JT Gravity And Random Matrix Ensembles

Edward Witten

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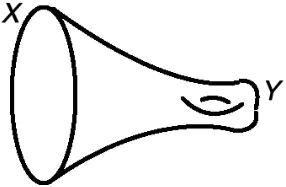

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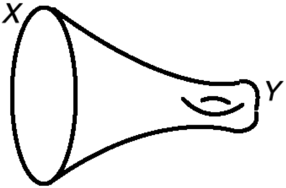

For given X, we sum over all of the Y’s, compatible with general principles.

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Apparently a true quantum mechanical system is dual to something complicated, while a simple bulk system is dual to an average of quantum systems, not to a specific, bona fide quantum system.

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In the work of S^3, it is important that JT gravity on an orientable manifold is not just 1-loop exact but tree-level exact, in the sense that the 1-loop correction is trivial. That is why everything reduces to computing volumes of moduli spaces, where the volumes are defined by classical formulas with no 1-loop correction.

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for a suitable function f. Each ensemble can be related to 2d geometry by expanding in perturbation theory and using 't Hooft's "ribbon graph" construction. The difference is that for a hermitian matrix, one gets orientable 2-manifolds only, but in the other cases, perturbation theory generates unorientable two-manifolds. That is because for the T-invariant ensembles, the propagator has a "twisted" term:

$$\left\langle H_{ij} H^{kl} \right\rangle = \delta_i^k \delta_j^l \pm \delta_j^k \delta_i^l.$$

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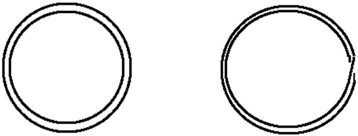

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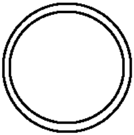

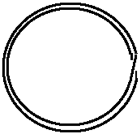

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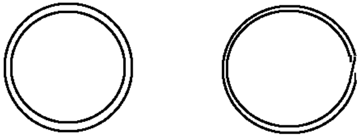

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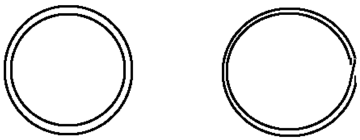

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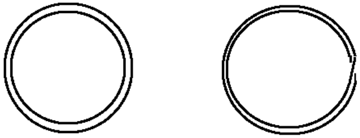

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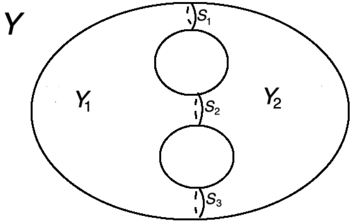

For each circle on which one glues, there are two moduli, namely a "length" parameter a and a "twist" parameter ϱ.

In the orientable case, the measure on moduli space that comes from JT gravity is the classical expression

$$\mu = \prod_i \mathrm{d}a_i \mathrm{d}\varrho_i$$

that one can read off from the classical action at tree-level.

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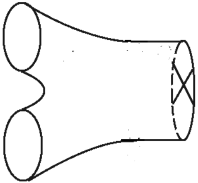

Such a boundary still has a length parameter a, but it has no gluing parameter ϱ.

Including the 1-loop correction, the measure on moduli space that comes from JT gravity is

$$\mu = \prod_i \mathrm{d}a_i \mathrm{d}\varrho_i \prod_\alpha \frac{1}{2} \coth \frac{a_\alpha}{4} \mathrm{d}a_\alpha,$$

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where a_α are the length parameters of the cross-caps. The factor $\prod_\alpha \frac{1}{2} \coth \frac{a_\alpha}{4}$ is the 1-loop correction. (This measure was first obtained in another way by P. Norbury.) Note that there is a divergence in the volume (obtained by integrating the measure μ over moduli space) because of a $\mathrm{d}a_\alpha / a_\alpha$ singularity for $a_\alpha \to 0$.

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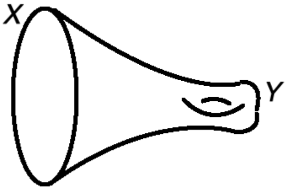

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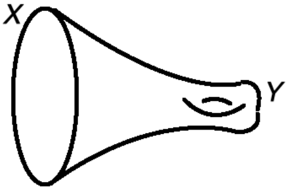

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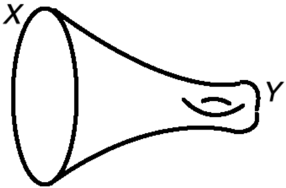

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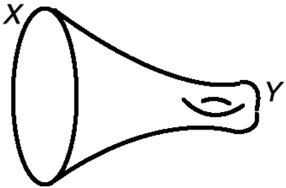

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that comes from the Schwarzian path integral. The JT path integral on such a manifold is supposed to compute a contribution to a correlator of matrix partition functions $\mathrm{Tr}\, e^{-\beta H}$ or (equivalently) matrix resolvents $\mathrm{Tr}\, \frac{1}{x-H}$.

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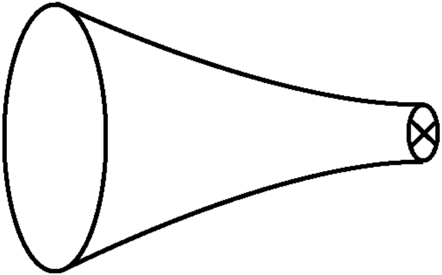

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The canonical form of a bifundamental of $U(L') \times U(L')$ is $C = \mathrm{diag}(\lambda_1, \lambda_2, \cdots)$ where the λ_i can be assumed to be all *positive*, in contrast to Dyson ensembles where the random matrix has positive or negative eigenvalues.

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$$\mu = \prod_i \lambda_i \prod_{j<k} |\lambda_j^2 - \lambda_k^2| \prod_{m=1}^{L'} \mathrm{d}\lambda_m .$$

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To completely specify the model, we need to know the matrix potential $exp(-L \mathrm{Tr}\, f(C^\dagger C))$ or (more usefully) the corresponding saddle point which describes the limiting distribution of eigenvalues for large L'.

For a generic model in which the eigenvalue distribution goes all the way down to $\lambda = 0$, one has

$$\rho(\lambda) \sim \frac{1}{\sqrt{\lambda}} \quad (\lambda \text{ small}).$$

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So this completes the specification of the model. (The same spectral curve, i.e. a model with the same density of states, has been studied by P. Norbury, with a different starting point than ours.)

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vanish in genus 0 (but not in higher orders). It turns out that this has a nice explanation in JT supergravity.

Like JT supergravity, JT supergravity is 1-loop exact on any 2-manifold, but it is tree-level exact on an orientable two-manifold.

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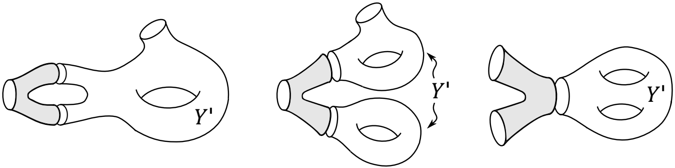

There are infinitely many choices of Σ, but there is a sum rule

$$1 = \sum_{\Sigma} f(\Sigma),$$

where $f(\Sigma)$ is a certain function of the moduli of Σ. (The earliest version of this formula is due to McShane.)

By inserting the identity $1 = \sum_{\Sigma} f(\Sigma)$ in the integral that defines the volume of the moduli space, Mirzakhani was able to get a recursion relation expressing the volume of the moduli space of hyperbolic structures on Y in terms of the analogous volume for a simpler surface Y'.

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$$\begin{aligned}\text{Matrix model anomaly} &= \text{super Schwarzian anomaly}\\ &\quad + \text{TFT anomaly coefficient.}\end{aligned}$$

In summary, I have tried to give an overview of the fact that the relation of JT gravity to a matrix model can be generalized to include time-reversal, fermions, and supersymmetry.