Bootstrap, present and future

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Bootstrap philosophy

Bound the space of consistent theories by imposing a minimal set of general principles - or consistency conditions - on a given set of observables.

Plan

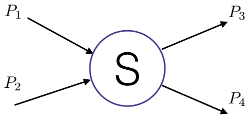
1. Typical questions in the bootstrap context.
2. What can the bootstrap teach us about String Theory/Quantum gravity?

Disclaimer: A beautiful and vast subject, developed by many people and this is not a thorough review.

Thanks to A. Bissi, Z. Komargodski, E. Perlmutter, L. Rastelli and S. Zhiboedov.

S-matrix bootstrap (Revival from the 60' [Penedones - Strings 2017])

- $2 \to 2$ scattering of the lightest particle in a massive Lorentz invariant QFT in $1+1$ dimensions.



$$p_i^2 = m^2$$
$$s = (p_1 + p_2)^2$$
$$t = 4m^2 - s, \quad u = 0$$

- The physical region corresponds to $s \geq 4m^2$.
- Think of s as complex, and extend $S(s)$ to an analytic function in the complex s-plane!

$$S_{phys}(s) = S(s + i\epsilon)$$

General principles

1. Analyticity: $S(s^*) = S^*(s)$
2. Crossing: $S(s) = S(4m^2 - s)$
3. Unitarity: $|S(s)|^2 \leq 1$ in the physical region.
4. Analytic structure in the $s-$plane:

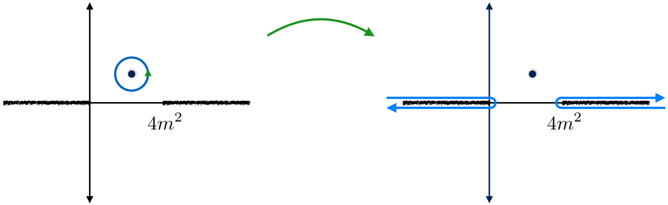
Particle production $\rightarrow$ a pole $S(s) \sim \frac{g^2}{s-M^2}$

Question: How large can g^2 be?

We can actually answer this!

Lesson: Complex analysis is very powerful!

Cauchy theorem $\rightarrow$ Dispersion relation!



$$S(s) = \frac{1}{2\pi i} \oint \frac{S(z)}{z-s} dz = \frac{1}{2\pi i} \int_{4m^2}^{\infty} \frac{Disc\ S(z)}{z-s} dz + \text{crossed}$$

Note we have used boundedness at infinity.

Modular bootstrap [Mazac's talk]

- Partition function of a 2D unitary CFT on the Euclidean torus of modulus τ:

$$Z(\tau,\tau^*)=\sum_{states} q^{h-c/24}\bar{q}^{\bar{h}-\bar{c}/24},\quad q=e^{2\pi i\tau},\quad \bar{q}=e^{-2\pi i\tau^*}$$

- Extend the domain $Z(\tau,\tau^*)\to Z(\tau,\bar{\tau})$ holomorphic in $H_+\times H_-$.

General principles

1. Sum over characters

$$Z(\tau,\bar{\tau})=\chi_{vac}(\tau,\bar{\tau})+\sum\chi_{h,\bar{h}}(\tau,\bar{\tau})$$

Unitarity implies $(h,\bar{h})\geq 0$ and positive multiplicities.

2. Modular invariace

$$Z\left(\frac{a\tau+b}{c\tau+d},\frac{a\bar{\tau}+b}{c\bar{\tau}+d}\right)=Z(\tau,\bar{\tau})$$

Question: How large can $h+\bar{h}$ of the first operator after $(0,0)$ be?

Conformal bootstrap [Simmons-Duffin - Strings 2018]

- Correlator of four identical scalar operators in a unitary CFT_d:

$$\langle\varphi(0)\varphi(z,\bar{z})\varphi(1)\varphi(\infty)\rangle = \mathcal{G}(z,\bar{z})$$

General principles

1. Decomposition in conformal blocks

$$\mathcal{G}(z,\bar{z}) = \sum_{\Delta,J=0,2,\cdots} a_{\Delta,J} g_{\Delta,J}(z,\bar{z})$$

2. Unitarity $\rightarrow \Delta \geq$ unitarity bound, $a_{\Delta,\ell} \geq 0$
3. Crossing symmetry

$$|1-z|^{2\Delta_\varphi}\mathcal{G}(z,\bar{z}) = |z|^{2\Delta_\varphi}\mathcal{G}(1-z,1-\bar{z})$$

Typical question

$$\varphi \times \varphi = 1 + c_T T_{\mu\nu} + c_\phi \phi + \cdots$$

Constraints on the dimension Δ_ϕ and OPE coefficient c_ϕ of intermediate operators.

- It is very useful to think of $z, \bar{z}$ as independent complex variables.
- Play again with complex analysis!

$$a_{\Delta,J} = \int_0^1 dz d\bar{z} K(J, z, \bar{z}) dDisc\left[\mathcal{G}(z, \bar{z})\right]$$

- Lorentzian inversion formula: A dispersion relation for CFT! [analogous of Froissart-Gribov] Again, boundedness is important.
- The CFT data is manifestly analytic in the spin!
- There exist non-local 'light-ray' operators in a Lorentzian CFT, corresponding to generic J.

We (mostly other people!) have made remarkable progress - analytic and numeric - on these three and related areas, but let's focus in a specific question:

What can the bootstrap teach us about string theory/quantum gravity?

Constraining EFT

Low energy effective action arising from a UV complete theory

$$\mathcal{L}_{eff} = \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi + \lambda_1\varphi^3 + \lambda_2\varphi^4 + \lambda_3(\partial_\mu\varphi)^4 + \cdots$$

What are the constraints imposed by the bootstrap? e.g. causality constraints imply $\lambda_3 > 0$.

Strategy 1

- From the effective action compute the S-matrix:

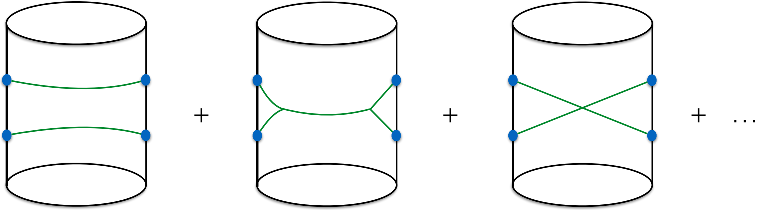
$$S_{eff} \to \text{Scattering Amplitudes}$$

- Then check consistency with unitarity, Regge boundedness, etc.

Recent beautiful 1+1 example constraining the flux tube effective action [Miro, Guerrieri, Hebbar, Penedones, Vieira]

Strategy 2

- Scattering in $AdS \to$ correlator in the boundary



- Then apply the conformal bootstrap to the boundary theory! [and I mean the full non-perturbative bootstrap]

You can study theories in AdS [*e.g.* Bissi's talk] or take the flat space limit [*e.g.* Ooguri's talk].

UV-complete theories of gravity

- Pure gravity is not a consistent theory - we have to UV complete it

$$\mathcal{L} = \sqrt{-g}\left(\frac{1}{2\kappa}R + \lambda_2 R^2 + \lambda_3 R^3 + \cdots\right)$$

- What is the most general consistent completion?

Bootstrap for non-local operators [Zhiboedov's talk]

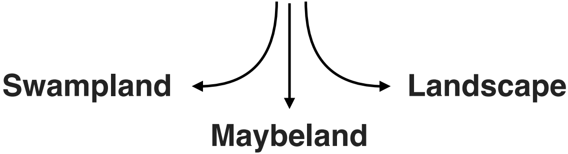
$$\mathcal{L} = \cancel{\sqrt{-g}\left(\frac{1}{2\kappa}R + \lambda_2 R^2\right)}$$

- Strategy 1 - Amplitudes of shockwaves.
- Strategy 2 - Conformal bootstrap for light ray operators.

For gravity on AdS_3 the modular bootstrap offers a beautiful tool, to study whether pure gravity is consistent! [Mazac's talk]

- The bootstrap seems the right tool to see if a 'nicely-looking' EFT belongs to the Swampland or the Landscape.
- But it gives only necessary conditions!

$$\mathcal{L}_{eff} = \frac{1}{2}\partial_\mu \varphi \partial^\mu \varphi + \lambda_1 \varphi^3 + \ldots$$



- We need to improve the bootstrap to give sufficient conditions!

Bootstrap to the next level

- Conformal bootstrap for operators with spin (already hapenning!)
- Can we encode the bootstrap conditions for an infinite number of four-point correlators? [*e.g.* how much can we shrink numerical islands?]
- Alternative: higher point correlators of a single operator:

$$\langle \varphi\varphi\varphi\varphi \rangle, \langle \varphi\varphi\varphi\varphi\varphi \rangle, \langle \varphi\varphi\varphi\varphi\varphi\varphi \rangle, \cdots$$

- Study bootstrap for non-local operators.:
  - Light-ray/ANEC operators (just starting!)
  - Wilson loops combined with local operators,
  - Line defects, surface operators,....

Gravitational bootstrap

- Can we use the bootstrap to constraint gravitational physics/the physics of black holes? (need to consider heavy external objects)
- Can we understand large N theories at finite temperature?
- Can we prove classical GR theorems?

Can the bootstrap do all this alone? It doesn't need to!

- Over the last years combining bootstrap with other techniques (susy localization, integrability, AdS/CFT, 4d/2d dualities, etc) has greatly deepen our understanding of QFT! [*e.g.* Beem's, Komargodski's talks]
- Let's keep combining bootstrap with other good ideas! [including GR ideas]

Back to quantum gravity

$$\mathcal{L}_{QG} = \mathcal{L}_{EH} + \text{completion}$$

- Is there a set of (bootstrap) conditions - general principles - that leads to a unique completion?
- Is this completion local or non-local? does it have higher spin fundamental d.o.f.? Is it string theory?

Q: What is string theory?

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A: The unique solution to bootstrap with that set of conditions!