Solving a 40-year-old Problem: 11D Superfields

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Strings 2021 Gong Show

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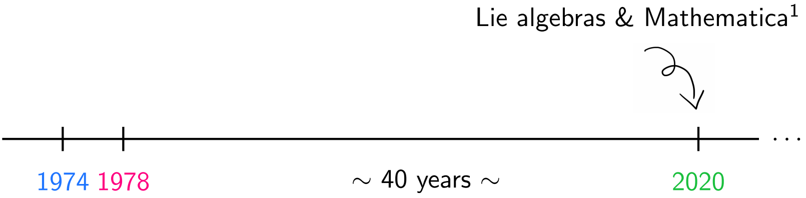
[arXiv: 1911.00807, 2002.08502, 2006.03609, 2007.05097, 2007.07390]

Motivation



- 1974: first 4D superfield written down! [Salam, Strathdee 1974]
- 1978: first 11D on-shell supergravity! [Cremmer, Julia, Scherk 1978]

  ⋮

- 2020: 11D superfield!!! [Gates, YH, SNHM, arXiv:2002.08502]

[1] LieART [Feger, Kephart, 2012], SUSYno [Fonseca, 2011]

The Question

In 11D, we have 32 Grassmann coordinate θ's. Since $\theta^2 = 0$,

$$\mathcal{V}(x,\theta) \;=\; v^{(0)}(x) + \sum_{n=1}^{32} v^{(n)}_{\alpha_1\cdots\alpha_n}(x)\,\theta^{\alpha_1}\cdots\theta^{\alpha_n}$$

Question:

How to write components / θ-monomials in $\mathfrak{so}(1,10)$ irreducible representations?

Difficulties:

- $2^{32} = 4,294,967,296$ total degrees of freedom
- many Fierz identities...
- Lorentz covariant θ-monomials at higher levels

Step 1: Young Tableaux

Totally antisymmetric product as YT:

$$\theta^{\alpha_1} \cdots \theta^{\alpha_n} \quad \Leftrightarrow \quad \underbrace{\{32\} \wedge \; \ldots \; \wedge \{32\}}_{n \text{ times}} \quad \Leftrightarrow \quad \begin{array}{|c|} \hline 32 \\ \hline \vdots \\ \hline 32 \\ -n+1 \\ \hline \end{array}$$

Theorem:

$$\text{YT} \quad \overset{\text{1-1}}{\Longleftrightarrow} \quad \text{Irreducible representation of } \mathfrak{su}(32)$$

Step 2: Branching Rules

$$\mathfrak{su}(32) \supset \mathfrak{so}(1,10) \quad \Rightarrow \quad \mathcal{R}_{\mathfrak{su}(32)} \xrightarrow{\text{branching rules}} \bigoplus \mathcal{R}_{\mathfrak{so}(1,10)}$$

Examples:

$$\{496\} \longrightarrow \{1\} \oplus \{165\} \oplus \{330\}$$

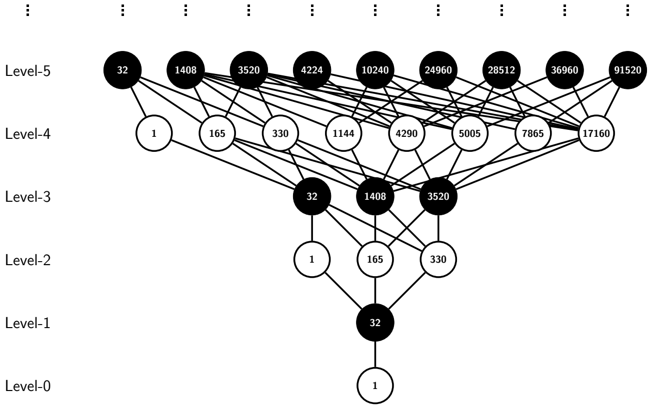
$$\Phi \qquad \Phi_{[\underline{abc}]} \qquad \Phi_{[\underline{abcd}]}$$

$$\{4{,}960\} \longrightarrow \{32\} \oplus \{1{,}408\} \oplus \{3{,}520\}$$

$$\Psi_{\alpha} \qquad \Psi_{[\underline{ab}]\alpha} \qquad \Psi_{[\underline{abc}]\alpha}$$

Dictionary & Graphical Rules in [Gates, YH, SNHM, arXiv:2006.03609]

11D Scalar Superfield



- Level-0 to level-32, symmetric about level-16
- Breitenlohner's method: $\mathcal{V}_{[\mathcal{R}]} = \mathcal{V} \otimes [\mathcal{R}]$ [Breitenlohner 1977]

11D Supergravity Surprise!

- $\mathcal{V} \Rightarrow$ linearized Nordström SG [Gates, YH, Jiang, SNHM 2019]
- Semi-prepotential candidate: $\mathcal{V} = \mathrm{D}^{\alpha}\,\mathcal{V}_{\alpha}$

Physical Component	Irrep	Level
graviton $h_{\underline{ab}}$	$\{65\}$, $\{1\}$	16
gravitino $\psi_{\underline{a}}{}^{\beta}$	$\{320\}$, $\{32\}$	17
3-form $B_{[3]}$	$\{165\}$	16

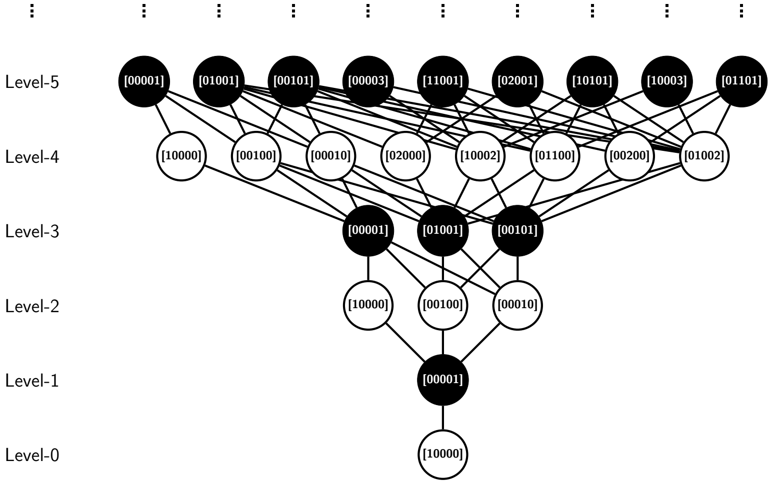
- Prepotential candidate: $\mathcal{V}_{\alpha}$
  contains 2-form $h_{[\underline{ab}]} = \{55\}$ at level-17 $\Rightarrow$ Poincaré vielbein

THANK YOU!!

Backup: More Background

- Our work leads to a formalism demonstrating a *manefest linear* realization of $\mathfrak{so}(1,10)$!
  - GS-formalism [Green and Sethi 1999]: started with a complex chiral spinor in $\mathfrak{so}(1,9) \Rightarrow$ coset symmetry $\mathfrak{so}(1,10)/\mathfrak{so}(1,9)$ realized in some *non-linear* manner
  - [Becker and Butter 2003]: coset symmetry $\mathfrak{so}(1,10)/\mathfrak{so}(1,3)$ realized in some *non-linear* manner
- A lot of previous work focused on modified SUGRA covariant derivatives, field strengths, Bianchi identities, etc.
  They are "orthogonal" to our current efforts.
- [Gates, Vashakidze 1987] showed the relationship between off-shell superfields and higher derivative terms in the *open string* effective action

Backup: 11D Adynkra



Backup: Poincaré vielbein & Gravitino

Decompositions of the inverse frame and gravitino fields in 11D yield

$$\begin{array}{ccccc} e_{\underline{a}}{}^{m} = & \{ h_{(\underline{ab})} + & \eta_{\underline{ab}} h + & h_{[\underline{ab}]} \} \, \eta^{\underline{bm}} \\ \{121\} & \{65\} & \{1\} & \{55\} \end{array}$$

where $h_{(\underline{ab})}$ is the conformal graviton, h is the trace, and $h_{[\underline{ab}]}$ is the two-form; and

$$\begin{array}{cccc} \tilde{\psi}_{\underline{a}}{}^{\alpha} = & \psi_{\underline{a}}{}^{\alpha} - & \frac{1}{11} (\gamma_{\underline{a}})^{\alpha\beta} \psi_{\beta} \\ \{352\} & \{320\} & \{32\} \end{array}$$

where $\psi_{\underline{a}}{}^{\alpha}$ is the conformal gravitino and $\psi_{\beta} \equiv (\gamma^{\underline{a}})_{\alpha\beta} \tilde{\psi}_{\underline{a}}{}^{\alpha}$ is the γ-trace.