Logarithmic Corrections to the Entropy of AdS Black Holes

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Strings 2021
Sao Paulo, June 22, 2021

U.S. DEPARTMENT OF ENERGY

Office of Science

Why logarithmic corrections to the entropy?

- The Universality of the Bekenstein-Hawking entropy formula

$$S = \frac{k_B c^3}{\hbar} \frac{A}{4G}$$

- Corrections/Physics: Gravity as an effective field theory (higher curvature) and logarithmic corrections (massless sugra fields)

$$S = \frac{A}{4G} + a \, \log\left(\frac{A}{G}\right) + \ldots$$

- Topic: The coefficient a in the AdS/CFT correspondence: Field Theory (micro)/ Gravity (macro).

Logarithmic terms: An IR window into UV physics

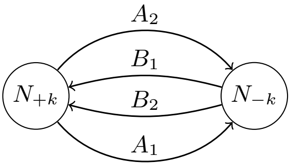
- The coefficient of the logarithmic corrections is determined macroscopically from the massless particle spectrum, insensitive to the UV completion of the theory.
- A litmus test: Any enumeration of quantum black hole microstate must agree with the macro result.
  - Sen and collaborators have checked various asymptotically flat black holes in string theory (precision Strominger-Vafa) [Sen '11] and challenged loop quantum gravity [Sen '12]. Quantum Entropy Formula [Sen '08].
  - Microscopic realization of Kerr/CFT [Pathak-Porfyriadis-Strominger-Varela '16].
  - AdS/CFT: Free energy of ABJM on S^3 [Bhattacharyya-Grassi-Mariño-Sen '12] $F_{S^3} \sim \mathrm{Ai}(N, k_a) \mapsto -\frac{1}{4}\log N$.
  - AdS black holes: The field theory answer must agree with massless sugra answer.

Outline

- The Topologically Twisted Index of 3d $\mathcal{N}=2$ Chern-Simons matter theories beyond large N
- Logarithmic corrections to $\mathrm{AdS}_4 \times SE_7$ black holes : $a \, \log\left(\frac{L}{\ell_P}\right)$
- The superconformal index of $\mathcal{N}=4$ SYM beyond large N
- Kerr/CFT and logarithmic corrections to AdS_5 black holes
- Open problems

ABJM Theory

- ABJM: A $U(N)_k \times U(N)_{-k}$ Chern-Simons-matter theory.



- The topologically twisted index as the supersymmetric partition function of the twisted theory on $S^1 \times S^2$ [Benini-Zaffaroni '15]:

$$Z(n_a, \Delta_a) = \mathrm{Tr}\,(-1)^F e^{-\beta\{\mathcal{Q},\mathcal{Q}^\dagger\}} e^{iJ_a\Delta_a}$$

  - Counts $\frac{1}{2}$-BPS states annihilated by the supercharge $\mathcal{Q}$.
  - J_a, generators of flavor symmetries.

The Topologically Twisted Index

- The topologically twisted index for ABJM theory [Benini-Hristov-Zaffaroni '15]:

$$Z(n_a,\Delta_a) = \prod_{a=1}^{4} y_a^{-\frac{1}{2}N^2 n_a} \sum_{I\in BAE} \frac{1}{\det \mathbb{B}} \times \frac{\prod_{i=1}^N x_i^N \tilde{x}_i^N \prod_{i\neq j}\left(1-\frac{x_i}{x_j}\right)\left(1-\frac{\tilde{x}_i}{\tilde{x}_j}\right)}{\prod_{i,j=1}^N \prod_{a=1,2}(\tilde{x}_j - y_a x_i)^{1-n_a}\prod_{a=3,4}(x_i - y_a\tilde{x}_j)^{1-n_a}}.$$

- Contour integral → Evaluation (Poles): $e^{iB_i} = e^{i\tilde{B}_i} = 1$ and $\mathbb{B}$ is the Jacobian

$$e^{iB_i} = x_i^k \prod_{j=1}^N \frac{(1-y_3\frac{\tilde{x}_j}{x_i})(1-y_4\frac{\tilde{x}_j}{x_i})}{(1-y_1^{-1}\frac{\tilde{x}_j}{x_i})(1-y_2^{-1}\frac{\tilde{x}_j}{x_i})},\quad e^{i\tilde{B}_j} = \tilde{x}_j^k \prod_{i=1}^N \frac{(1-y_3\frac{\tilde{x}_j}{x_i})(1-y_4\frac{\tilde{x}_j}{x_i})}{(1-y_1^{-1}\frac{\tilde{x}_j}{x_i})(1-y_2^{-1}\frac{\tilde{x}_j}{x_i})}.$$

- An exact expression in N which can be computed numerically.

Logarithmic Correction to the Topologically Twisted Index

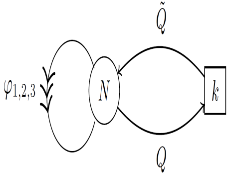
- In the large-N limit, the $k=1$ index takes the form

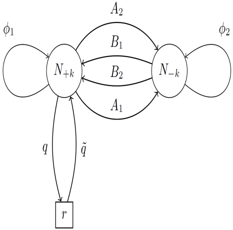
$$\log Z = -\frac{N^{3/2}}{3}\sqrt{2\Delta_1\Delta_2\Delta_3\Delta_4}\sum_a \frac{n_a}{\Delta_a} + N^{1/2} f_1(\Delta_a, n_a) - \frac{1}{2}\log N + f_3(\Delta_a, n_a) + \mathcal{O}(N^{-1/2}),$$

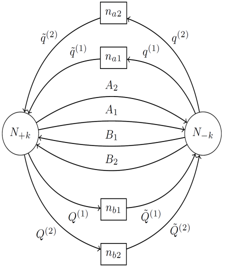
- The leading term reproduces the Bekenstein-Hawking entropy of extremal $\mathrm{AdS}_4 \times S^7$ magnetic black holes [Benini-Hristov-Zaffaroni '15].
- The $-\frac{1}{2}\log N$ term is the field theory logarithmic correction [Liu-PZ-Rathee-Zhao '17] .

Universality of Logarithmic Corrections

- 3d $\mathcal{N}=2$ Chern-Simons matter Theories: $S^7 \to V^{5,2}, N^{0,1,0}, Q^{1,1,1}$
- The universality of $-\frac{1}{2}\log N$ term [PZ-Xin '20].







Logarithmic terms in one-loop effective actions

- One-loop effective action is equivalent to computations of determinants for kinetic operators, A:

$$\frac{1}{2}\,\ln \mathrm{det}' A = \frac{1}{2}\sum_i{}'\,\ln \lambda_i$$

- The heat kernel $(\sum_i e^{-\tau\lambda_i})$ contains information about both the non-zero modes and the zero modes, ϵ is a UV cutoff.

$$-\frac{1}{2}\,\ln \mathrm{det}' A = \frac{1}{2}\int_\epsilon^\infty \frac{d\tau}{\tau}\left(\mathrm{Tr}K(\tau) - n_A^0\right)$$

- At small τ, the Seeley-DeWitt expansion for the heat kernel

$$\mathrm{Tr}K(\tau) = \frac{1}{(4\pi)^{d/2}}\sum_{n=0}^\infty \tau^{n-d/2}\int d^dx\,\sqrt{g}\,a_n(x,x).$$

Logarithmic terms in one-loop effective actions

- Since non-zero eigenvalues of a standard Laplace operator A scale as L^{-2}, it is natural to redefine $\bar{\tau} = \tau/L^2$.
- The logarithmic contribution to $\ln \mathrm{det}' A$ comes from the term $n = d/2$ (integrated trace anomaly):

$$-\frac{1}{2}\,\ln \mathrm{det}' A = \left(\frac{1}{(4\pi)^{d/2}}\int d^d x\,\sqrt{g}\,a_{d/2}(x,x) - n_A^0\right)\mathrm{log} L + \ldots .$$

- On very general grounds (diffeomorphism), the coefficient $a_{d/2}$ vanishes in odd-dimensional spacetimes.

Logarithmic terms: Key Facts

- Robustness: Independent of UV cutoff ϵ.
- In odd-dimensional spaces (11d Sugra) the coefficient of the $\log$ can only come from zero modes or boundary modes.

$$\log Z[\beta,\ldots] = \sum_{\{D\}} (-1)^D (\beta_D - 1) n_D^0 \log L + \Delta F_{\text{Ghost}}.$$

- Subtract the zero modes (-1) and add them appropriately due to integration over zero modes (β_D).
- The ghost contributions are treated separately.

Final Result

- Quantizing C_3 requires 2-form ghost which contributes

$$(2-\beta_2)n_2^0 \log L.$$

- Using $\beta_2 = 7/2$ and $n_2^0 = 2$ [non-extremal branch]:

$$\log Z[\beta, \ldots] = -3 \log L + \cdots .$$

- The AdS/CFT dictionary: $L/\ell_P \sim N^{1/6}$

$$S = \cdots - \frac{1}{2} \log N + \cdots ,$$

- Perfectly agrees with the microscopic result!!!

Universality of Logarithmic Corrections: Gravity

- Similar results for asymptotically $\mathrm{AdS}_4 \times SE_7$ black holes with $SE_7 = \{S^7, V^{5,2}, N^{0,1,0}, Q^{1,1,1}, M^{1,1,1}\}$
- Every seven-dimensional, compact Einstein manifold of positive curvature has vanishing first Betti number, $R_{mn} = 6m^2 g_{mn} \Rightarrow \Delta_1 \geq 6m^2$.
- A universal macroscopic result that matches the field theory [PZ-Xin '20]:

$$S = S_{BH} \quad -\frac{1}{2}\log N + \cdots ,$$

Wrapped M5 branes in AdS/CFT

- Entries in AdS_4/CFT_3 from M2's and M5's.

$\text{AdS}_4/\text{CFT}_3$	from M2-branes	from M5-branes
M-theory set-up	N M2-branes probing Cone(SE_7)	N M5-branes wrapped on M_3
Dual Field theory	Known only for special examples of SE_7	Systematic algorithm applicable to general M_3
Gravity dual	$AdS_4 \times SE_7$	Warped $AdS_4 \times M_3 \times \tilde{S}^4$
Symmetry	Isometry of SE_7 ($\supset U(1)_R$)	Only $U(1)_R$
L^2/G_4	$\frac{N^{3/2}\pi^2}{\sqrt{27/8\text{vol}(SE_7)}}$	$\frac{2N^3\text{vol}(M_3)}{3\pi^2}$
L/ℓ_P	$\propto N^{1/6}$	$\propto N^{1/3}$

- Logarithmic corrections match on the field theory (3d-3d) and the gravity side [Gang-Kim-PZ '19, Benini-Gang-PZ '19]:

$$\log Z_{1-loop} = (g-1)(1-b_1)\log N$$

Classical Entropy Formula in AdS

- Sen's Classical Entropy function [Sen '05] yields, under some symmetry considerations, the Wald entropy (higher curvature contributions) of the black hole in flat space.
- The leading Bekenstein-Hawking entropy of asymptotically $AdS_{4,5}$ black holes can be computed from the near-horizon geometry using Sen's Classical Entropy function [Morales-Samtleben '06, Goulart '15] [Ghosh-PZ '20,Ghosh-Godet-PZ '21?].
- The Quantum Entropy Formula [Sen '08] has been successful in asymptotically flat black holes [Sen '14]. Applications for asymptotically AdS black holes?
- We have reproduced the field theory prediction from the asymptotic AdS_4 region not from the near-horizon AdS_2 region.

A puzzle: Quantum Entropy Formula

- The near horizon geometry: $AdS_2 \times M_9$, M_9 is a S^2 bundle over S^7 with $\{n_a\}$.
- Graviton zero modes from two sources: a graviton in AdS_2 and gauge fields corresponding to Killing vectors of M_9.
- There is a 1-form zero mode in AdS_2; AdS_{2M} admits a M-form zero mode.
- Contributions: graviton, gravitino, 3-form and 1-form ghost lead to the total logarithmic correction $(N \sim L^6)$:

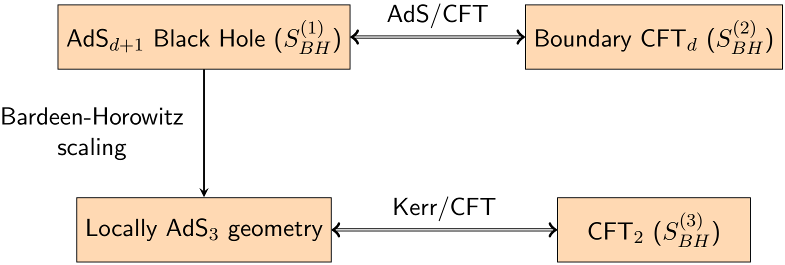
$$\Delta F = \left(-45 + 36 - \frac{3}{2} - \frac{3}{2}\right) \log L = -12 \log L \sim -2 \log N.$$

- Simultaneously obtained [Liu-PZ-Rathee-Zhao '17, Jeon-Lal '17].
- The quantum entropy formula counts near horizon degrees of freedom, it requires a revision in AdS.
- $\Longrightarrow$ Explorations in other AdS black holes: AdS_5.

Microscopic Description: Rotating, Electrically Charged Black Holes in $AdS_{4,5,6,7}$

- The microscopic entropy of AdS_5 black holes 2018:
  - Cabo-Bizet-Cassani-Martelli-Murthy
  - Choi-Kim-Kim-Nahmgoong
  - Benini-Milan
- AdS_4: Choi-Hwang-Kim [1908.02470]; Nian-PZ [1909.07943]; Bobev-Crichigno [1909.05873]; Benini-Gang-PZ [1909.11612]
- AdS_6: Choi-Kim [1904.01164]
- AdS_7: Kantor-Papageorgakis-Richmond [1907.02923]; Nahmgoong [1907.12582].
- Given that entropy $(A/(4G))$ is such a universal quantity .... Do you really need the full power of AdS/CFT (UV complete setting) or is there an *effective low energy description*?
- Lessons from Strominger-Vafa: D-brane technology versus Brown-Henneaux.

Kerr/CFT



$$ds^2 = \alpha_1 \left[-\widetilde{r}^2\, d\tau^2 + \frac{d\widetilde{r}^2}{\widetilde{r}^2} \right] + \Lambda_1(\theta) \left[d\widetilde{\phi} + \alpha_2\, \widetilde{r}\, d\tau \right]^2 + \Lambda_2(\theta) \left[d\widetilde{\psi} + \beta_1(\theta) d\widetilde{\phi} + \beta_2(\theta)\, \widetilde{r}\, d\tau \right]^2 + \alpha_3\, d\theta^2$$

- Kerr/CFT: [Guica-Hartman-Son-Strominger '08]; AdS black holes: [Lu-Mei-Pope '08, Chow-Cvetic-Lu-Pope '08]
- Bekenstein-Hawking entropy of AdS$_{4,5,6,7}$ [Nian-PZ '20], [David-Nian-PZ '20], [David-Nian '20].

The superconformal index (see Sameer's talk)

$$\mathcal{I}(p,q;v) = \mathrm{Tr}_{\mathcal{H}(S^1\times S^3)}\left[(-1)^F\, e^{-\beta\{\mathcal{Q},\mathcal{Q}^\dagger\}} v_a^{Q_a} p^{J_1+\frac{r}{2}} q^{J_2+\frac{r}{2}}\right],$$

- Counts $\frac{1}{16}-$BPS states for $\mathcal{N}=4$ SYM theory and $\frac{1}{4}-$BPS states for generic $\mathcal{N}=1$ SCFT's.
- Q_a are the charges of states that commute with the super charge $\mathcal{Q}$
- r is the R-charge
- The fugacities p and q are associated to the two angular momenta $J_{1,2}$ of S^3.

Evaluation beyond large N

- Cardy-like limit ($|\tau| \to 0$), exact in N through SU(N) Chern-Simons matrix model, saddle point approximation [GonzálezLezcano-Hong-Liu-PZ '20].

$$\mathcal{I}(\tau;\Delta) = \mathcal{I}(\tau;\Delta)\Big|_{\text{Main Saddle Point}} + (\text{ other saddles})$$

$$\log \mathcal{I}(\tau;\Delta)\Big|_{\text{Main Saddle Point}} = -\frac{\pi i(N^2-1)}{\tau^2}\prod_{a=1}^{3}[\Delta_a]_\tau + \log N + \mathcal{O}(e^{-1/|\tau|}).$$

- Large-N, any τ, BA approximation [GonzálezLezcano-Hong-Liu-PZ '20].

$$\mathcal{I}(\tau;\Delta) = \mathcal{I}(\tau;\Delta)\Big|_{\text{Basic BA}} + (\text{other BA solutions})$$

$$\log \mathcal{I}(\tau;\Delta)\Big|_{\text{Basic BA}} = -\frac{\pi i(N^2-1)}{\tau^2}\prod_{a=1}^{3}[\Delta_a]_\tau + \log N + \mathcal{O}(N^0),$$

- Log for other gauge groups [Amariti-Fazzi-Segati '20].
- EFT for the SCI: Log as degeneracy of vacua [Cassani-Komargodski '21, Ardehali-Murthy '21].

Logarithmic term from near-horizon CFT_2

- Kerr/CFT provides a CFT_2 with n global $U(1)$ symmetries whose conserved charges are P^i and associated chemical potentials are μ_i:

$$Z(\tau,\bar{\tau},\vec{\mu}) = \mathrm{Tr}\, e^{2\pi i\tau L_0 - 2\pi i\bar{\tau}\bar{L}_0 + 2\pi i\mu_i P^i}\,,$$

$$\rho(E_L,E_R,\vec{p}) = \int d\tau\, d\bar{\tau}\, d^n\mu\, Z(\tau,\bar{\tau},\vec{\mu})\, e^{-2\pi i\tau E_L + 2\pi i\bar{\tau}E_R - 2\pi i\mu_i p^i}\,.$$

- Saddle point reproduces the Bekenstein-Hawking entropy $A/(4G)$.
- Logarithmic corrections from Kerr/CFT[David-GonzálezLezcano-Nian-PZ '21]. Toward quantum Kerr/CFT?

$$\Delta S_{CFT_2} = -\frac{1}{2}\log\det\left(\partial_i\partial_j S_{eff}\right) = \log N$$

- Applies for the AdS_5 rotating black string [Hosseini-Hristov-Tachikawa-Zaffaroni '20].

Outlook

- AdS$_4$ versus AdS$_5$: Zero modes as obstructions for near-horizon approaches.
- The degrees of freedom do not live locally at the horizon. Corrections to the Sen's Quantum Entropy Formula, extra hair in AdS.
- Some aspects of Hawking Radiation à la Callan-Maldacena (from near-horizon CFT$_2$ [Nian-PZ '20]).
- The IIB gravity computation of $\log N$.
- Higher curvature corrections? Recent work [Bobev-Charles-Hristov-Reys '20]. Back to Sen's Entropy function formalism [Ghosh-Godet-PZ '21?].
- AdS/CFT *de facto* solves many black hole puzzles.
  Let's see that explicitly!