What’s New With Q?

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The Road Ahead

Disclaimer: Supersymmetry is an enormous subject. I make no claim at being complete. I apologize in advance for the variety of fascinating papers that are not discussed below

Themes:

- Classification of superconformal field theories
- Exact calculations in SUSY QFT
- Connections between SUSY and Non-SUSY dynamics

The Classification Problem

Goal: In each spacetime dimension d>2, classify possible superconformal field theories.

Why is it interesting? :

- SCFTs are highly symmetric universality classes. Understanding them is a starting point to classifying the possible dynamics of QFTs

- SCFTs exhibit extremely rich mathematical structure. New SCFTs might lead to new mathematics

- The special case d=3 is related holographically to AdS vacua crucial to the string landscape

The Nahm Classification

Foundational algebraic results due to Nahm mean that the range of spacetime dimensions where SCFTs are possible is limited

If we demand that our theories admit an energy-momentum tensor, then # Poincare supercharges, N_Q, is also bounded [Cordova-Dumitrescu-Intriligator]

d=6: N_Q=8, 16. Highly constrained by consistency (e.g. anomaly cancellation). Plausible conjectured classification for N_Q=16. Constructive progress using string theory for N_Q=8

d=5: N_Q= 8. New examples and proposed classifications for simple theories (low rank) using string theory techniques

d=4: N_Q=4, 8, 12, 16. Plausible conjectured classification for N_Q=16. Recent results for simple theories (low rank) with smaller N_Q

d=3: N_Q=2, 4, 6, 8, 10, 12, 16. Classification wide open/wild (even for big N_Q)

The Status of Maximal SUSY

- **d=6:** N_Q= 16 theories admit a conjectural ADE classification [Witten]. The A series is constructed by M5-branes.

Can motivate the ADE classification by reducing on a circle to 5d SYM. On the Coulomb branch, the monopole strings and W-boson particles must uplift to the same 6d string which requires simply laced group

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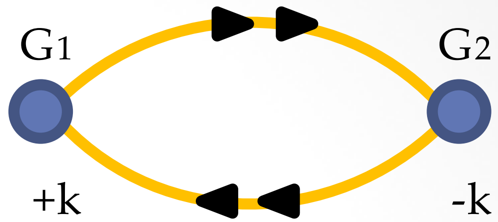
- **d=4:** N_Q= 16 conjectural classification in terms of gauge theories, (i.e. N=4 SYM) Can it be made into a theorem?

Every N_Q= 16 theory has an exactly marginal complex coupling τ with fixed Zamolodchikov metric [Papadodimas]: $ds^2 \sim \frac{|d\tau|^2}{Im(\tau)^2}$

Try to prove that there is a weak-coupling limit (tricky … depends on the topology of the coupling space i.e. the duality group)

Maximal SUSY Zoo in 3d



Bagger-Lambert-Gustavsson:

$$BLG_k = \frac{SU(2)_k \times SU(2)_{-k}}{Z_2}$$

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Maximal susy for any k. Small k can be thought of as the worldvolume theory of two M2 branes. But for general k there is no known string theory embedding!

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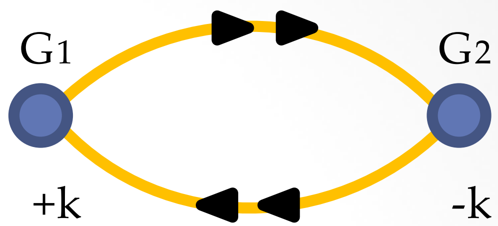

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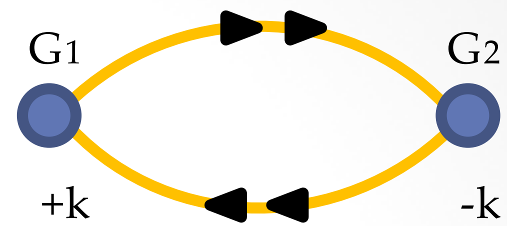

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List both incomplete (exceptional groups!) and redundant (dualities!)

[Bashkirov-Kapustin, Agmon-Chester-Pufu]: $BLG_3 + Free\ Theory = ABJM_{3,1}$

The Exotic World of 4d N=3

- In late 2015 [Gacía-Etxebarria-Regalado] constructed the first known examples of 4d SCFTs with $N_Q = 12$
- These theories are intrinsically strongly-coupled (no marginal parameters are possible) [Aharony-Evtikhiev, Cordova-Dumitrescu-Intriligator]

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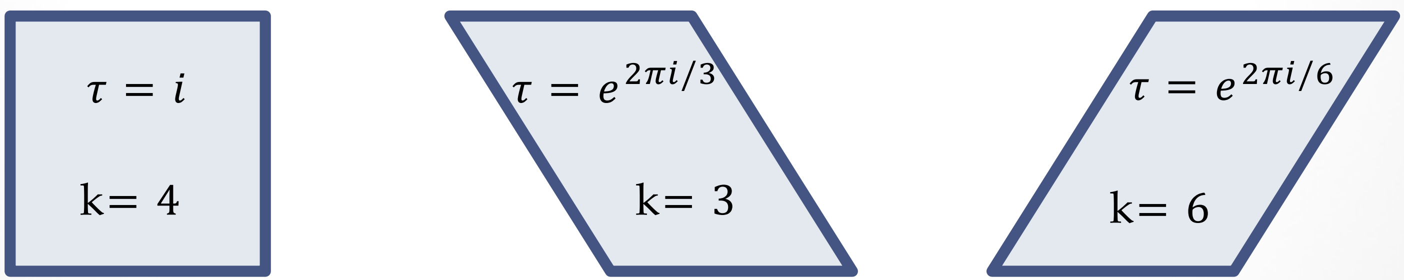
- Construction involves F-theory on $(C^3 \times T^2)/Z_k$ (can be thought of as an F-theory uplift of ABJM geometry). For k=1,2 the τ parameter of the torus can be general. For k=3,4,6 this construction is possible if τ is frozen.



- Probing with D3 branes leads to interacting 4d SCFTs with $N_Q = 12$. For large numbers of D3 branes these theories have an AdS_5 dual where the transverse geometry is a quotient of S^5 and the quotient also acts as a duality in spacetime [Aharony-Tachikawa]

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Sparse results on the dynamics of these theories

- **Chiral Algebras:** [Nishinaka-Tachikawa] In rank one this leads to exact computations of protected OPE coefficients and a formula for the central charge.

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- **Moduli Space:** [Nishinaka-Tachikawa, Argyres-Martone] In rank one C^3/Z_k, with a chiral operator of dimension k

- Also results on **superconformal indices** ([Imamura-Yokoyama, Burton-Pini-Pomoni]) and **superconformal bootstrap** ([Lemos-Liendo-Meneghelli-Mitev])

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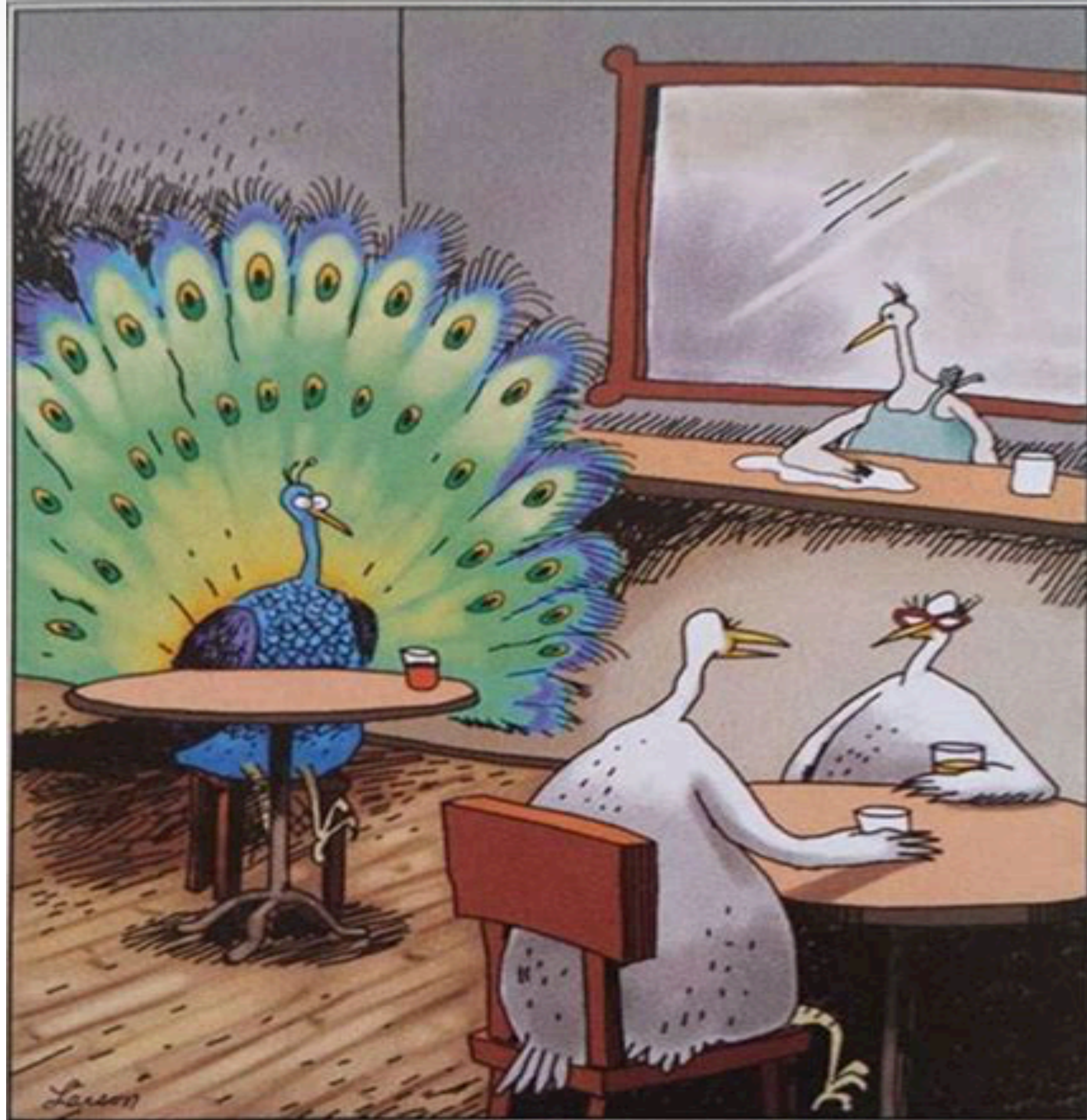
4d N=3 theories seem like a natural candidate for a classification program. The moduli space of vacua is necessarily flat

Assuming no relations among the chiral operators (not clear if this is correct!) we are led to a quotient of a vector space by a complex reflection group. Weyl groups give N=4 examples, other groups may give genuinely N=3 theories

Classification Requires a Taste for the Exotic



“Don’t encourage him, Sylvia.”

The $N_Q = 8$ Universe: 6d

Over the past few years there has been dramatic progress on 6d (1,0) theories (on moduli space there are 2-form gauge fields B and $dB = * dB$)

F-Theory Constructions: classification of possible geometries that can support a 6d SCFT [Heckman-Morisson-Rudelius-Vafa]

Dual Holographic Solutions: All AdS_7 solutions of type II string theory are known [Appruzi-Fazzi-Rosa-Tomasiello]

QFT Analysis: strategy is to deform to the tensor branch and try to understand consistency conditions [Seiberg, Bhardwaj]. This led to progress on anomalies [Ohmori-Shimizu-Tachikawa-Yonekura, Intriligator, Cordova-Dumitrescu-Intriligator]

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Open Problems: Known interacting examples always have tensor branches…why? How many distinct interacting theories are there which, at low-energy, look like a single tensor? (F-theory predicts exactly one! The small E_8 instanton)

Applications of 6d SCFTs

What's at Stake: Compactification on a manifold M produces field theories in lower spacetime dimension. We can trade some problems involving the dynamics of the lower dimensional theory for problems involving the geometry of M.

We can use our knowledge of 6d, together with geometry, to organize our thoughts about SCFTs in lower spacetime dimensions (Famous example, class S construction of 4d N=2 theories. [Gaiotto])

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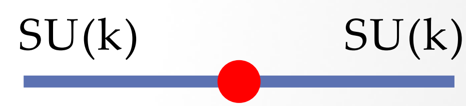
Reduce on a surface to get 4d SCFT



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Results: moduli, indices, anomalies, symmetry enhancement, duality

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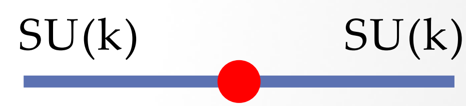

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Open Problem: Are 6d SCFTs parents for all lower-dimensional SCFTs?

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Field Theory: After relevant deformation, one often finds a non-abelian gauge theory and hypermultiplets. [Seiberg, Intriligator-Morrison-Seiberg]

Geometry/String-Theory: M-theory on a local geometry where a complex surface collapses [Intriligator-Morrison-Seiberg]. Or in Type IIB, webs of (p,q) 5-branes [Aharony-Hanany].

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Puzzle: What are the correct low-energy consistency conditions on the field theory? On the moduli space of vacua in the gauge theory we find an abelian theory. Should this make sense for all vacua, or just a subregion?

For the field theory to be consistent with known string constructions we must use the weaker (subregion) constraint. Explored in [Jefferson-Kim-Vafa-Zafrir]

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The 6d to 5d connection has been explored [Hayashi-Kim-Lee-Yagi, Del Zotto-Heckman-Morrison, Jefferson-Katz-Kim-Vafa]. Known 5d theories with ≤ 2 non-abelian gauge groups arise from compactification from 6d

Recent Results in 5d

New results about these theories have appeared in [Chang-Fluder-Lin-Wang]

Consider the [Seiberg] exceptional series. At low energies these are SP(2N) gauge theory with $N_F < 8$ fundamental hypermultiplets

In the UV the gauge coupling becomes strong and we reach a critical point

The topological current $J_{Top} \sim * \, Tr(F \wedge F)$ together with the IR flavor symmetry acting on the hypers enhances in the UV to the exceptional group E_{N_f+1}

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Using both the **superconformal bootstrap** and exact results for **sphere partition functions** they extract estimates for the two-point functions

$$< TT > \sim C_T \ , \qquad\qquad < JJ > \sim C_J$$

For instance in the rank one theory with E_8 flavor symmetry one finds

$$C_T(E_8) \big/ C_T(hyper) \approx 153 \qquad\qquad C_J(E_8) \big/ C_J(hyper) \approx 243$$

The results also suggest that, among all theories with E_8 flavor symmetry, this theory minimizes C_J

The $N_Q = 8$ Universe: 4d

The problem of classifying 4d SCFTs has reached the stage of big data. For instance [Chacaltana-Distler-Trimm-Zhu] recently constructed 49,836 isolated SCFTs by compactifying the E_8 (2,0) theory on three-punctured spheres

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Coulomb Branch: Directly constrain the [Seiberg-Witten] geometry. Works best for low-rank theories. Systematized by [Argyres, Lottito, Long, Lu, Martone, Whittig] (Next talk!!)

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Higgs Branch: There is a tight interplay between the geometry of the Higgs branch and the protected set of local operators encoded in the chiral algebra of [Beem-Lemos-Liendo-Peelaers-Rastelli-van Rees]

The chiral algebra is a VOA constructed from (twisted) correlators of local operators restricted to a 2d plane. The Higgs branch is an associated variety constructed by truncating to commutative algebra [Beem-Rastelli]

A_{2n-1} AD Theory $\sim$ C^2/Z_n Higgs Branch $\sim$ [Bershadsky-Polyakov] VOA

Calculations in SUSY QFT

Exact calculations are the bread and butter of supersymmetry

- Superconformal index of the Argyres-Douglas CFT [Song's Talk!]
- Organizing principles for supersymmetric partition functions
- Relations between moduli spaces of vacua and critical exponents of local operators [Hellermans's Talk!]
- 3d dualities and boundary conditions [Paquette's Talk!]
- Exact calculations using deconstructed field theories
- Relation between OPE coefficients and scattering amplitudes [Chester/Pufu's Talk!]
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An Organizing Principle for Localization Calculations

Over the past decade there has been significant interest in computing supersymmetric partition functions and indices [many many papers]

- **3d theories on M_3:** S^3 (correlation functions, F-theorem); $S^2 \times S^1$ (superconformal index); $\Sigma_g \times S^1$ (AdS_4 black hole entropy)….

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An important clue is that in the special case where M_3 is a product $\Sigma_g \times S^1$ then we can interpret the partition function as an observable in a 2d topological A-model on the surface Σ_g

Natural to interpret other partition functions of 3d or 4d theories as observables in 2d theory. This organizes the different geometries and background fields into correlation functions of 2d local operators

Geometry → Local Operators

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In a partition function of a d-dimensional theory on $\Sigma_g \times T^{d-2}$ we may want to introduce fluxes for background gauge fields (R or flavor symmetries)

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One such flux is associated to the geometric fibration of the circles over Σ_g.

Using this idea we can change a partition function on a circle fibration (e.g. S^3) to a partition function on the product (e.g. $S^2 \times S^1$)

The price we pay is that we introduce a disturbance localized at a point in the surface (e.g. the S^2)

Geometry → Local Operators

There is also a 2d local operator that allows us to insert orbifold singularities in the fibration, i.e. non-trivial Seifert fibers (to appear!)

A 2d local operator uplifts to a defect operator in higher dimensions: in 3d a line, in 4d a surface

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3d Statement: The partition function of a 3d N=2 theory on any Seifert 3-manifold (roughly a circle bundle over a Riemann surface with orbifold singularities) can be understood as a correlation function of line defects on the geometry $S^2 \times S^1$ where the lines wrap the circle

4d Statement: The partition function of a 4d N=1 theory on torus bundle over a Riemann surface can be understood as a correlation function of surface defects on the geometry $S^2 \times T^2$ where the surface defects wrap the torus

Organizes a huge class of partition functions!

(2,0) Deconstruction

The 6d maximally symmetric (2,0) theory plays a starring role in the story of SCFTs

But what is this theory? How can we study it?

Various Ideas: light cone quantization, large N techniques, reduction to 5d, **dimensional deconstruction** [Arkani-Hamed-Cohen-Kaplan-Karch-Motl]

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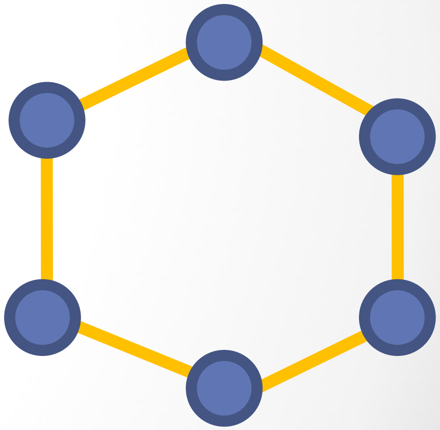

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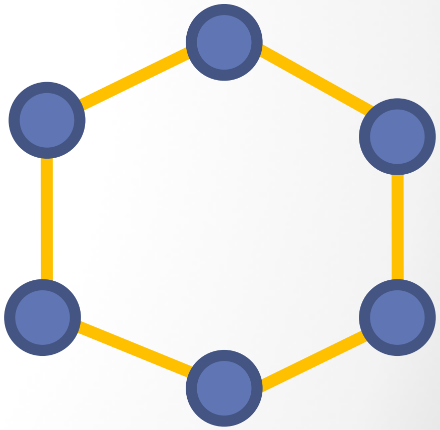

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At long distances the resulting theory is believed to describe the 6d (2,0) theory of k M5 branes on a torus with sides R_i (Note that the 4d theory is UV complete)

Nice idea, but how can we make it quantitative?

Quantitative Deconstruction

[Hayling-Papageorgakis-Pomoni-Rodriguez-Gomez] provide quantitative checks on the deconstruction idea using supersymmetric partition functions

Half BPS Operators: The partition function of half-bps primary operators in the (2,0) is [Bhattacharyya-Minwalla]:

$$Z = \Pi_{n=1}^{k} \frac{1}{1-x^n}$$

(One can guess this by looking at coordinates on the moduli space of vacua)

This is reproduced by the higgs branch primary operators in the quiver

Quantitative Deconstruction

[Hayling-Papageorgakis-Pomoni-Rodriguez-Gomez] provide quantitative checks on the deconstruction idea using supersymmetric partition functions

Half BPS Operators: The partition function of half-bps primary operators in the (2,0) is [Bhattacharyya-Minwalla]:

$$Z = \Pi_{n=1}^{k} \frac{1}{1 - x^n}$$

(One can guess this by looking at coordinates on the moduli space of vacua)

This is reproduced by the higgs branch primary operators in the quiver

These days we have to do better than half-BPS operators!

S^4 Partition Function: Using exact localization we can compute the S^4 partition function in 4d and the $S^4 \times T^2$ partition function in 6d. These match exactly in the deconstruction limit (should be sensitive to the string spectrum in 6d)

Striking check! Motivation to work harder and learn more about the (2,0) theory

Connections Between SUSY and Non-SUSY Dynamics

Lots of recent progress in 3d non-supersymmetric QFT (see e.g Benini's talk) These results have cousins in the supersymmetric world. For instance

- **3d N=4 mirror symmetry:** U(1) gauge theory with 2 hypers of charge 1 is self-dual and flows to a theory with enhanced SU(2)xSU(2) global symmetry [Intriligator-Seiberg]

- **Non Susy Analog:** U(1) gauge theory with 2 electrons of charge 1 is self-dual and flows to a theory with SU(2)xSU(2) global symmetry [Xu-You, Hsin-Seiberg]

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It is natural to ask if such susy/non-susy dualities can be smoothly connected.

In 3d this has been shown for large N Chern-Simons matter dualities in [Jain-Minwalla-Yokoyama, Gur-Ari-Yacoby] and explored at small N in [Kachru-Mulligan-Torroba-Wang] using soft supersymmetry breaking

Can we use this idea to guess the IR behavior of gauge theories in 4d?

Soft SUSY Breaking

We add a susy breaking mass term to a susy Lagrangian to remove the scalars

$$L = L_{susy} + M^2\,|\phi|^2$$

Ideally, the mass operator should be positive definite and part of a short multiplet (so we can track it) and preserve all symmetries so that all 't Hooft anomalies are maintained

This problem of 4d soft susy breaking was explored in the heyday of duality [Aharony-Sonnenschein-Peskin-Yankielowicz, Alvarez-Gaume-Distler-Kounnas-Marino-Zamora, Arkani-Hamed-Rattazzi, Luty-Rattazzi, Nelson-Strassler, Abel-Buican-Komargodski]

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- The regime of parametric control is small M (using supersymmetry)
- The regime of most interest is large M (the non-supersymmetric theory)

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We will use the supersymmetric model to propose candidate phases for the non-susy theory. We cannot be certain which, if any, holds dynamically. But by construction any phases that we find this way will match all 't Hooft anomalies

Adjoint QCD

An ideal test case is 4d adjoint QCD with two Weyl flavors of adjoint quarks λ^i. The index i is a doublet under a chiral SU(2) global symmetry

Lore is that this theory confines and undergoes chiral symmetry breaking via a chiral condensate

$$< Tr(\lambda^i \lambda^j) > \neq 0$$

This leads to a non-linear sigma model of nambu-goldstone bosons on CP^1

Is this really correct? If so, what are the couplings in the sigma model?

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In [Cordova-Dumitrescu] (online now!) we construct candidate phases for this theory (with gauge group SU(2)) by embedding it into pure N=2 SYM. This adds a complex adjoint scalar ϕ. The IR of the N=2 theory is solved ([Seiberg-Witten]). There is a moduli space of vacua controlled by

$$u = < Tr(\phi^2) >$$

At generic u the IR is a U(1) vector multiplet. At $u = \pm\Lambda^2$ there are additional massless fields and the IR is N=2 SQED

A Deconfined Phase at Small M

The UV scalar mass operator $|\phi|^2$ can be matched in the supersymmetric theory and leads to a potential on the moduli space [Luty-Rattazzi]

$$|\phi|^2 \sim i(a\,\overline{a}_D - \overline{a}\,a_D)$$

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The physics of this phase is quite exotic:

- The gaugino is a doublet under the global SU(2) so chiral symm is unbroken
- The free photon means the theory is deconfined (and in a Coulomb phase!)
- The gauge coupling of the photon is at the self-dual value $\tau = i$. An unbroken discrete r-symmetry of the UV acts in the IR by S-duality

This candidate phase matches all 't Hooft anomalies (closely related to a recent conjecture by [Anber-Poppitz, Senthil-Bi]).

Does it persist as the soft mass M is increased?

Monopole Condensation

At $u = \pm\Lambda^2$ the theory is SQED with a charge one hyper h^i in a doublet of SU(2)
The scalar potential for h^i and the vector multiplet scalar σ is

$$V \sim e^2|h|^4 + |h|^2|\sigma|^2 + M^2(|\sigma|^2/e^2 - |h|^2 + \Lambda\, Re(\sigma))$$

If we naively extrapolate to large M (speculative! dangerous!) this suggests that h^i will eventually condense.

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The embedding in SQED also allows us to determine subtle topological couplings which have interesting implications for the physics. One highlight:

The 2d confining string of adjoint QCD is a 2d topological insulator

We can give the fermions a mass and flow to pure YM with zero theta angle. The mass term leaves the bulk 4d physics gapped

Eventually the string undergoes a phase transition to a trivial phase

Conclusions

- The universe of supersymmetric field theory is vast and growing. Each theory is a potential playground for mathematical physics
- New exact results in supersymmetry are teaching us about the guts of quantum field theory
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Thanks for Listening!