field description of a large N quantum system.

The ramp in SYK

The SYK model is a quantum mechanics of N Majorana fermions with Hamiltonian

$$H = \sum_{a_1 < \ldots < a_4} J_{a_1 \ldots a_4} \, \psi_{a_1} \psi_{a_2} \psi_{a_3} \psi_{a_4}$$

We can write the spectral form factor (start with $\beta = 0$) using two replicas of the system:

$$|Z(iT)|^2 = \underbrace{\mathrm{Tr}\left[e^{-iHT}\right]}_{\text{“}L\text{''}} \cdot \underbrace{\mathrm{Tr}\left[e^{iHT}\right]}_{\text{“}R\text{''}}$$

Both “L” and “R” have the same random couplings $J_{a_1 \ldots a_4}$. We will consider the disorder average of the product

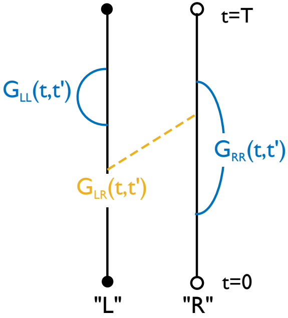
$$\langle |Z(iT)|^2 \rangle \equiv \int dJ_{a_1 \ldots a_4} e^{-\frac{N^3}{J^2} J^2_{a_1 \cdots a_4}} |Z(iT)|^2.$$

This can be computed by an effective theory of collective fields:

$$\langle |Z(iT)|^2 \rangle = \int \mathcal{D}G_{ij}(t,t')\mathcal{D}\Sigma_{ij}(t,t')e^{-N\,I[G,\Sigma]}$$

Here $\Sigma_{ij}(t,t')$ are Lagrange multipliers enforcing the relation

$$G_{ij}(t,t') = \frac{1}{N}\sum_a \psi_a^{(i)}(t)\psi_a^{(j)}(t').$$



1. Action:

$$I = -\log \mathrm{Pf}\left(\partial_t \delta_{ij} - \Sigma_{ij}\right) + \frac{1}{2}\int_0^T \int_0^T dt dt' \left(\Sigma_{ij} G_{ij} - \frac{J^2}{4} s_{ij} G_{ij}^4\right)$$

   where $s_{LL} = s_{RR} = -1$ and $s_{LR} = 1$.

2. Ansatz:

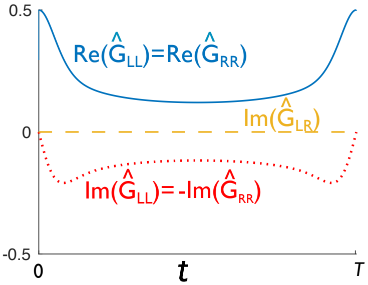
$$G_{ij}(t, t') = \hat{G}_{ij}(t - t'), \qquad \Sigma_{ij}(t, t') = \hat{\Sigma}_{ij}(t - t').$$

3. Saddle point equations:

$$\hat{G}_{ij}(\omega_n) = -\left[i\omega_n \delta_{ij} + \hat{\Sigma}_{ij}(\omega_n)\right]^{-1}, \qquad \hat{\Sigma}_{ij}(t) = s_{ij} J^2 \hat{G}_{ij}^3(t)$$

Two types of saddle point

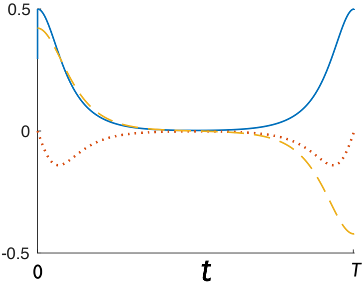
Uncorrelated saddle pt.



- $G_{LR} = 0, \; \hat{G}_{LL} \to \frac{const.}{(iT \sin \frac{\pi t}{T})^{\frac{1}{2}}}$
- $N\times$ action $\to -2S_0$
- describes "slope" region

$$\langle |Z(iT)|^2 \rangle \supset \frac{e^{2S_0}}{T^3}$$

Correlated saddle pt.



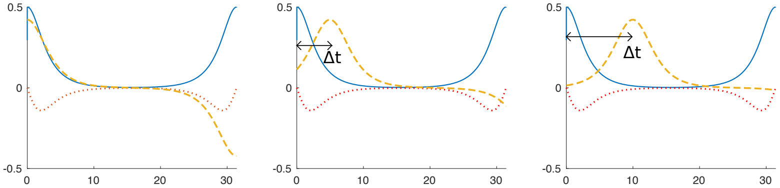
- exp. decaying in t
- action $\to 0$
- two zero modes (see below)
- describes "ramp" region

$$\langle |Z(iT)|^2 \rangle \supset T$$

The important zero mode

Given one solution with G_{LR} nonzero, can find a family:

$$\hat{G}_{LR}(t) \to \hat{G}_{LR}(t - \Delta t).$$



The parameter Δt is an exact Goldstone zero mode for the spontaneous breaking

$$(\text{time translation})_L \times (\text{time translation})_R \to (\text{time translation})_{\text{diag.}}$$

The integral over this zero mode gives the ramp factor of T

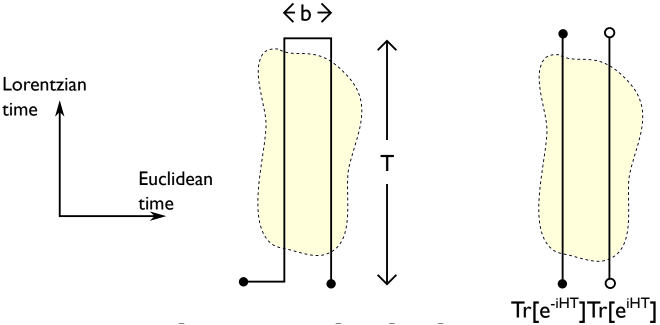
$$\int_0^T d(\Delta t) = T$$

Where does the correlated saddle pt. come from?

Consider an auxiliary problem

$$\mathrm{Tr}[e^{-bH}e^{-iHT}e^{-bH}e^{iHT}] = \mathrm{Tr}[e^{-2bH}]$$

and cut and paste the solution from the long Lorentzian portions of the contour (need to add image to make correctly antiperiodic).



Answer to first problem is independent of T, so saddle point action along Lorentzian part is zero.

The second zero mode

The parameter b was arbitrary. This is a second zero mode, reflecting that all energies contribute equally to the ramp in $|Z(iT)|^2$.

To focus on a given energy band, can study instead a microcanonical version

$$Y_{E,\Delta}(T) \equiv \int_{\epsilon+i\,\mathbb{R}} d\beta e^{\beta E+\beta^2\Delta^2} Z(\beta+iT)$$

This fixes b in terms of E.

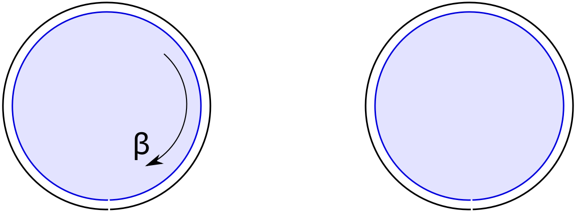
Translating to gravity

At low temperatures, there is a correspondence betwen the SYK model and Jackiw-Teitelboim (JT) gravity

$$I_{JT} = -\frac{S_0}{2\pi}\int \sqrt{g}R - \frac{1}{2}\int \sqrt{g}\phi(R+2) + \text{boundary terms}$$

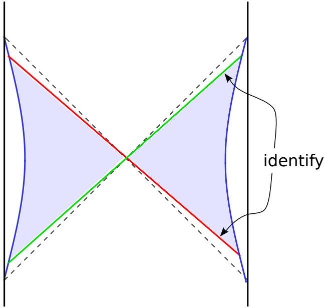
Solutions to SYK can be compared to solutions to JT by matching the correlation functions G of SYK fermions to those of free 2d fermions in the bulk JT space.

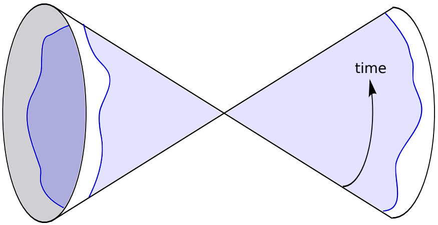
$Z(\beta)Z(\beta) =$ two separate hyperbolic disks (Euclidean BHs)



$Z(\beta + iT)Z(\beta - iT) =$ two separate complexified hyperbolic disks

Can one find a solution in JT gravity that represents the nontrivial "ramp" saddle points?





$$ds^2 = -\sinh^2(\rho)dt^2 + d\rho^2, \qquad t \sim t + T.$$

The action is zero, and we get a factor of T from a zero mode. This is the JT gravity version of the SYK ramp.

Is the double cone singular?

When t is compactified $t \sim t + T$ in Rindler space:

$$ds^2 = -\rho^2 dt^2 + d\rho^2$$

we have to say what happens at $\rho = 0$.

Things can go badly: a naive regularization

$$ds^2 = -(\rho^2 + \epsilon^2)dt^2 + d\rho^2$$

is singular as $\epsilon \to 0$, $R = -2\epsilon^2/(\epsilon^2 + \rho^2)^2$. Worse, due to purely Lorentzian periodicity, the partition function of even a free field on this space is ill-defined.

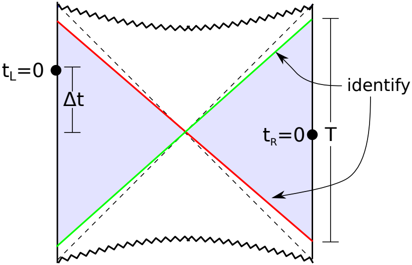
Contour in microcanonical transform suggests:

$$ds^2 = -(\rho + i\epsilon)^2 dt^2 + d\rho^2$$

this space is smooth, and in fact $R = 0$. In at least some cases, quantum field theory seems OK on this type of space.

More general black holes

Can consider a similar solution for any stationary black hole in any dimension.



BHs with all charges and angular momenta will contribute equally, so we should sum over them (subject to the energy constraint).

So there are solutions in the supergravity theory dual to $\mathcal{N} = 4$ SYM. This leads to puzzles (e.g. in this case expect an erratic ramp, not a smooth one).

Summary

In the SYK model, we can understand the ramp as arising from a nontrivial saddle point that correlates the two systems.

This saddle can be translated to JT gravity.

In more general gravity theories, one can also consider similar "double cone" saddle points. The interpretation is not clear but see short talk on Friday for some discussion of this.