

String theory compactifications with sources

Alessandro Tomasiello

Strings 2019

Introduction

Internal D-brane or O-plane **sources**
important in string theory compactifications

- in AdS/CFT they realize flavor symmetries
- O-planes seem necessary for **de Sitter** and for Minkowski beyond CY

[Gibbons '84, de Wit, Smit, Hari Dass '87,
Maldacena, Nuñez '00...]

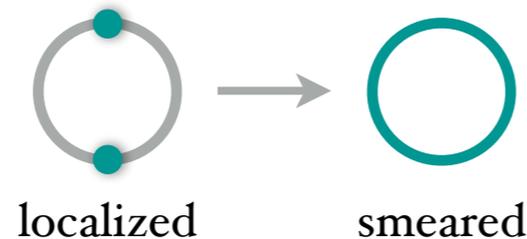
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[Gibbons '84, de Wit, Smit, Hari Dass '87,
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- it has been hard to find examples; often people have resorted to 'smearing'



[Acharya, Benini, Valandro '05,
Graña, Minasian, Petrini, AT '06,
Caviezel, Koerber, Körs, Lüst, Wrase, Zagermann '08,
Andriot, Goi, Minasian, Petrini '10...]

However, O-planes should sit at fixed loci of involutions

⇒ they shouldn't be smeared by definition.

Plan:

I. Progress in finding solutions

II. How we introduce localized sources

III. de Sitter?

I. Geometry of solutions

- Systematic classification of **BPS solutions**:
more successful than ad hoc Ansätze

- old methods: G -structures; gen. complex geometry, pure spinors

[Strominger '86, Gauntlett, Pakis '02...]

[Graña, Minasian, Petrini, AT '05...]

- Conceptual origin: calibrations. Type II, for example:

‘calibration conjecture’:

[Martucci, Smyth '05,
Lüst, Patalong, Tsimpis '10...]

collective **D-brane** calibration

$$(d + H \wedge) \Phi = (\iota_K + \tilde{K} \wedge) F$$

[AT '11]

$$d\Omega = -\iota_K * H + \underbrace{(\Phi, F)}_6$$

[Legrandi, Martucci, AT '18]

NS₅-brane calibration

pairing

- practically, the D-brane equation is enough for $d \geq 4$

$$\left. \begin{array}{l} \text{AdS}_d \\ \text{Mink}_d \end{array} \right] \times M_{10-d}$$

⇒ pure spinor equations

[Graña, Minasian, Petrini, AT '05]

⇒ matrix pure spinor equations for **extended** susy

[Passias, Solard, AT '17;
Passias, Prins, AT '18;
+ Macpherson, in progress]

- In general more calibration equations [eg KK-monopole] needed for sufficiency

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• Supersymmetry breaking?

- For Minkowski: sometimes possible to break susy by adding **one term** to pure spinor equations

[Legramandi, AT, in progress]

- Via consistent truncations

[Passias, Rota, AT, '15...]

- Direct solution of EoM, with some lessons from the susy case

[Cordova, De Luca, AT, '18]

- some recent solution classes:

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- AdS₇ in IIA: $S^2 \rightarrow I$ 

+ susy-breaking twins

[Apruzzi, Fazzi, Rosa, AT '13
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[Rota, AT '15; Passias, Prins, AT '18;
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$$(\text{top. } S^2) \rightarrow \text{KE}_4, \Sigma_g \times \Sigma_{g'}$$

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$\mathcal{N} = (0, 8), (0, 7) : F_4$ and G_3 superalg.

[Dibitetto, Lo Monaco, Petri, Passias, AT '18]

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[Dibitetto, Lo Monaco, Petri, Passias, AT '18]

Almost all analytic.

For ex.
$$e^{-2A} ds_{M_6}^2 = -\frac{1}{4} \frac{q'}{xq} dx^2 - \frac{q}{xq' - 4q} D\psi^2 + \frac{\kappa q'}{3q' - xq''} ds_{\text{KE}_4}^2$$

[dual to CS-matter theories]

$q(x) = \text{deg. 6 pol.}$

generalizes

[Guarino, Jafferis, Varela '15] (anal.)

[Petrini, Zaffaroni '09; Lüst, Tsimpis '09...] (num.)

formally similar to

[Gauntlett, Martelli, Sparks, Waldram '04] in IId

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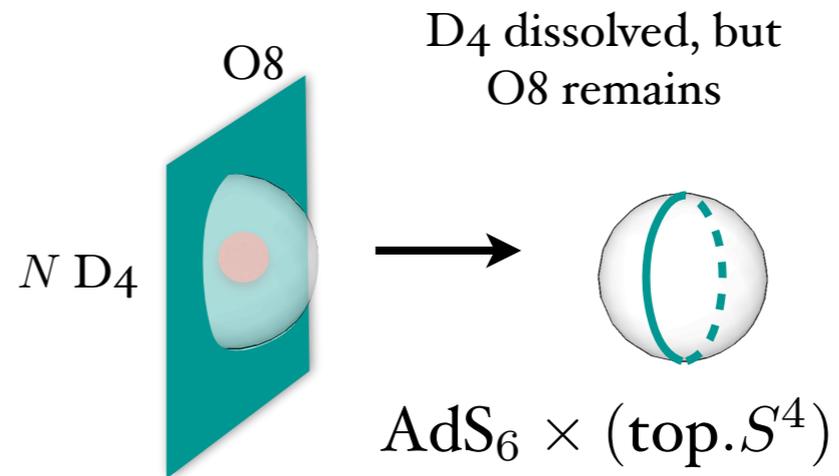
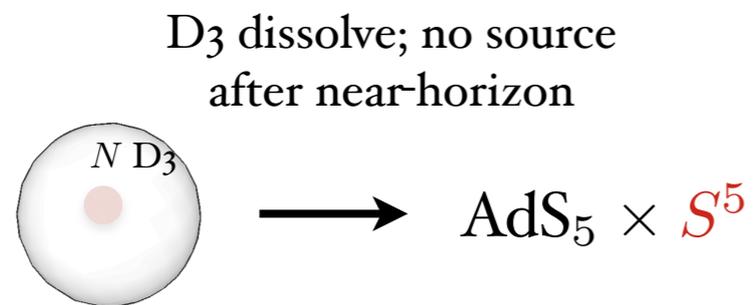
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[Gauntlett, Martelli, Sparks, Waldram '04] in IIB

- relations between different cases often suggest 'correct' coordinates
- we will now see that all these admit possible sources...

II. Including sources

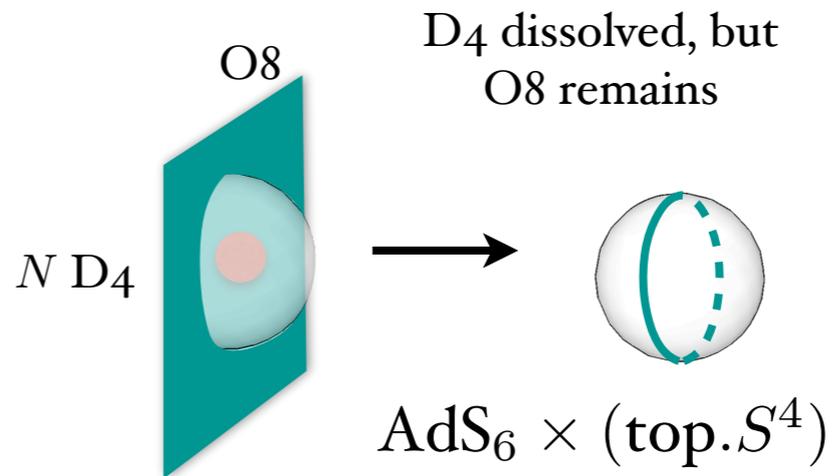
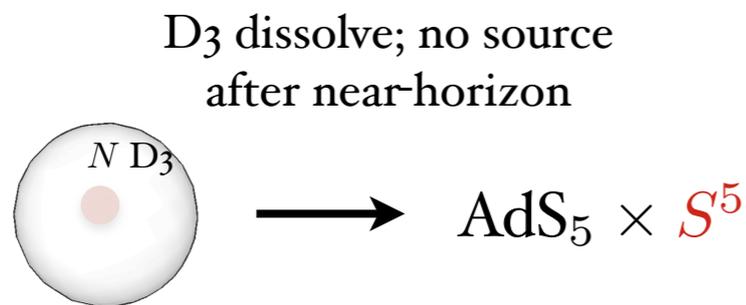
- Many AdS solutions have near-horizon origin



[Youm '99,
Brandhuber, Oz '99]

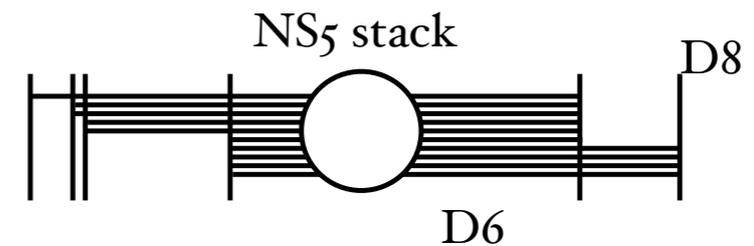
II. Including sources

- Many AdS solutions have near-horizon origin



[Youm '99, Brandhuber, Oz '99]

- Unclear if all AdS are near-horizon limits
- Intersecting brane solutions are rare anyway



- Better strategy: start from analytic classes, explore boundary conditions for sources

- Sources create **singularities** where supergravity breaks down

backreaction
on flat space:

$$ds_{10}^2 = H^{-1/2} ds_{\parallel}^2 + H^{1/2} ds_{\perp}^2$$

$0, \dots, p$ $p+1, \dots, 9$
 ↙ ↘
 H H
 ↖ ↗
 harmonic function in \mathbb{R}_{\perp}^{9-p}

$$e^{\phi} = g_s H^{(3-p)/4}$$

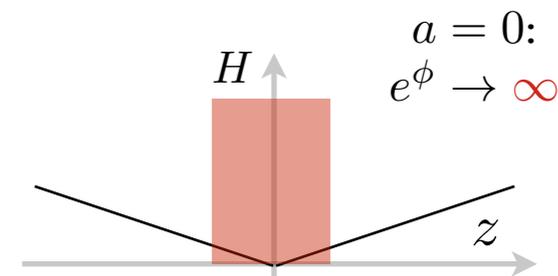
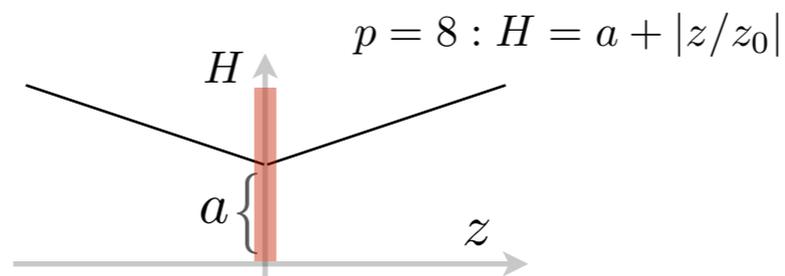
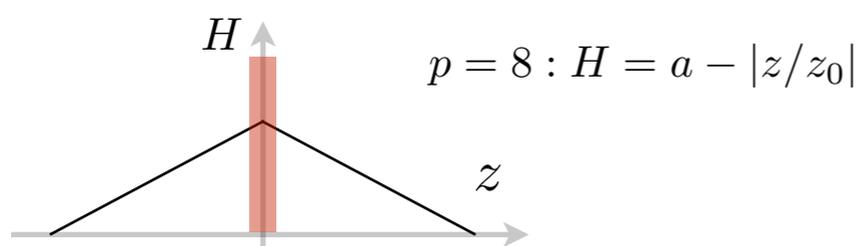
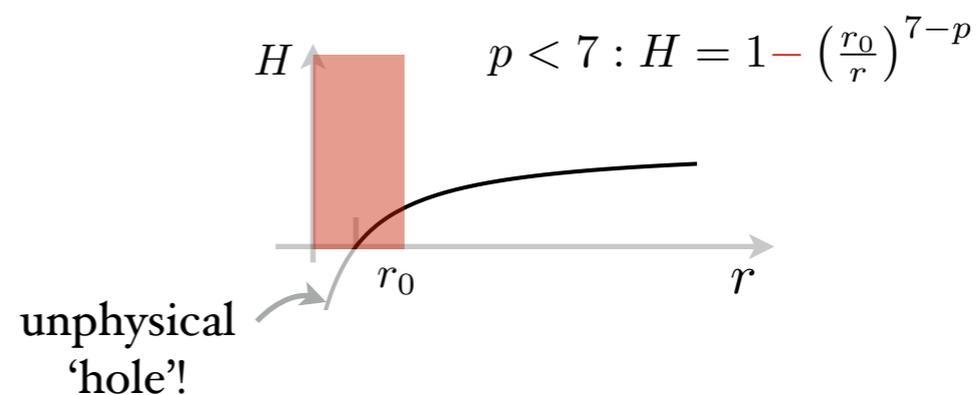
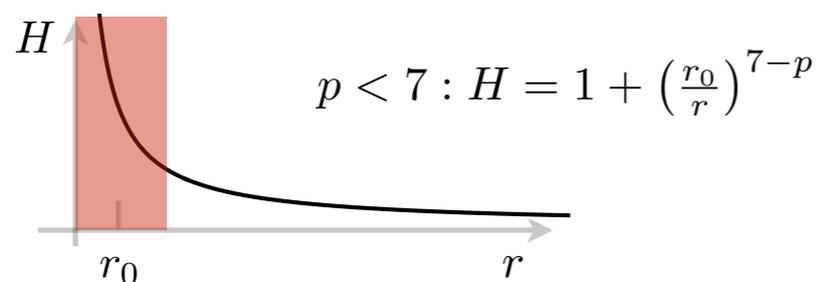
$$ds_{\perp}^2 = dr^2 + r^2 ds_{S^{8-p}}^2$$

- supergravity artifacts: they should be **resolved** in appropriate duality frame

D-branes

O-planes

[O_{p-} : tension=charge= -2^{p-5}]



- Example: AdS₇ in IIA. **All** solutions:

$$\frac{1}{\pi\sqrt{2}}ds^2 = 8\sqrt{-\frac{\alpha}{\ddot{\alpha}}}ds_{\text{AdS}_7}^2 + \sqrt{-\frac{\ddot{\alpha}}{\alpha}} \left(dz^2 + \frac{\alpha^2}{\dot{\alpha}^2 - 2\alpha\ddot{\alpha}} ds_{S^2}^2 \right)$$

↑
interval

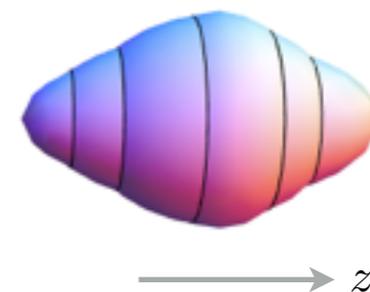
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$$B = \pi \left(-z + \frac{\alpha\dot{\alpha}}{\dot{\alpha}^2 - 2\alpha\ddot{\alpha}} \right) \text{vol}_{S^2}$$

$$F_2 = \left(\frac{\ddot{\alpha}}{162\pi^2} + \frac{\pi F_0 \alpha \dot{\alpha}}{\dot{\alpha}^2 - 2\alpha\ddot{\alpha}} \right) \text{vol}_{S^2}$$



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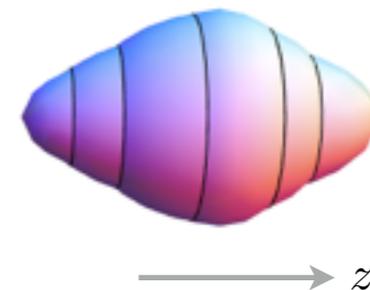
- Each BPS solution has a non-susy 'evil twin':

$$\frac{1}{\pi\sqrt{\cancel{2}}}ds^2 = \cancel{8} \sqrt{-\frac{\alpha}{\ddot{\alpha}}} ds_{\text{AdS}_7}^2 + \sqrt{-\frac{\ddot{\alpha}}{\alpha}} \left(dz^2 + \frac{\alpha^2}{\dot{\alpha}^2 - \cancel{\alpha}\ddot{\alpha}} ds_{S^2}^2 \right)$$

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some are **unstable**

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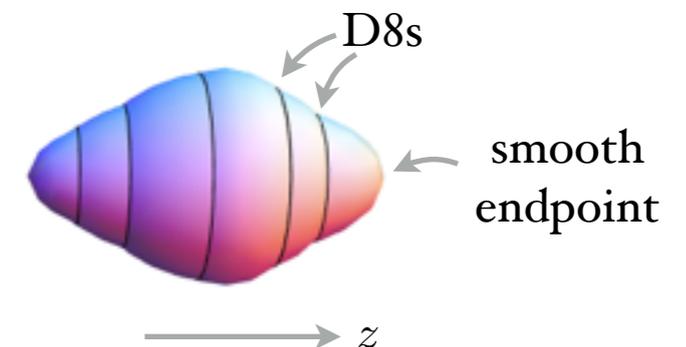
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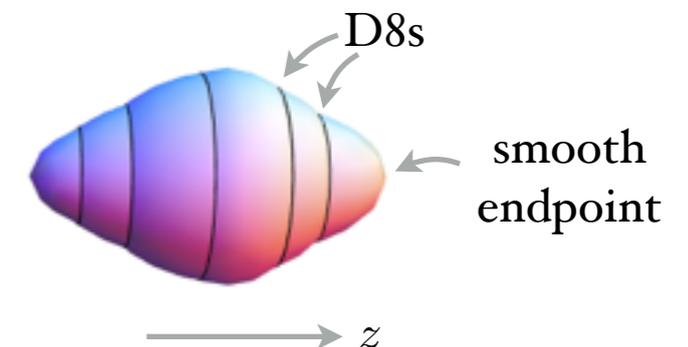
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what happens with other boundary conditions?

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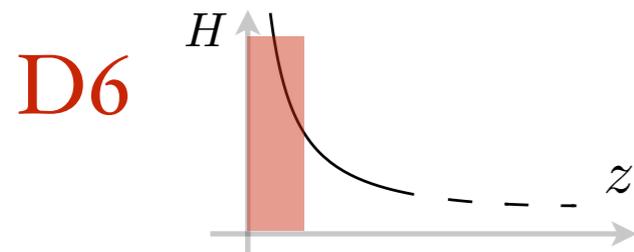
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transverse \mathbb{R}^3

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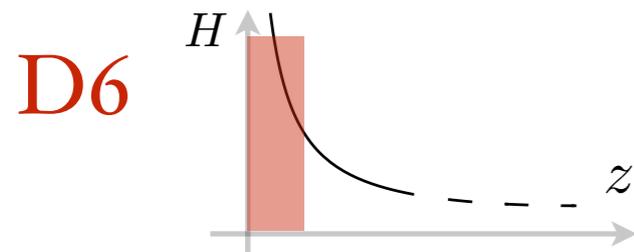
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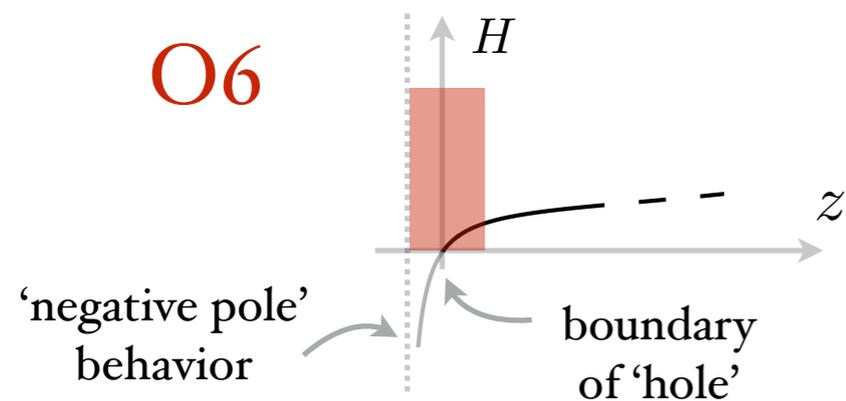
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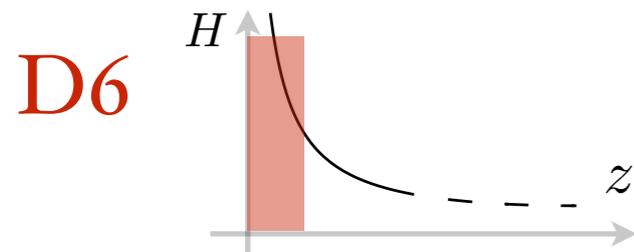
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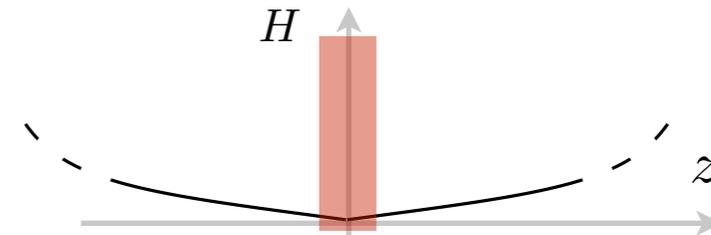
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[Bah, Passias, AT '17]

transverse \mathbb{R}

$$ds_{10}^2 \sim z^{-1/2}(ds_{\text{AdS}_7}^2 + ds_{S^2}^2) + z^{1/2}dz^2$$

O8



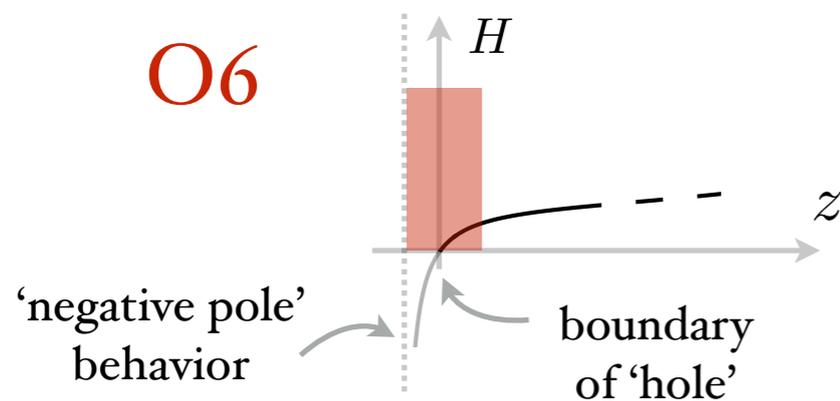
$$e^{\phi} \xrightarrow{z \rightarrow 0} \infty$$

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transverse \mathbb{R}^3

$$ds_{10}^2 \sim z^{-1/2}ds_{\text{AdS}_7}^2 + z^{1/2}(dz^2 + ds_{S^2}^2)$$

O6



$$\frac{1}{\pi\sqrt{2}}ds^2 = 8\sqrt{-\frac{\alpha}{\ddot{\alpha}}}ds_{\text{AdS}_7}^2 + \sqrt{-\frac{\ddot{\alpha}}{\alpha}}\left(dz^2 + \frac{\alpha^2}{\dot{\alpha}^2 - 2\alpha\ddot{\alpha}}ds_{S^2}^2\right)$$

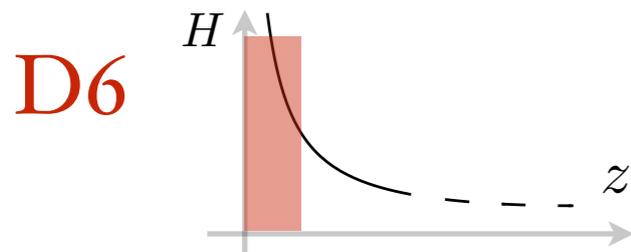
compare locally with

$$ds_{10}^2 = H^{-1/2}ds_{\parallel}^2 + H^{1/2}ds_{\perp}^2$$

- $\alpha \rightarrow 0$

transverse \mathbb{R}^3

$$ds^2 \sim z^{1/2}ds_{\text{AdS}_7}^2 + z^{-1/2}(dz^2 + z^2ds_{S^2}^2)$$

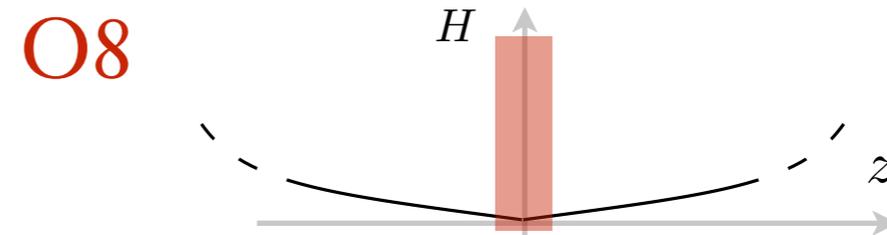


- $\alpha \rightarrow 0, \dot{\alpha} \rightarrow 0$

[Bah, Passias, AT '17]

transverse \mathbb{R}

$$ds_{10}^2 \sim z^{-1/2}(ds_{\text{AdS}_7}^2 + ds_{S^2}^2) + z^{1/2}dz^2$$

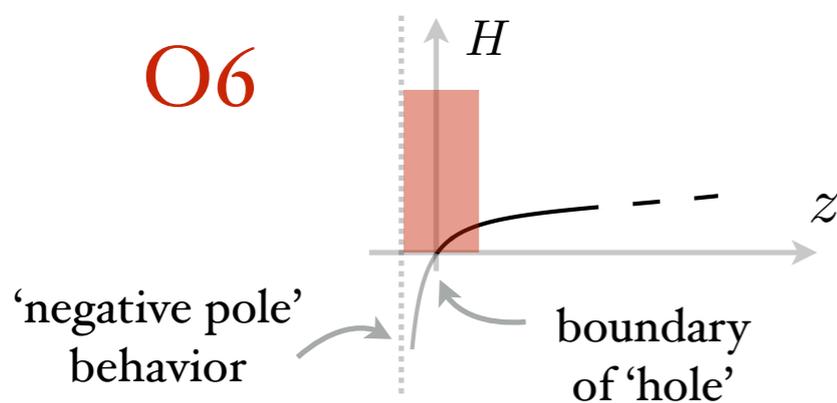


$$e^{\phi} \xrightarrow{z \rightarrow 0} \infty$$

- $\ddot{\alpha} \rightarrow 0$

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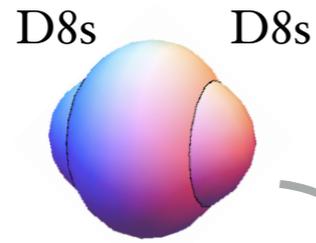
- Not always so easy...

- Supergravity artifacts, but same local behavior as solutions in flat space

• Holographic checks work with all sources

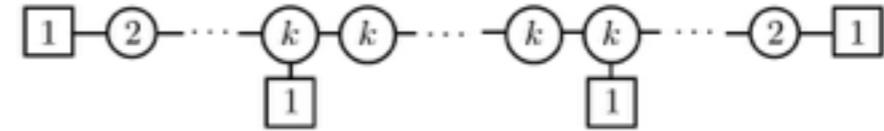
[Cremonesi, AT '15]
[Apruzzi, Fazzi '17]

Examples



integral over
internal dimensions
[Henningson, Skenderis '98]

dual quiver theory [SU gauge groups]



susy, grav. &
R-symmetry anomalies

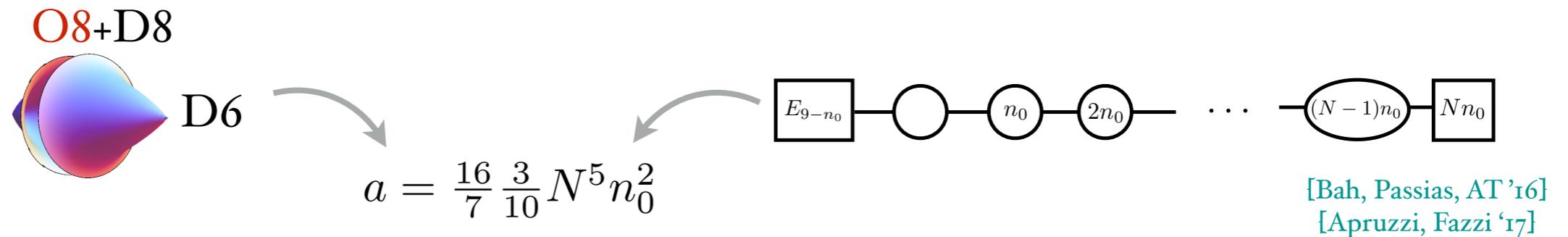
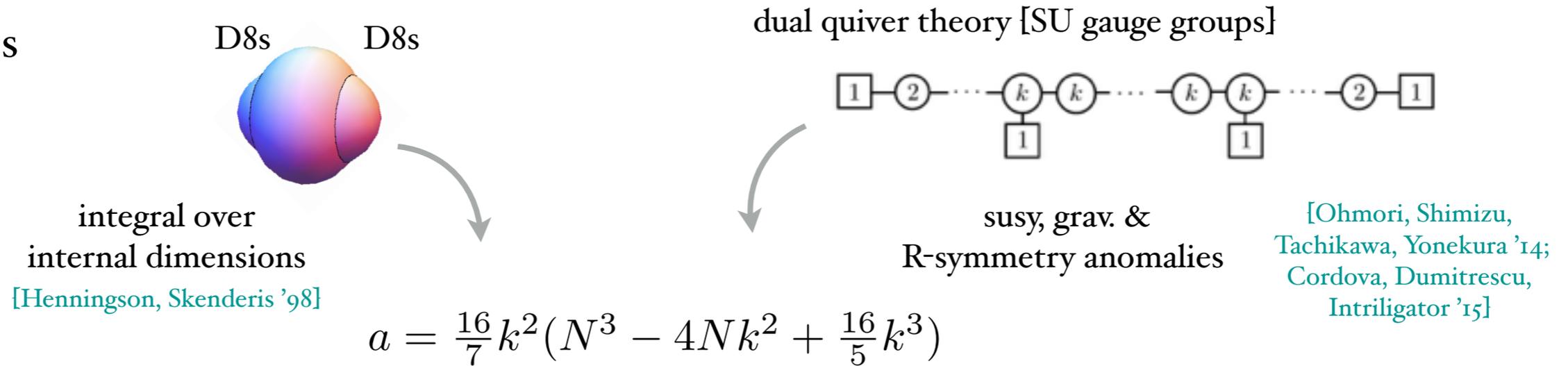
[Ohmori, Shimizu,
Tachikawa, Yonekura '14;
Cordova, Dumitrescu,
Intriligator '15]

$$a = \frac{16}{7} k^2 (N^3 - 4Nk^2 + \frac{16}{5} k^3)$$

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[Cremonesi, AT '15]
[Apruzzi, Fazzi '17]

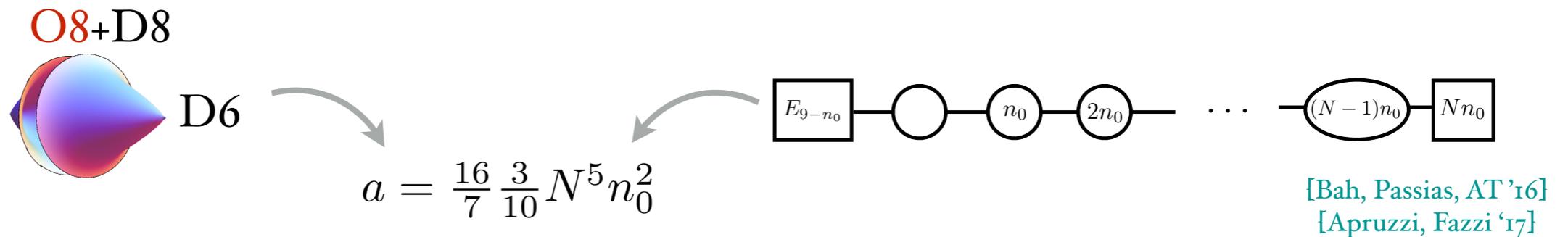
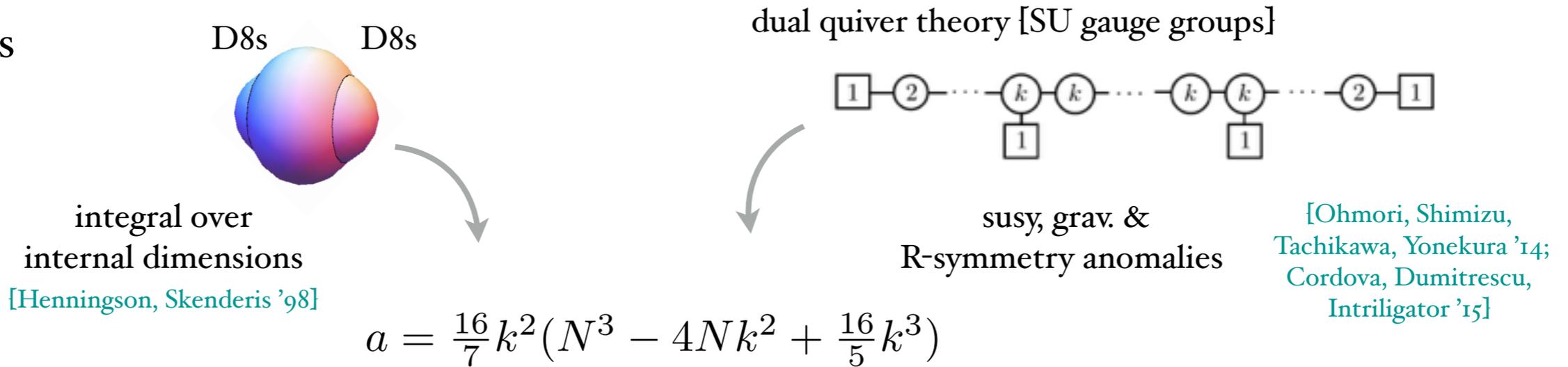
Examples



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[Cremonesi, AT '15]
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Examples

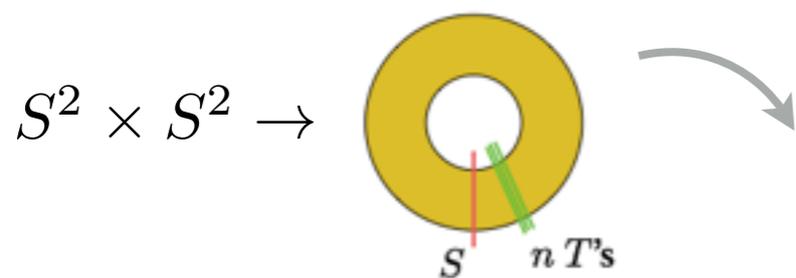


• Holographic check of S-folds:

sort of alternative to sources.
I was skeptical, but:

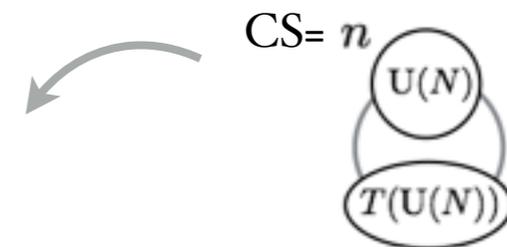
AdS₄ solution: [Inverso, Trigiante, Samtleben '16]

CFT₃ dual: [Assel, AT '18]



free energy =

$$\frac{1}{2}N^2 \ln \left(\frac{1}{2} \left(n + \sqrt{n^2 - 4} \right) \right)$$



see also [Garozzo, Lo Monaco, Mekareeya '18]

- Sources can be introduced in most classes

- AdS₇ in IIA: $S^2 \rightarrow I$

sources: D8, D6, O8, O6

- AdS₅ in IIA: $(\text{top. } S^3) \rightarrow \Sigma_g + \text{“punctures”}$

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- AdS₄ in IIA $(\text{top. } S^3) \rightarrow H_3, S^3$

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O8 $(\text{top. } S^2) \rightarrow \text{KE}_4, \Sigma_g \times \Sigma_{g'}$

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$\mathcal{N} = (0, 8), (0, 7) : F_4$ and G_3 superalg.

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- Other notable classes that admit sources:

- AdS₆ in IIB: (p, q) -fivebranes

[D'Hoker, Gutperle, Karch, Uhlemann '16...]

- AdS₅ in 11d: M₅

[Gaiotto, Maldacena '09...]

- AdS₄ $\mathcal{N} = 4$ in IIA: NS₅, D₅

[...Assel, Bachas, Estes, Gomis '11, '12]

- AdS₃ in F-theory

[Couzens, Lawrie, Martelli, Schäfer-Nameki '17; Haghighat, Murthy, Vandoren, Vafa '15]

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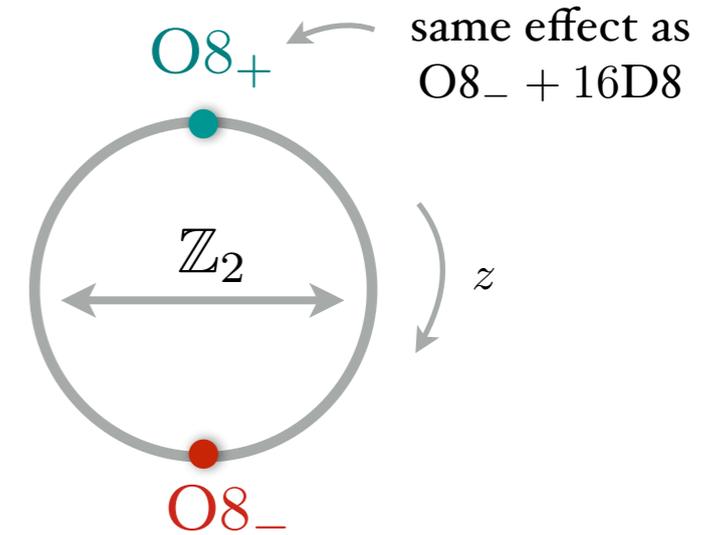
- Let's see if we can use this progress as inspiration for de Sitter...

dS

- Simplest model [Córdova, De Luca, AT '18]

$$ds^2 = e^{2W(z)} ds_{dS_4}^2 + e^{-2W(z)} (dz^2 + e^{2\lambda(z)} ds_{M_5}^2)$$

compact hyperbolic



Minkowski: [Dabholkar, Park '96, Witten '97, Aharony, Komargodski, Patir '07]

see also [Silverstein, Strings 2013 talk]

dS

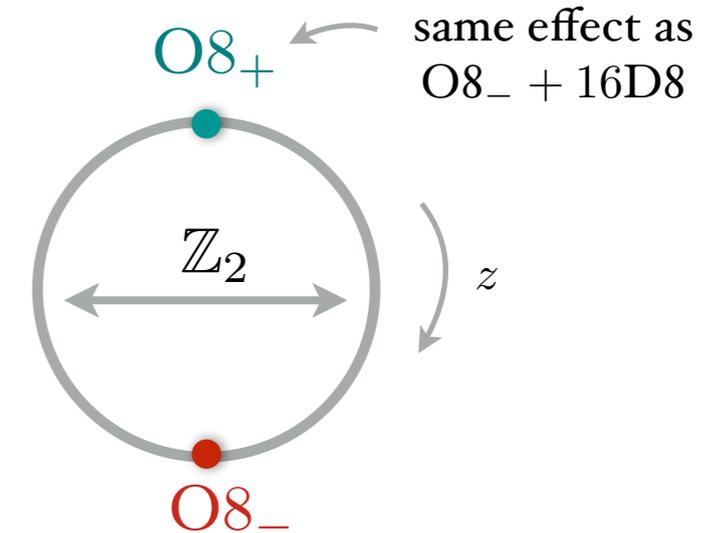
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Boundary condition at $O8_+$

$$e^{W-\phi} f'_i|_{z \rightarrow 0^+} = -1 \quad f_i = \{W, \frac{1}{5}\phi, \frac{1}{2}\lambda\}$$



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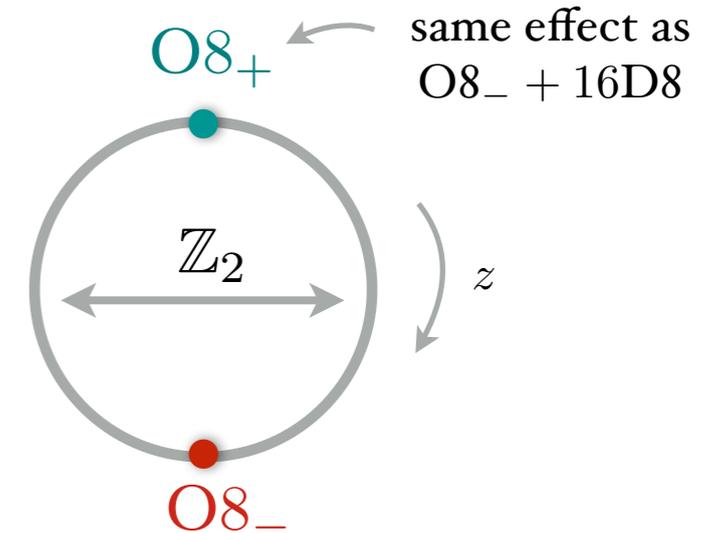
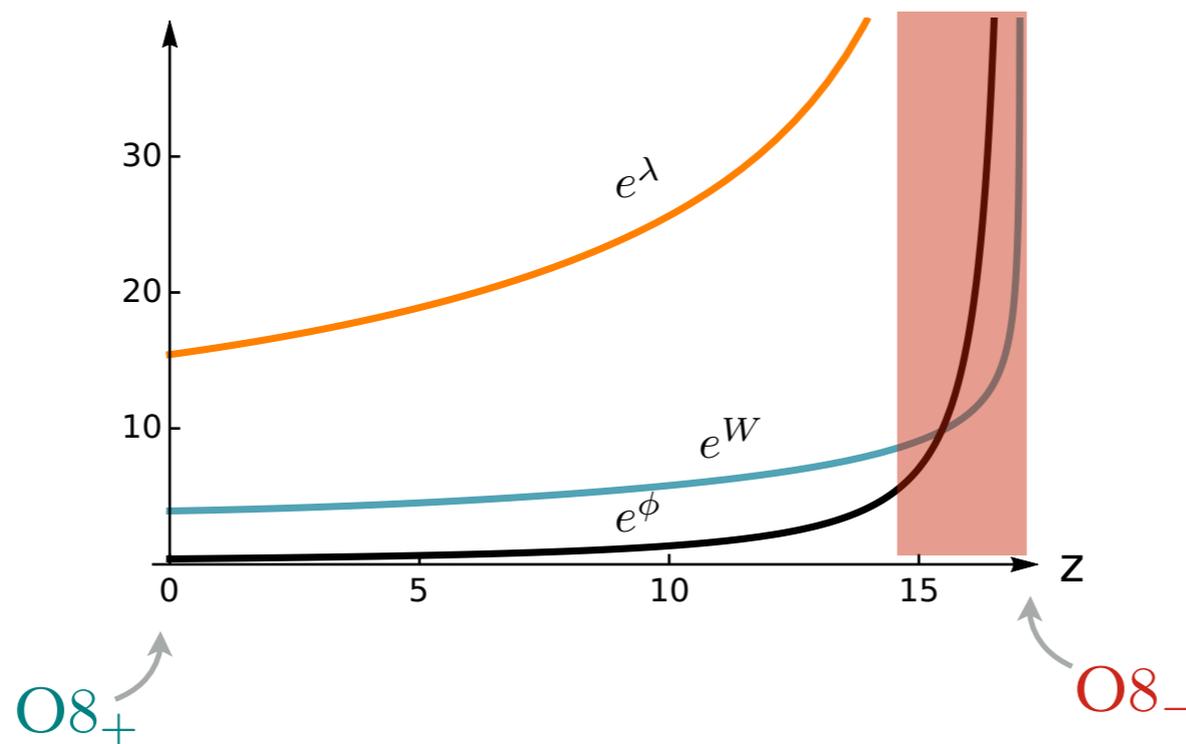
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Numerical evolution:
we manage to reach

$$e^{f_i} \sim |z - z_0|^{-1/4}$$



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same as $O8_-$ in flat space
[even the coefficients work]

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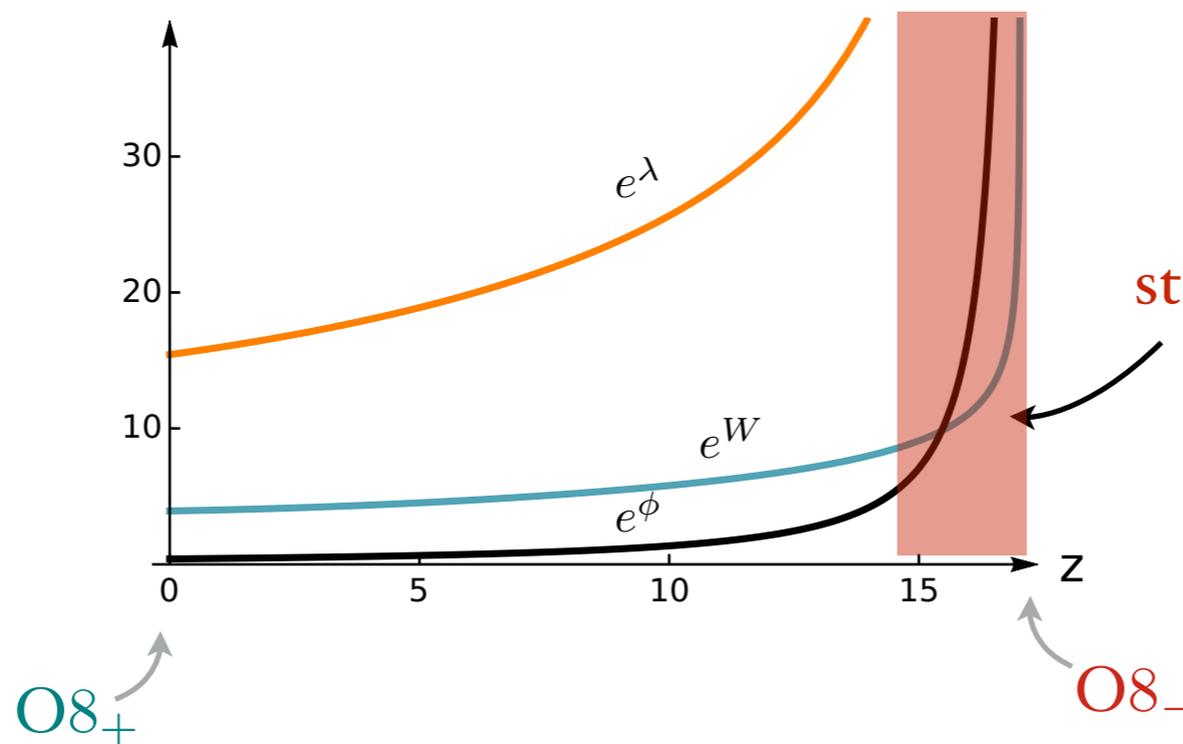
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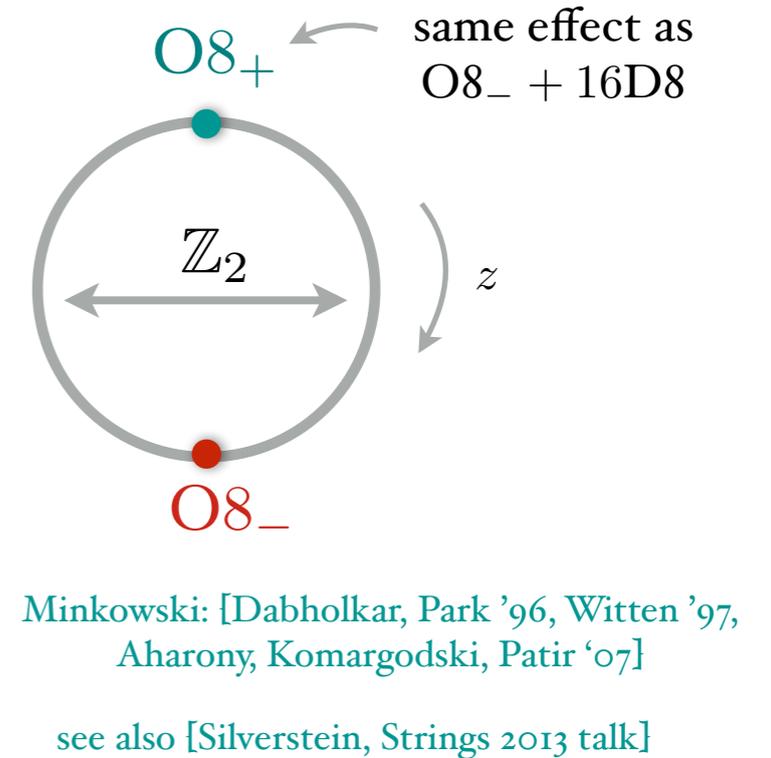
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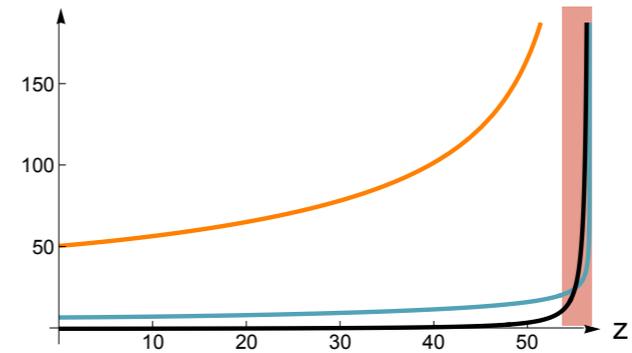
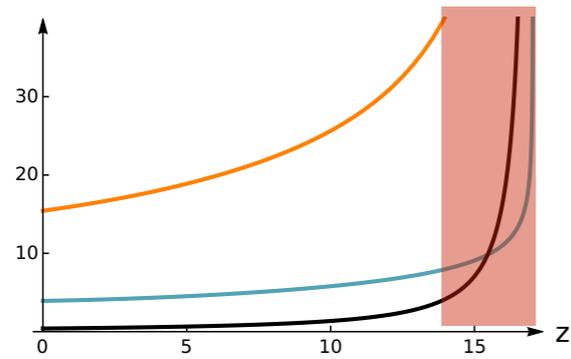


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● Rescaling symmetry:

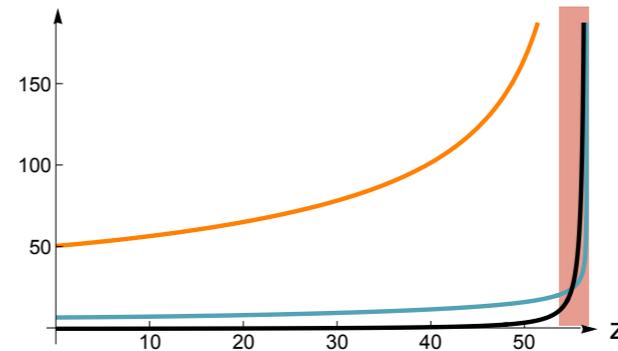
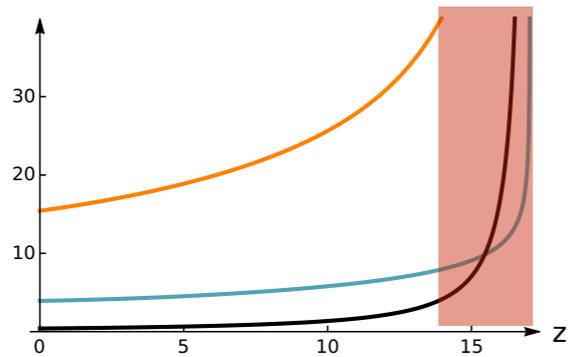
$$g_{MN} \rightarrow e^{2c} g_{MN}, \phi \rightarrow \phi - c$$



it makes strong-coupling region small, but it doesn't make it disappear.

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- In the O8_ region stringy corrections become dominant

$$\dots \gg e^{-2\phi} R^4 \gg e^{-2\phi} R$$

$$\hat{\hat{R}}^4$$

supergravity action is **least important term**;
ideally in this region we'd switch to another duality frame.

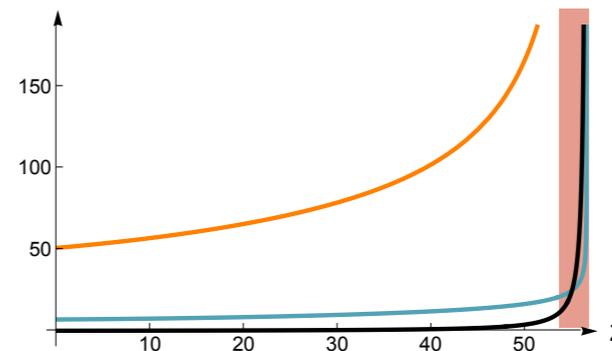
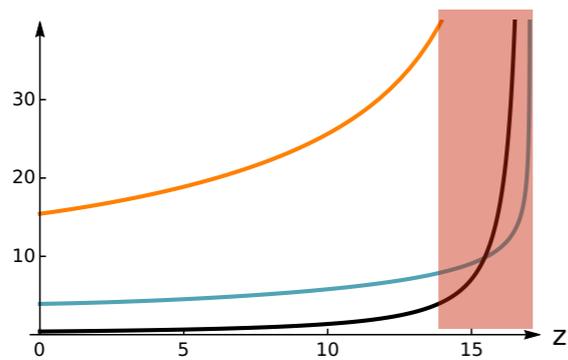
In other words: full string theory will fix c

it has been \sim argued that supergravity contributes to this

[Cribiori, Junghans '19]

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[Cribiori, Junghans '19]

- Hope that this solution is sensible comes from similarity with flat-space O8₋
 (which we know to exist in string theory)

- We also tried: $O8_+ - O6_-$

[Córdova, De Luca, AT, work in progress]

$$ds^2 = e^{2W} ds_{dS_4}^2 + e^{-2W} (dz^2 + e^{2\lambda_3} ds_{M_3}^2 + e^{2\lambda_2} ds_{S^2}^2)$$

surrounds the $O6$

$$H = h_1 dz \wedge \text{vol}_2 + h_2 \text{vol}_3$$

$$F_2 = f_2 \text{vol}_2$$

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$$F_0 \neq 0$$

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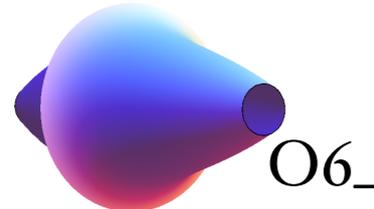
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$$\begin{aligned} H &= h_1 dz \wedge \text{vol}_2 + h_2 \text{vol}_3 \\ F_2 &= f_2 \text{vol}_2 \\ F_4 &= f_{41} \text{vol}_3 \wedge dz + f_{42} \text{vol}_4 \\ F_0 &\neq 0 \end{aligned}$$

- we already know one such solution for $\Lambda < 0$:

from a **non-susy AdS₇ solution** with $O8_+$ and $O6_-$

$$\alpha = 3k(N^2 - z^2) + n_0(z^3 - N^3)$$



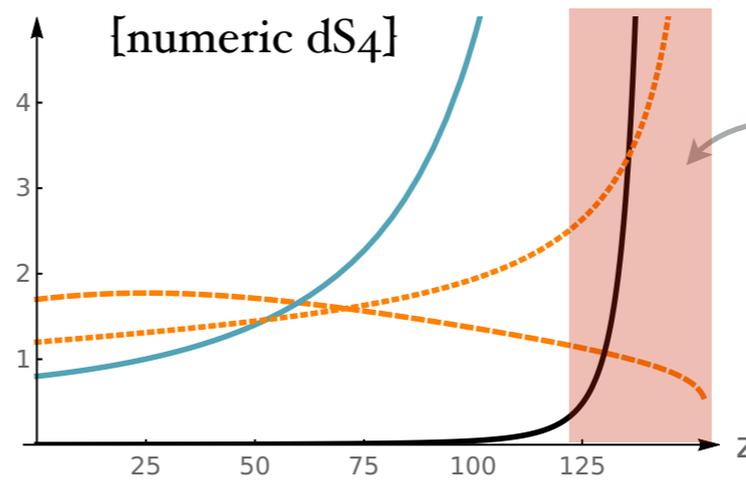
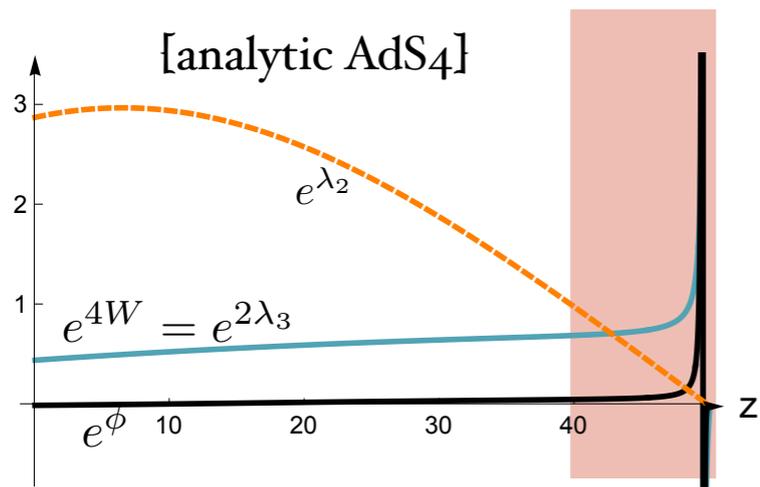
$$\frac{1}{\sqrt{\pi}} ds^2 = 12 \sqrt{-\frac{\alpha}{\ddot{\alpha}}} ds_{\text{AdS}_7}^2 + \sqrt{-\frac{\ddot{\alpha}}{\alpha}} \left(dz^2 + \frac{\alpha^2}{\dot{\alpha}^2 - \alpha \ddot{\alpha}} ds_{S^2}^2 \right)$$

\downarrow
 $\text{AdS}_4 \times H_3$ ← compact hyperbolic

- we slowly modified it numerically, bringing Λ up

$$ds^2 = e^{2W} ds_{dS_4}^2 + e^{-2W} (dz^2 + e^{2\lambda_3} ds_{M_3}^2 + e^{2\lambda_2} ds_{S^2}^2)$$

[functions rescaled for clarity]

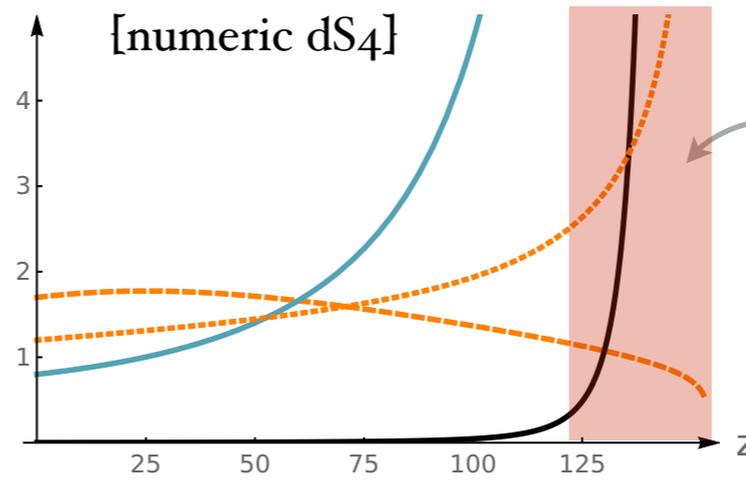
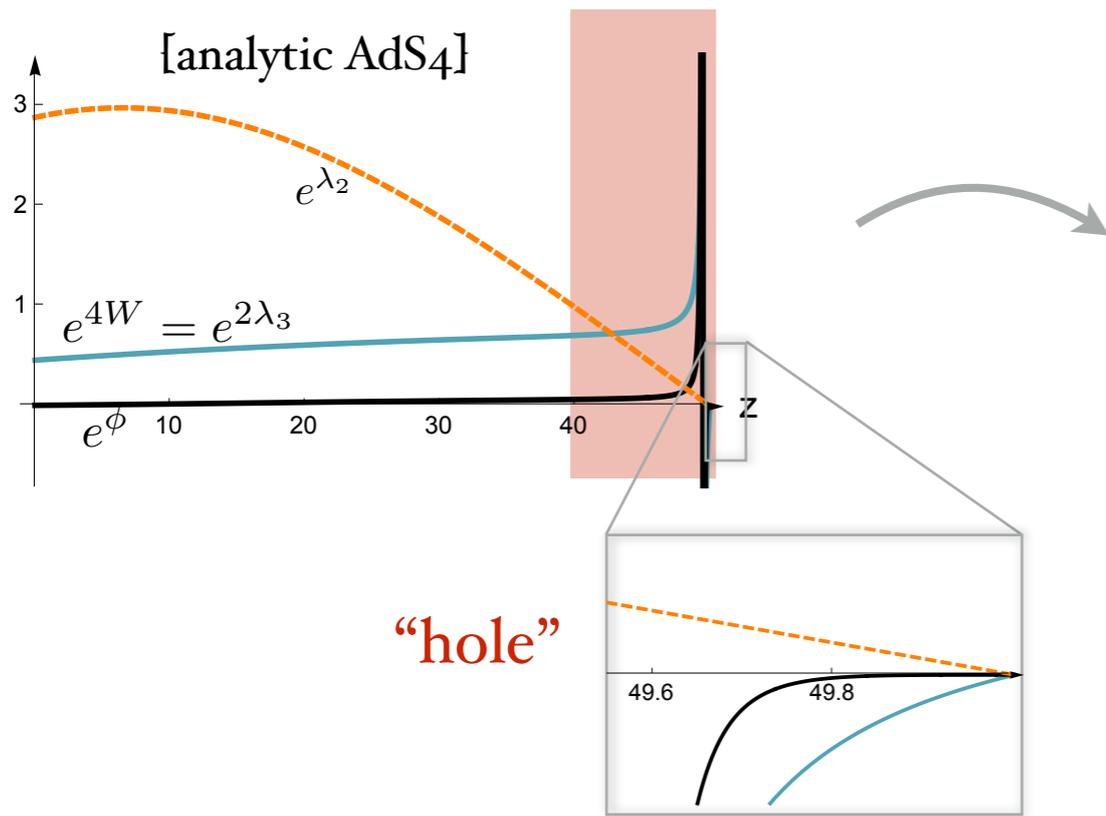


We still obtain the O6 boundary.

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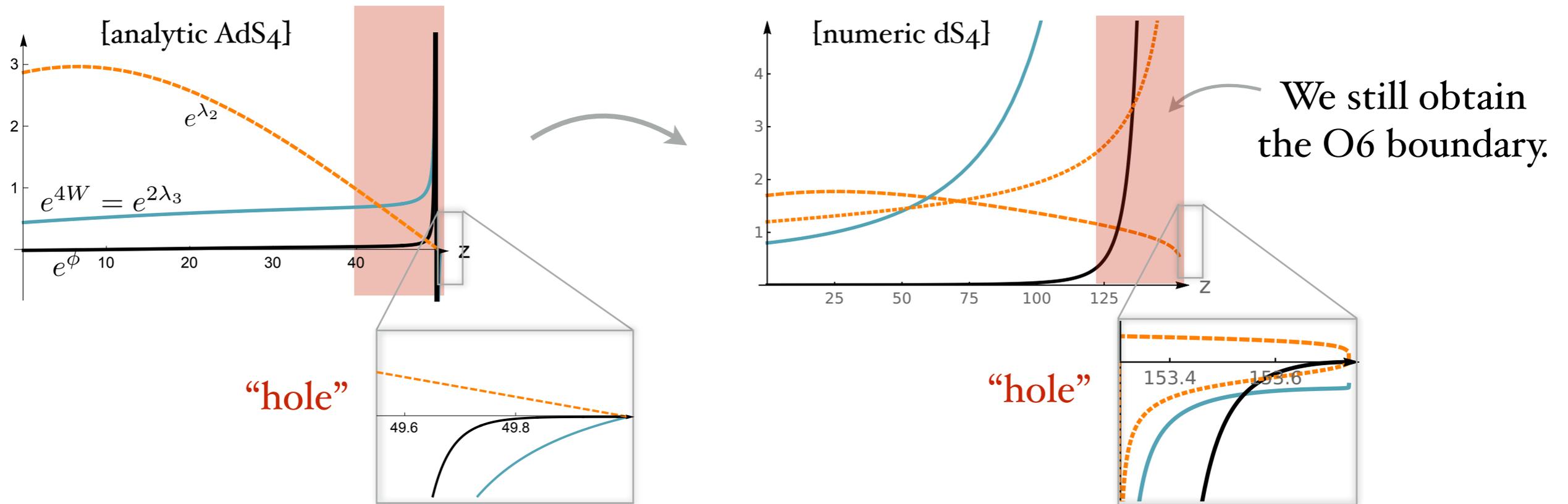
- Recall: for AdS solution we can analytically ‘inside the hole’

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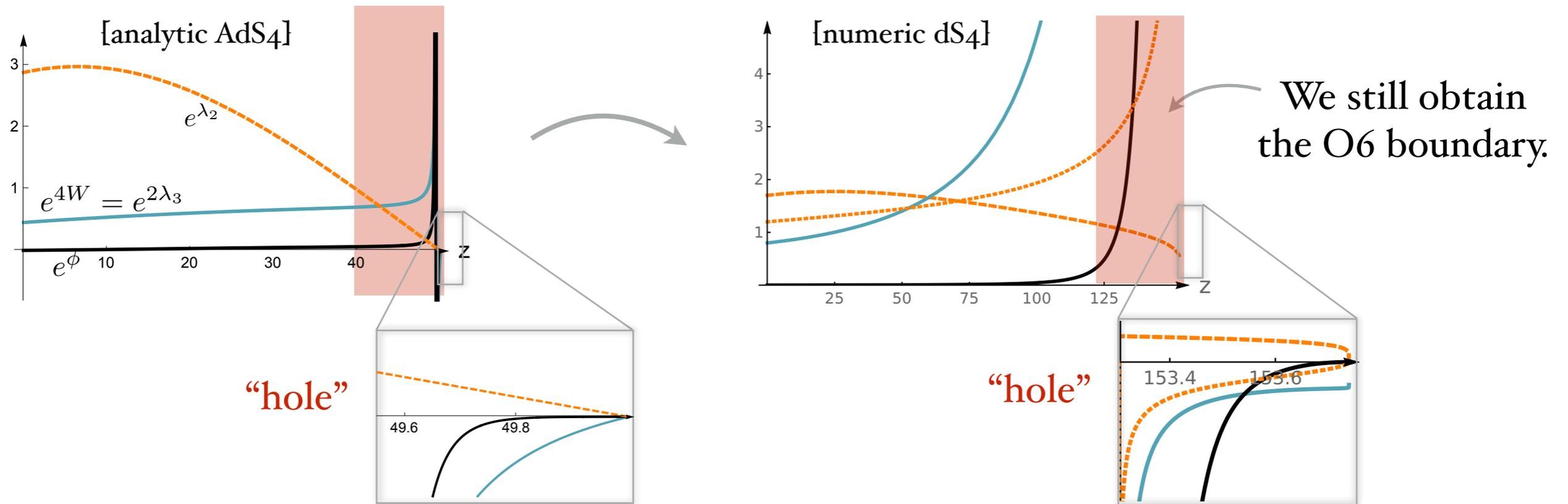
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- A perhaps more physical procedure: probe analysis

perhaps following

[Sen '96, ... Saracco, AT, Torroba '13]

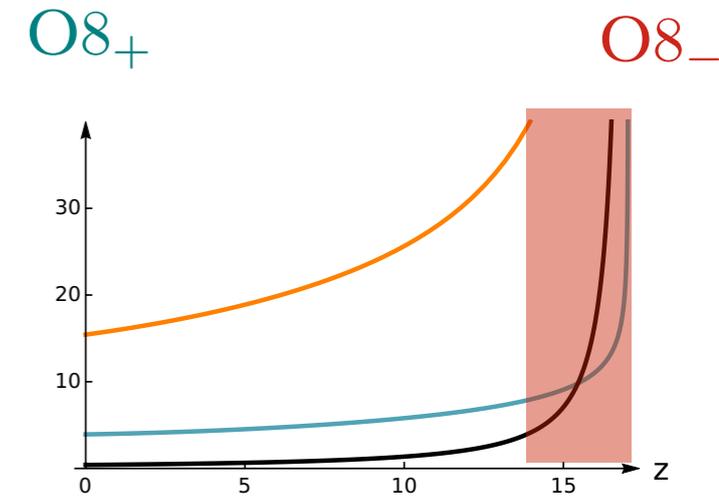
Conclusions

- A lot of progress in AdS solutions
 - often **localized O-plane** sources are possible
 - holography works even in their presence
 - sometimes non-supersymmetric
- Time to look for de Sitter
 - Using numerics, we find dS solutions with O8-planes in relatively simple setup
 - Also O8-O6 solutions
 - There are regions where supergravity breaks down.
Inevitable! If you want solutions with O-planes.
We better learn how to deal with them.

Backup slides

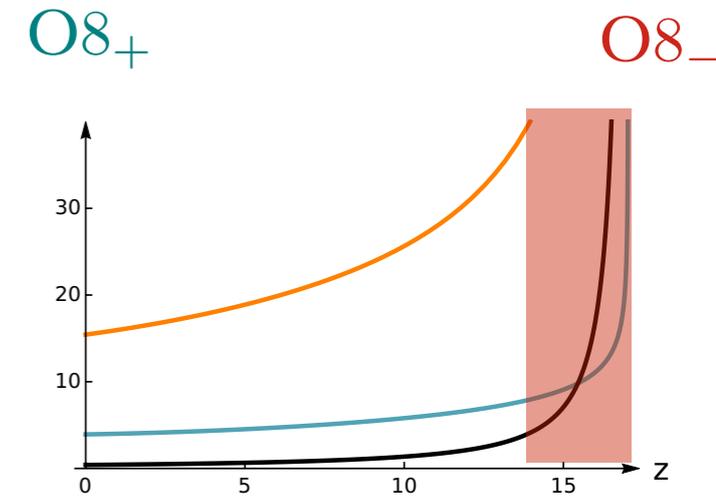
Possible criticism of the O8-O8 model

• O8₊: $\partial_z^2 \left(\begin{array}{c} \nearrow \\ \leftarrow \end{array} \right) = -\delta \Rightarrow e^{W-\phi} f'_i |_{z \rightarrow 0^+} = -1$



Possible criticism of the O8-O8 model

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- Near O8₋, supergravity **breaks down**;
we shouldn't take its EoMs seriously.

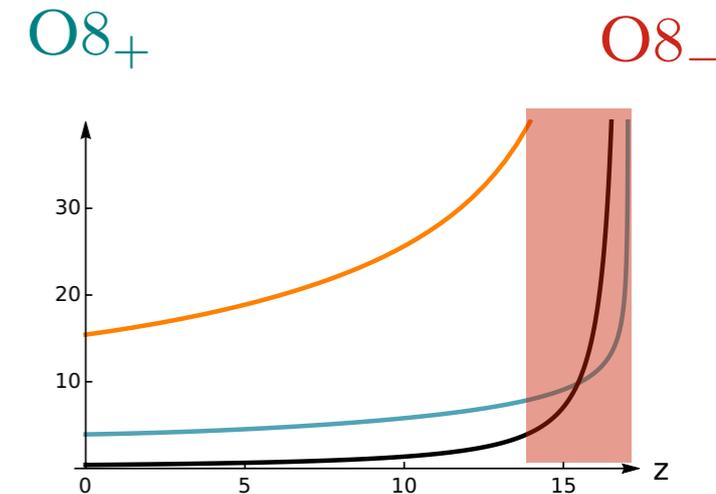


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Let's do it anyway...



• if we extrapolate from O8₊ with $a \neq 0$: $\partial_z^2 \left(\begin{array}{c} \searrow \\ \leftarrow \end{array} \right) = \delta \Rightarrow e^{W-\phi} f'_i|_{z \rightarrow z_0^+} = 1$

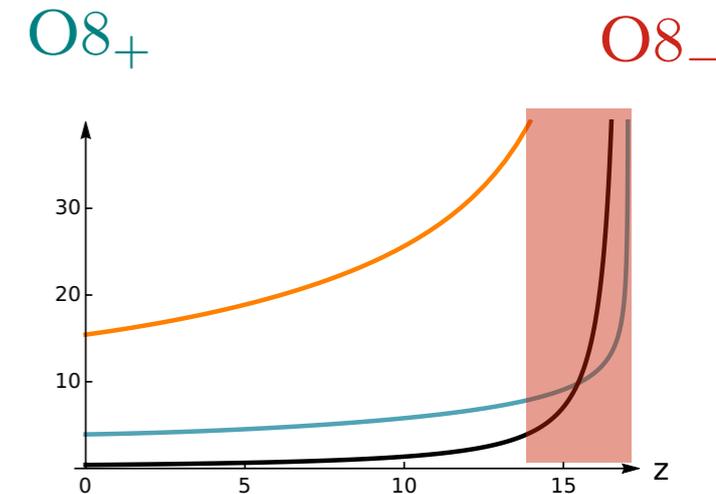
$e^{W-\phi} \sim |z - z_0|, f_i \sim \log |z - z_0| \Rightarrow$ so this works ✓

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$$e^{W-\phi} \sim |z - z_0|, \quad f_i \sim \log |z - z_0| \quad \Rightarrow \quad \text{so this works } \checkmark$$

- but if we rewrite it $f'_i = e^{\phi-W}$

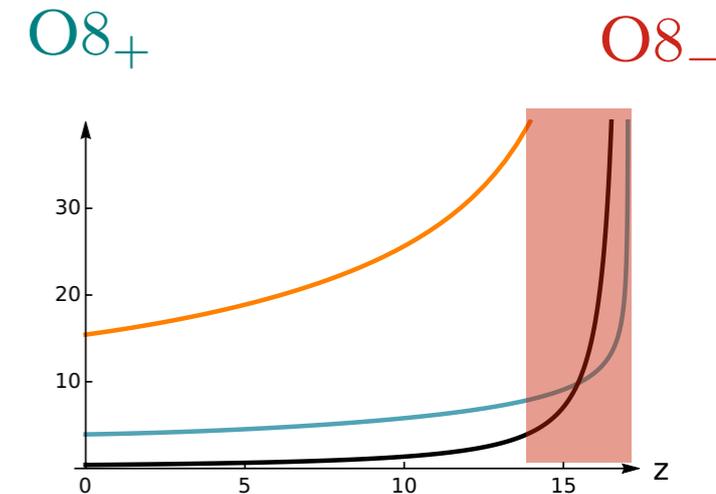
works at leading $\frac{1}{|z-z_0|}$ order, but not with subleading constant.

Possible criticism of the O8-O8 model

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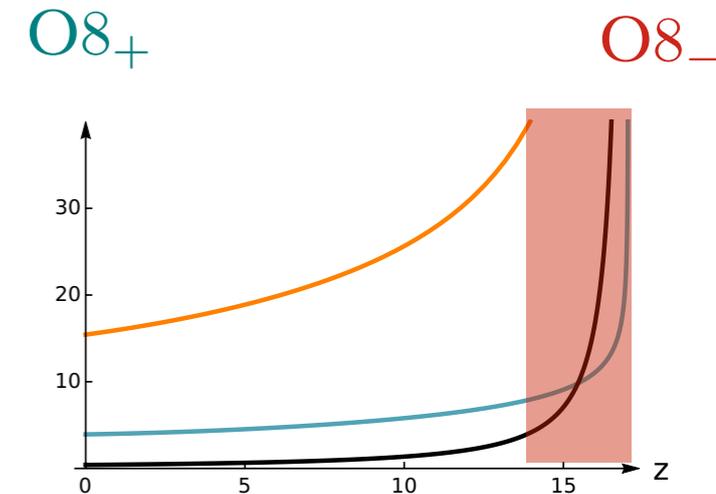
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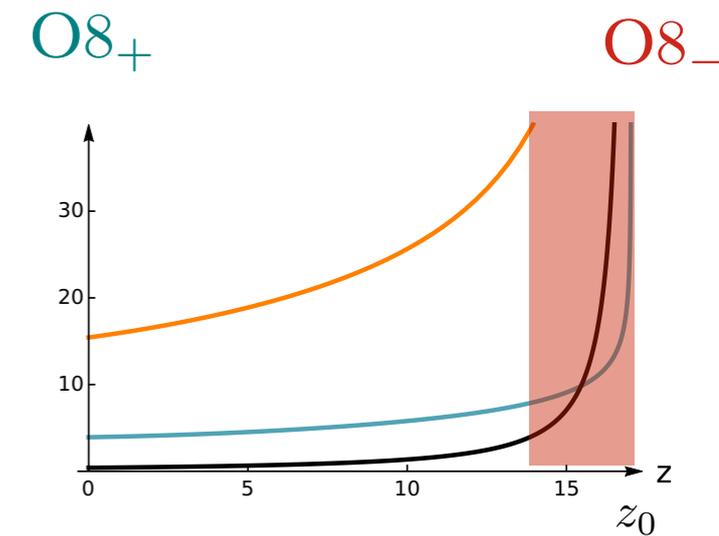
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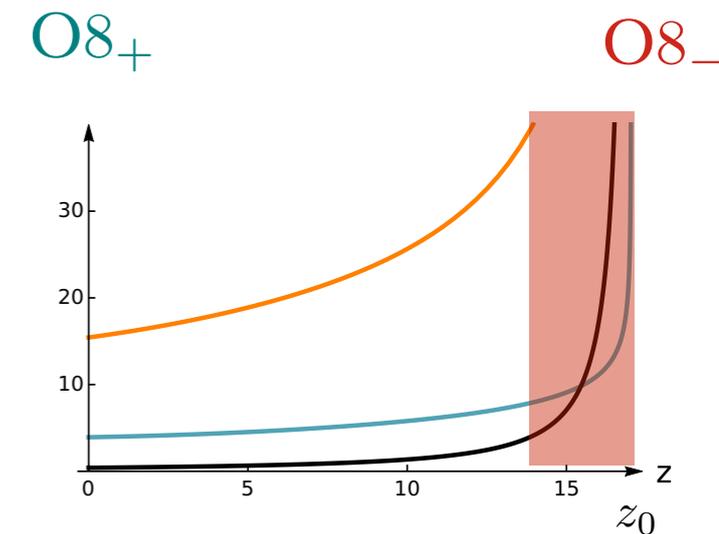
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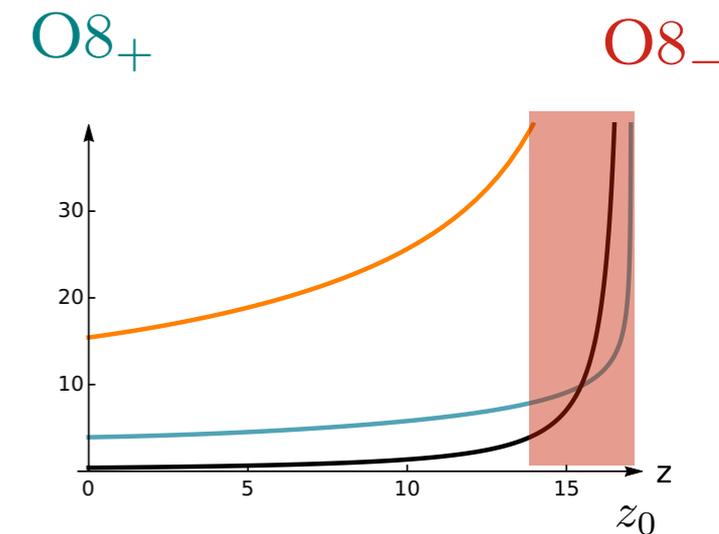
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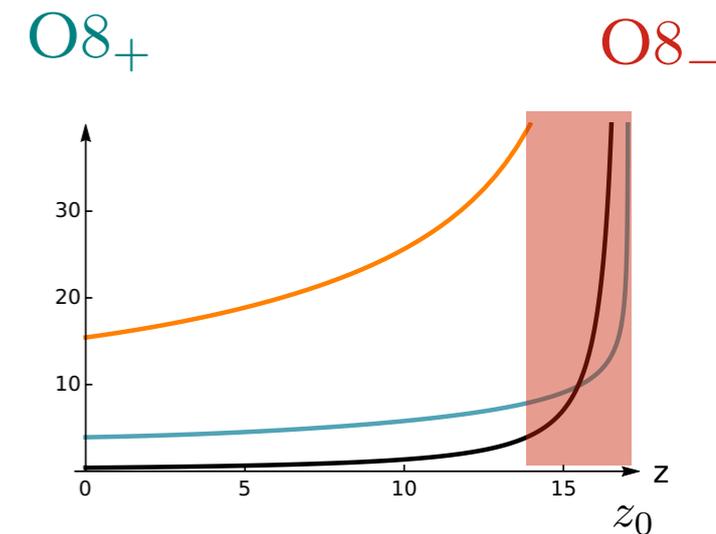
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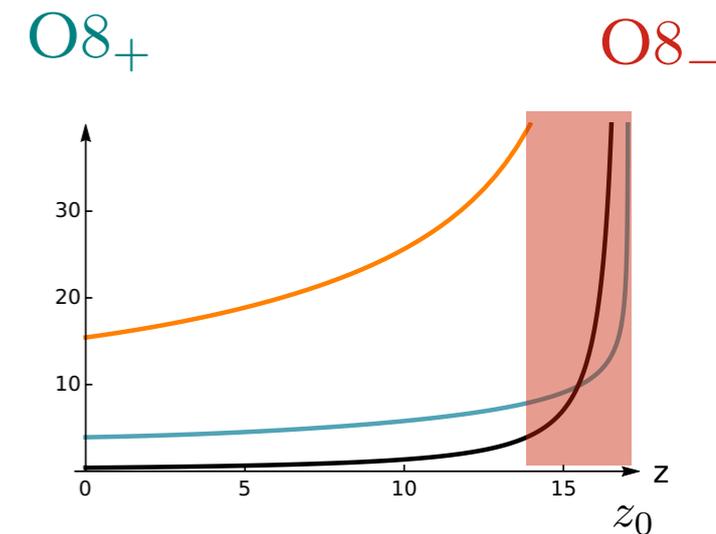
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