Quantum Gravity Constraints and their Stringy Realisation

- Weak Gravity and Swampland Distance Conjecture:
  1808.05958, 1810.05169, 1901.08065
  w/ Seung-Joo Lee and Wolfgang Lerche
- Emergent Strings:
  1904.06344 w/ S-J Lee and W Lerche
- Swampland Bounds:
  1905.13213 w/ S-J Lee

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Quantum Gravity Conjectures

Which EFT can be coupled to a fundamental theory of QG?

Swampland of inconsistent EFTs

$\leftrightarrow$

Landscape of consistent QGs

Image: www.physics.harvard.edu/node/873

Swampland Conjectures of general scope, but not sharply proven.

String theory as a framework for QG allows to test explicit conjectures

- Quantitative check of swampland conjectures and sharper formulation
- Study manifestations of swampland conjectures in string geometry

Some QG Conjectures

see talks by Reece, Palti; reviews [Palti'19] [Brennan,Carta,Vafa'17]

1. No Global Symmetries: [Banks,Dixon'88], [Banks,Seiberg'13], [Harlow,Ooguri'18]
   Gauge symmetries cannot become global in presence of gravity.
   What goes wrong in limit $g_{\rm YM} \to 0$?

2. Completeness Conjecture: [Polchinski'03]
   The full charge lattice should be populated.
   Violated in open string sector - where do the states come from?

3. Weak Gravity Conjecture: [Arkani-Hamed,Motl,Nicolis,Vafa'06]
   Tower of charged particles with $q^2 g_{\rm YM}^2 \geq \# M^2$
   What is the correct numerical bound and which states obey it?

4. Swampland Distance Conjecture: [Ooguri,Vafa'06]
   Infinite tower of states becomes massless at infinite distance.
   What's the nature of the theory at infinite distance?

Quantum Gravity Conjectures

Address these questions in situations with $N = 1$ SUSY in 6d and 4d:

F-theory on elliptic CY3 and CY4	$\Longleftrightarrow$	Heterotic theory on K3 and CY3

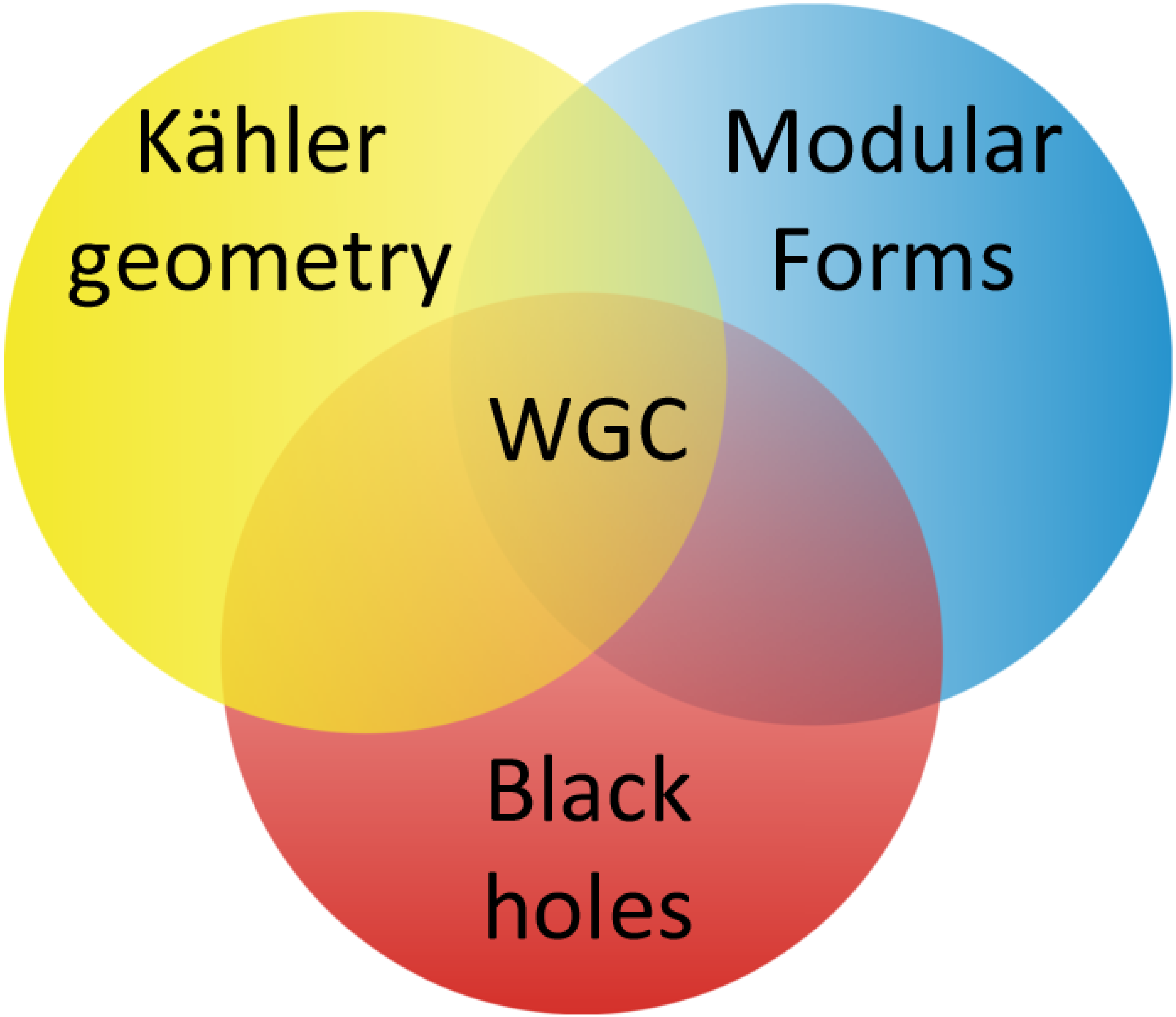
In the geometric infinite distance limit[a]

$$g_{\rm YM} \to 0 \quad \text{while} \quad M_{\rm Pl}^4 \quad \text{fixed}$$

we

- prove the Distance Conjecture
- and the Weak Gravity Conjecture including the effect of scalar fields
- and identify the relevant tower of **non-BPS states**: $g_{\rm YM}^2 q_k^2 \geq M_k^2$.

[a] With caveats in 4d: Limit of 'Class A' - see [Lee,Lerche,TW'19].


Kähler geometry
Modular Forms
WGC
Black holes

General theme: [Lee,Lerche,TW'18 '19]

Emergence of a dual weakly coupled, asymptotically tensionless critical string at infinite distance in moduli space

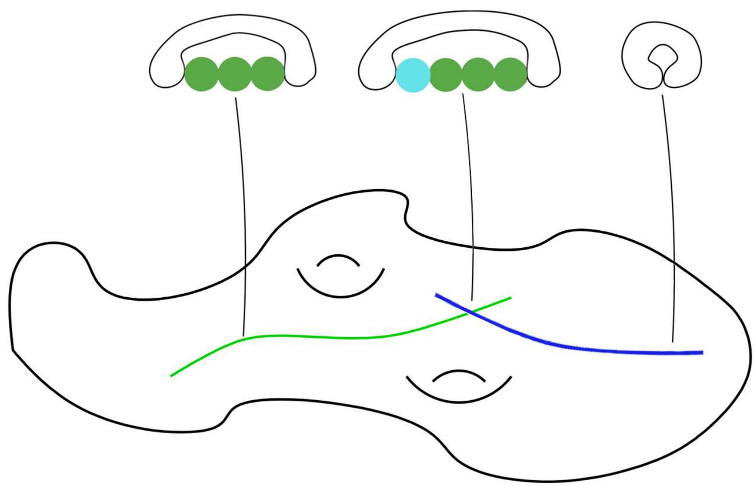
- heterotic string (for most of this talk)
- Type II string (see end of this talk)

Duality and **stringy-ness**	$\Longleftrightarrow$	**Quantum Gravity Conjectures**

cf. [Ooguri,Vafa'06]

F-theory in 6d

F-theory in 6d $\Longleftrightarrow$ Type IIB on $\mathbb{R}^{1,5} \times B_2$ with 7-branes



- B_2 = compact Kähler surface
  $\Longrightarrow$ dynamical gravity
- 7-branes on complex curve $C \subset B_2$
  $\Longrightarrow$ gauge symmetry

Couplings: (IIB Einstein frame, $\ell_s = 1$)

$$\mathbf{M_{Pl}^4 = 4\pi vol_J(B_2)}, \qquad \mathbf{\frac{1}{g_{YM}^2} = \frac{1}{2\pi} vol_J(C)}$$

Mostly focus on gauge group $U(1)$:

C = 'height pairing of rational section on ell fibration over B_2'

cf reviews [TW'18][Cvetič,Lin'18]

Gravity and U(1)s

What happens if take $g_{\mathrm{YM}} \to 0$ at M_{Pl} finite?

Swampland intuition:

In presence of gravity, no global symmetries are possible. [Banks,Dixon'88]

General expectation: [AMNV'06], [Ooguri,Vafa'06], ...

- Offensive limit should be at infinite distance (beyond reach).
- Effective theory must break down (quantum gravity censorship).

Substantial corpus of related works see talk by Palti

[Kläwer,Palti'16] [Heidenreich,Reece,Rudelius'16,'18'19] [Montero,Shiu,Soler'16] [Palti'17]

[Grimm,Palti,Valenzuela'18] [Hebecker et al. 16,'19] [Andriolo,Junghans,Noumi,Shiu'18]

[Blumenhagen,Kläwer,Schlechter,Wolf'18], [Lüst,Palti'18], [Grimm,Li,Palti'18]

[Corvilain,Grimm,Valenzuela'18] [Marchesano,Wiesner'19] [Font,Herraez,Ibanez'19]

[Erkinger,Knapp'19] [Grimm,V.d.Heisteeg'19] ...

Step 1:

Recovering a dual heterotic string in limit

$g_{\text{YM}} \to 0$

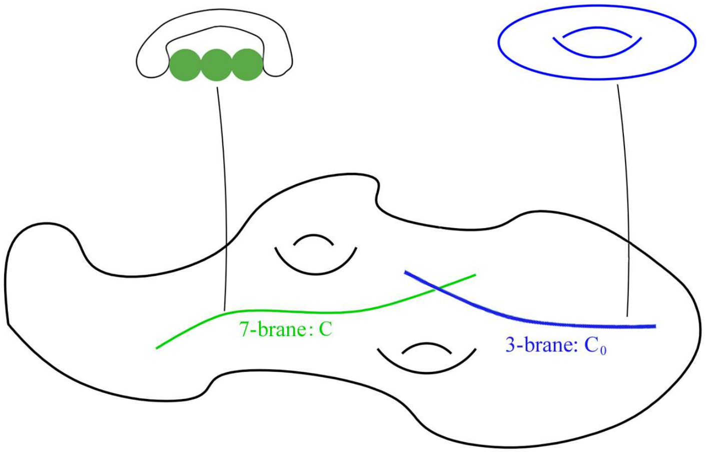
Global limit in Kähler geometry

$$\frac{1}{g_{\mathrm{YM}}^2} \sim \mathrm{vol}_J(C) = t \to \infty \quad \text{while} \quad M_{\mathrm{Pl}}^4 \sim \mathrm{vol}_J(B_2) \equiv 1$$

Theorem: [Lee,Lerche,TW'18]

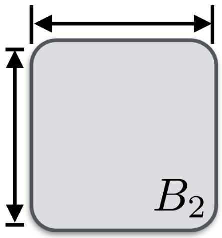
There exists a rational curve C_0:

$$C_0 \cdot C \neq 0 \quad \text{and} \quad \mathrm{vol}_J(C_0) \sim \frac{1}{t} \to 0$$



General intuition

"On finite volume surface, if one direction gets big, normal direction must get very small". Proof for base B_2 via Mori's cone thm. $\to C_0 = \mathbb{P}^1$, $C_0^2 = 0$



Quantum Gravity Conjectures

1) No global symmetries.

$\Rightarrow$ *The limit* $g_{\text{YM}} \to 0$ *must be at infinite distance in moduli space.*

Indeed this is the case here.

Result: Limit $t \to \infty$ at distance $\Delta \sim \log(t) \to \infty$
(w.r.t. to metric on tensor moduli space).

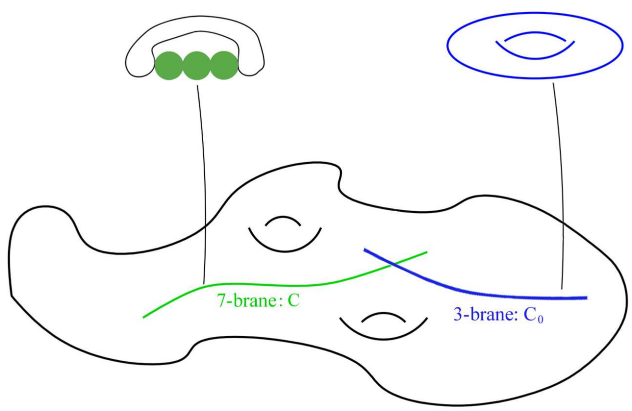
2) Swampland Distance Conjecture

Infinitely many (charged!) states should become massless at exponential rate.

Tensionless solitonic string

from D3 on C_0

tension $\sim T \sim \text{vol}_J(C_0) \sim \frac{1}{t} \sim e^{-\Delta}$



Tensionless strings

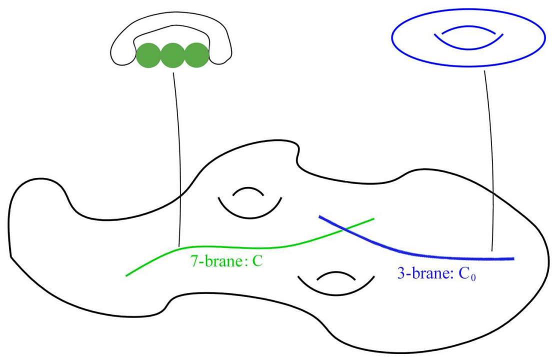
Key fact: [Lee,Lerche,TW'18]

asymptotically tensionless string

=

weakly coupled, critical heterotic string

cf.[Harvey,Strominger'95][Maldacena,Strominger,Witten'97]



Analysis of worldsheet theory of D3 on $C_0 = \mathbb{P}^1$ with $C_0^2 = 0$

- Twisted reduction of $N = 4$ SYM with varying gauge coupling along C_0
  [Martucci'14] [Haghighat,Murthy,Vafa,Vandoren'15] [Lawrie,Schafer-Nameki,TW'16]

  $\Rightarrow$ 2d $N = (0, 4)$ effective theory with

  (4+4) left-moving scalars
  (4+4) right-moving scalars + fermionic partners

- At intersection $C_0 \cap C$:
  isolated 3-7 string modes **charged** under 7-brane gauge group

  $\Rightarrow$ 16 left-moving (charged) fermions

Tensionless 6d strings

F-theory bases B_2 admitting the limit $g_{\rm YM} \to 0$:

Class 1)

B_2 is Hirzebruch

$$p: \quad \mathbb{P}^1_f = C_0 \;\to\; \begin{array}{c} \mathbb{F}_a \\ \downarrow \\ \mathbb{P}^1_b \end{array}$$

Exists perturbative heterotic dual on K3 $\mathcal{K}$ [Morrison,Vafa'96]

$$r: \quad T^2 \;\to\; \begin{array}{c} \mathcal{K} \\ \downarrow \\ \mathbb{P}^1_b \end{array}$$

$$(g_s^h)^2 = \frac{\mathrm{vol}_J(\mathbb{P}^1_f)}{\mathrm{vol}_J(\mathbb{P}^1_b)} \to 0 \quad \text{in tensionless limit}$$

Class 2) B_2 is blowup of Hirzebruch

- Dual heterotic string on $\mathcal{K}$ with extra NS 5-branes on heterotic side
- Away from defects heterotic string 'quasi-perturbative' (justified ex post!)

Step 2:

Proving the WGC via heterotic string

Previous results in literature:

- Spectral flow implies WGC for pert. het string. [AMNV'06]
- Sublattice WGC; proven for toroidal orbifolds
  [Heidenreich,Reece,Rudelius'15/16]

New results [Lee,Lerche,TW'18]

- Determine index of sublattice satisfying the WGC
- Valid beyond perturbative situations
- Explicit match with black hole solutions

The elliptic genus

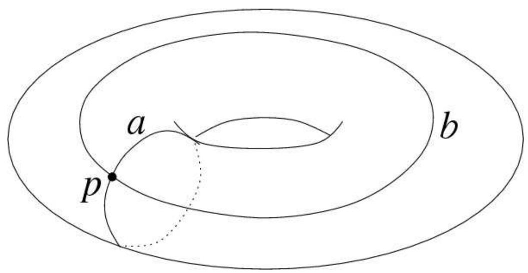
Elliptic genus of 6d het. string $\leftrightarrow$ Subsector of charged non-BPS states

[Schellekens,Warner'86] [Witten'87], ...

$$Z_{\mathcal{K}}(\tau, z) \;\equiv\; \mathrm{Tr}_R \left[(-1)^F F^2 q^{H_L} \bar{q}^{H_R} \xi^J\right]$$

$q = e^{2\pi i \tau}$: τ complex structure of worldsheet T^2

$\xi^J = e^{2\pi i z J}$: fugacity w.r.t. flavour symmetry - e.g. $U(1)$



$$Z_{\mathcal{K}}(\tau, z) = q^{-1} \sum_{n \geq 0} N(n, r)\, q^n\, \xi^r$$

n: excitation level of string r: U(1) charge

The elliptic genus

Ell. genus = (Ratio of) **modular forms** of

weight -2 and $U(1)$ fugacity index m

$$\varphi_{\mathbf{w},\mathbf{m}}\left(\frac{a\tau+b}{c\tau+d},\frac{\zeta}{c\tau+d}\right) = (c\tau+d)^{\mathbf{w}}e^{2\pi i\frac{\mathbf{m}\,c}{c\tau+d}\frac{\zeta^2}{2}}\varphi_{\mathbf{w},\mathbf{m}}(\tau,\zeta)$$

$$\varphi_{\mathbf{w},\mathbf{m}}(\tau,\zeta+\lambda\tau+\mu) = e^{-2\pi i\mathbf{m}(\frac{\lambda^2}{2}\tau+2\frac{\lambda\zeta}{2})}\varphi_{\mathbf{w},\mathbf{m}}(\tau,\zeta)$$

- $U(1)$ index m $\leftrightarrow$ t'Hooft anomaly of worldsheet [Schellekens,Warner'86] [Benini,Eager,Hori,Tachikawa'13] [Del Zotto,Lockhart'16] [...]

Here: $\boxed{m=\frac{1}{2}C\cdot C_0}$ $\quad C_0$: het string $\quad C$: gauge 7-brane

- $\boxed{\mathrm{Het.\,5-branes}\Rightarrow\mathbf{quasi-modular}}$ (incl. $E_2(\tau)$) [Lee,Lerche,TW'18]

The elliptic genus

$$Z_{\mathcal{K}}(\tau,z) = q^{-1}\sum_{n\geq 0} N(n,r)\, q^n\, \xi^r = \frac{\Phi_{10,m}(\tau,z)}{\eta^{24}(\tau)}$$

- $\Phi_{10,m}(\tau,z)$: (quasi-)modular Jacobi form of weight $w=10$

  [Eichler,Zagier]

Every such form can be expanded

$$\varphi_{w,m}(\tau,z) = \sum_{\ell\in\mathbb{Z} \bmod 2m} h_\ell(\tau)\Theta_{m,\ell}(\tau,z)$$

$$\Theta_{m,\ell}(\tau,z) = \sum_{k\in\mathbb{Z}} q^{(\ell+2mk)^2/4m}\, \xi^{\ell+2mk}$$

Subsector with $\ell=0$ contains: $n=mk^2$ $\quad r=2mk \equiv$ U(1) charge $\mathfrak{q}_k$

$\Longrightarrow$ sublattice of charge lattice with $\mathfrak{q}_k = 2m\,k\,,k\in\mathbb{Z}$

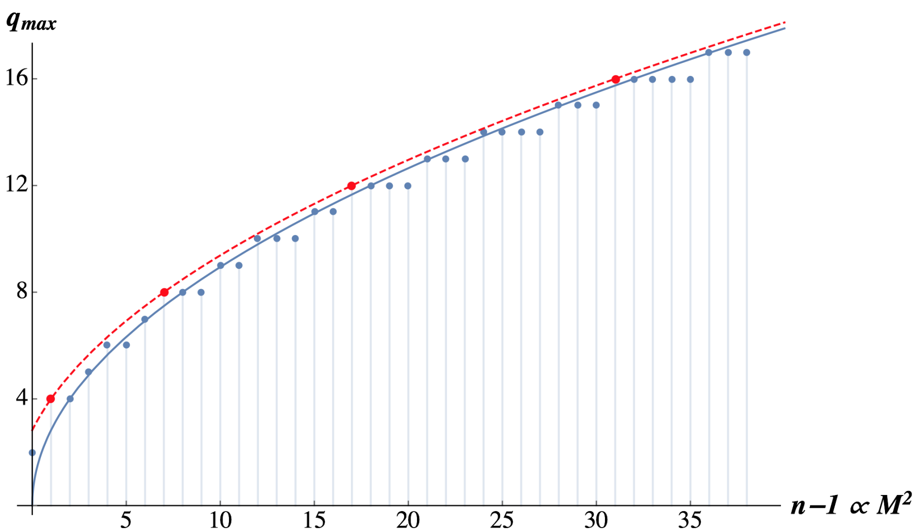
$\mathfrak{q}_k^2 = 4m\,n(k)$ [Lee,Lerche,TW'18]

The elliptic genus

Index 6d of string non-BPS excitations $\Longleftrightarrow$ Gopakumar-Vafa invariants of 5d BPS states

[Klemm,Mayr,Vafa'96] [Haghighat,(Iqbal,Kozaz),[Klemm], Lockhart,Vafa'(13)[14]],...

6d F-theory ell. genus $\longleftrightarrow$ 5d M-theory BPS invariants $\longleftrightarrow$ 4d Type IIA prepotential



Example:

[Lee,Lerche,TW'18]

$m = 2$, on $B = dP_2$

- Closed expression!
- $\mathfrak{q}_k^2 = 4m\, n(k)$ for $\mathfrak{q}_k = 2m\, k$

$\Rightarrow$ **Completeness Hypothesis satisfied** ✓

Weak Gravity Conjecture

'Charged black holes must be able to decay.' [AMNV'06]

$\Rightarrow \exists!$ 'super-extremal' state w.r.t. charged extremal black hole

$$\frac{\mathfrak{q}^2 g_{\rm YM}^2}{M^2}\Big|_{\rm state} \overset{!}{\geq} \frac{Q^2 g_{\rm YM}^2}{M^2}\Big|_{\rm B.H.}$$

Relevant black holes in weak coupling $g_{\rm YM} \to 0$: [Lee,Lerche,TW'18]

Dilatonic Reissner-Nordström BH

- $S = \int_{\mathbb{R}^{1,5}} \sqrt{-g}R + \frac{1}{2} d\phi \wedge * d\phi + \frac{1}{4 g_{\rm YM}^2} e^{\alpha\phi} F_{\mu\nu}F^{\mu\nu}$

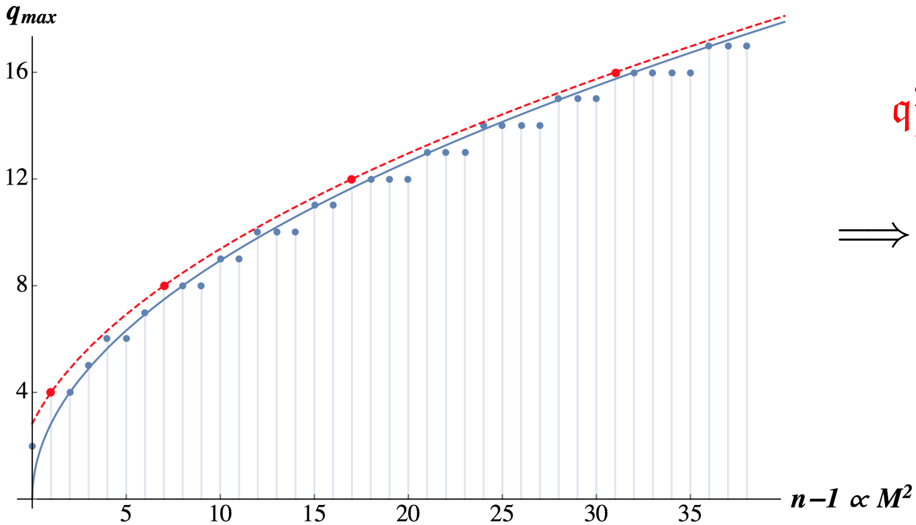
- WGC bound for decay of dilatonic RN black hole:

$$q^2 g_{\rm YM}^2 \overset{!}{\geq} \frac{M^2}{M_{\rm Pl}^{d-2}} \left(\frac{d-3}{d-2} + \frac{\alpha^2}{4} \right)$$

[Heidenreich,Reece,Rudelius'15'19][Palti'17]

- Explicit analysis of 6d SUGRA in weak coupling limit: $\alpha = 1$ [Lee,Lerche,TW'18]

Weak Gravity Conjecture



$\mathfrak{q}_k^2 = 4m\,n(k)$ for $\mathfrak{q}_k = 2m\,k$

$$\Longrightarrow \quad \mathfrak{q}_k^2\, g_{\rm YM}^2 = \frac{M_n^2}{M_{\rm Pl}^4} + 4m\, g_{\rm YM}^2$$

$$\mathfrak{q}_k^2\, g_{\rm YM}^2 > \frac{M_n^2}{M_{\rm Pl}^4} \quad \text{as} \quad t \to \infty$$

$\Longrightarrow$ **WGC for sublattice of rank** $2m = C \cdot C_0$ [Lee,Lerche,TW'18]

cf. [AMNV'06] [Heidenreich,Reece,Rudelius'15'16][Montero,Shiu,Soler'16],...

Valid in weak coupling limit!

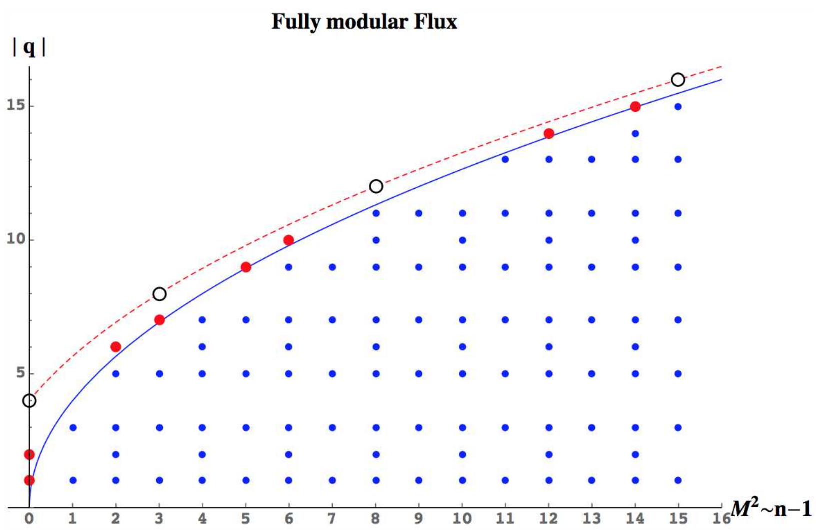
$$M_n^2 = 8\pi T(n-1)\,, \qquad T = 2\pi \quad \text{vol}(C_0)$$

$$M_{\rm Pl}^4 = 4\pi\,\text{vol}(B_2) \equiv 4\pi\,, \qquad \frac{1}{g_{\rm YM}^2} = \frac{1}{2\pi}\text{vol}(C)\,, \qquad \text{vol}(C)\text{vol}(C_0) \to 2m\,.$$

The 4d elliptic genus

Extension to 4d N=1 substantially more involved [Lee,Lerche,TW'19]

Only a <u>shifted</u> sublattice manifest in elliptic genus ($w = -1$!)



Example: $m = 4$

- Super-extremal states:

  $\mathfrak{q}_k = 2mk \pm \ell\,,$

  $n_k - 1 = k(mk \pm \ell) \qquad \ell \geq 1$

  $\Rightarrow \mathfrak{q}_k^2 = 4m(n_k - 1) + \ell^2 \quad k \in \mathbb{Z}$

- blue line: $\mathfrak{q}_k^2 = 4m(n_k - 1)$ (WGC-line)

- WGC still satisfied by tower of states ✓ cf [Andriolo,Junghans,Noumi,Shiu'18]
- Missing sublattice states present in partition function? .

  cf [Heidenreich,Reece,Rudelius'16,'19]

Emergent Strings @ ∞ distance

Example 1: Asymptotic regime $g_{\rm YM} \to 0$ in F-theory

Solitonic string (D3 on *rational* curve) $\xrightarrow[\text{distance}]{\text{infinite}}$ Tensionless critical heterotic string in new duality frame

$\downarrow$

tower of light particles ✓

Example 2: Type IIB theory on K3 [Lee,Lerche,TW'19]

In asymptotic weak coupling regime for 2-forms cf. [Aspinwall,Morrison'95],. . .

Solitonic string (D3 on *elliptic* curve) $\xrightarrow[\text{distance}]{\text{infinite}}$ Critical fundamental Type IIB string in dual frame

$\downarrow$

tower of light particles ✓

Strings and Swampland Bounds

$\exists$? Swampland of *anomaly-free supergravities w/out UV completion*?

- In 6d $N = (1, 0)$ SUGRA, anomaly constraints particularly powerful
- Anomaly-free families of SUGRAs with infinite rank gauge group
  [Kumar,Morrison,Taylor'10], ...

Extra constraints from consistency of sector of BPS strings [Kim,Shiu,Vafa'19]

$$\sum_i c_i \overset{!}{\leq} c_L \qquad c_i : \text{WS current due to spacetime gauge group } G_i$$

$\Longrightarrow$ Puts many of these theories with non-abelian gauge groups in the 'swampland' [Kim,Shiu,Vafa'19]

Particularly mysterious remains the abelian gauge sector

Potential candidate for swampland(?):
6d $N = (1, 0)$ SUGRA with infinitely many $U(1)_i$ $(T \geq 9)$ [Park,Taylor'11]

Strings and Swampland Bounds

1) Demanding unitarity of **6d supergravity** strings:

Upper bound on $N_{U(1)}$ **in 6d N=(1,0) supergravity**

$$N_{U(1)} \leq 22$$ [Lee,TW'19]

2) In F-theory:

$U(1)$	$\longleftrightarrow$	rational section of elliptic fibration
$N_{U(1)}$	$\longleftrightarrow$	rank of Mordell-Weil group

Explicit analysis of solitonic heterotic string:

- All $U(1)$ factors in 6d must be embeddable into $E_8 \times E_8$.[a]
- **New math 'theorem'**: [Lee,Regalado,TW'18],[Lee,TW'19]
  The maximal rank of the Mordell-Weil group of an elliptically fibered Calabi-Yau 3-fold is 16.[b]

[a] Proven for $T \geq 1$; for $T = 0$: $\mathrm{rk}(U(1) \leq 24$ proven, and $\mathrm{rk}(U(1) \leq 16$ argued.
[b] If $h^{1,1}(B) = 1$, $\mathrm{rk}(\mathrm{MW}) \leq 24$ proven, and ≤ 16 argued.

Summary

Quantum Gravity Conjectures realized in string theory
by conspirational interplay of

- duality/emergence,
- geometry of compactification spaces,
- modularity.

What is the underlying fundamental principle?

Emergent Strings @ ∞ distance

Type IIB theory on K3 $\Rightarrow$ 6d $N=(2,0)$ theory

$$\mathcal{M}_{(\mathbf{5},21)} = O(\Gamma_{\mathbf{5},21})\backslash O(\mathbf{5},21)/O(\mathbf{5}) \times O(21)$$ [Aspinwall,Morrison'95],...

- **5 non-compact directions** $S_i \to \infty$ $\Longleftrightarrow$ weak coupling limits for tensors T_i

$$S = \int_{\mathbb{R}^{1,5}} \left(\frac{1}{2} M_{\mathrm{Pl}}^4 \sqrt{-|g|} R + \frac{2\pi}{4} (S_i^2 d\mathrm{T}_i \wedge * d\mathrm{T}_i + S_i^{-2} d\tilde{\mathrm{T}}_i \wedge * d\tilde{\mathrm{T}}_i) + \dots \right)$$

- string coupling to T_i $\overset{S_i \to \infty}{\Longrightarrow}$ tensionless weakly coupled critical string in dual frame

For geometric weak coupling limit in Kähler moduli space: [Lee,Lerche,TW'19]

- $S_3 \sim t = \mathrm{vol}(C) \to \infty$
- elliptic curve C^0
  $\mathrm{vol}(C^0) \sim \frac{1}{t} \to 0$

$$r: \quad \mathbf{C^0} \to K3 \to \mathbb{P}^1_b$$

solitonic string from D3 on C^0 $\overset{S_3 \to \infty}{\Longrightarrow}$ fundamental Type IIB string on dual K3