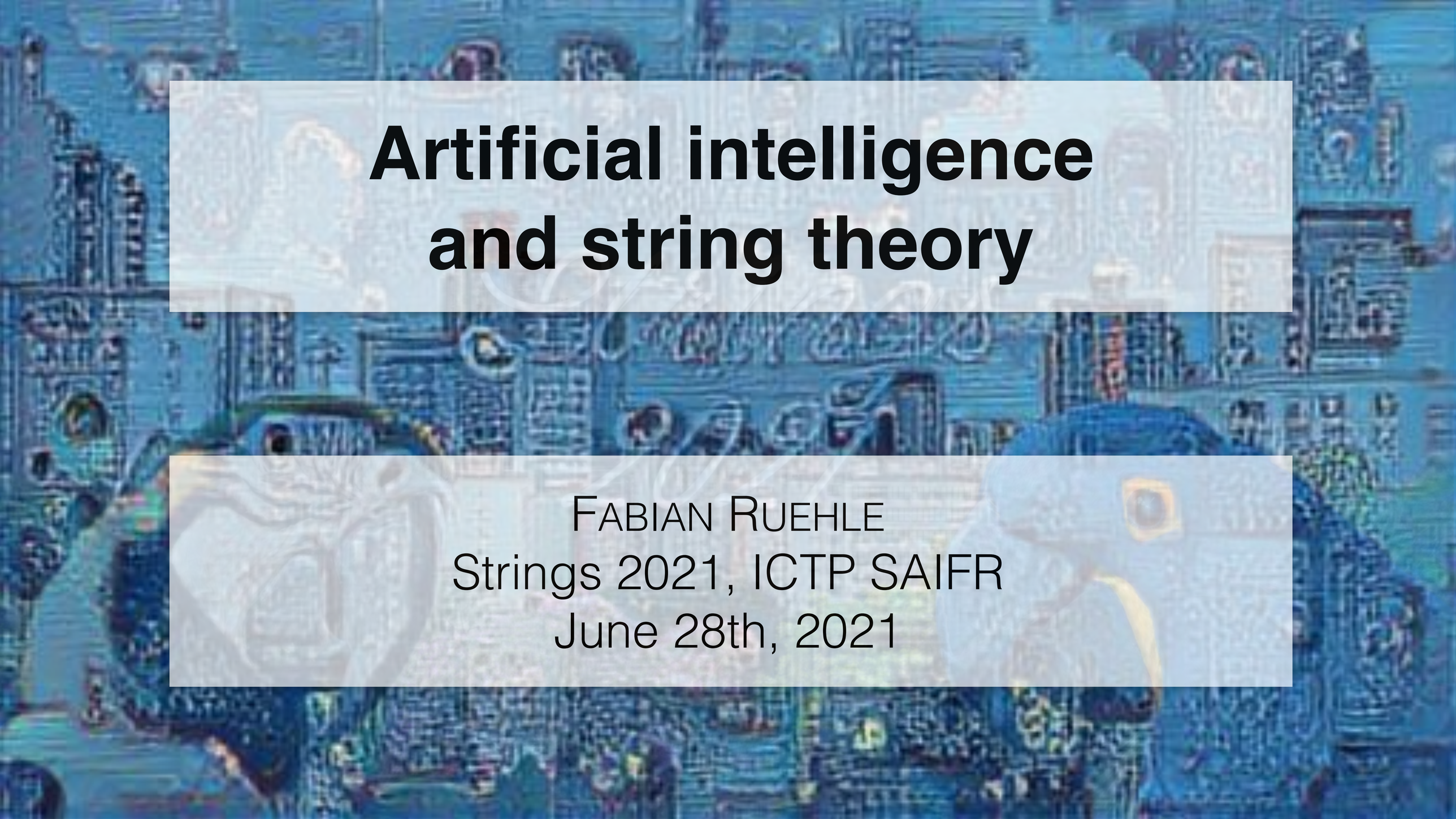

Artificial intelligence and string theory

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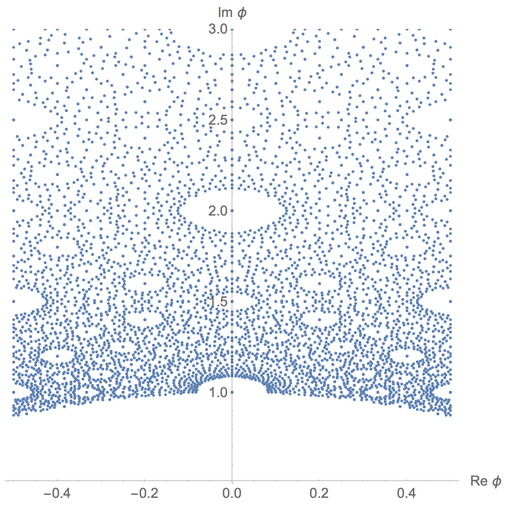
Strings 2021, ICTP SAIFR

June 28th, 2021

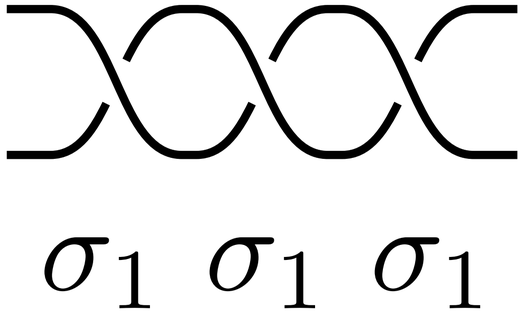
Are NNs appropriate tools for String Theory?

- NNs: $h_W : \mathbb{R}^n \to \mathbb{R}^m$ (affine trafos + non-linear functions) are universal approximators, i.e. they can learn in theory learn any function
- For currently not fully understood reasons parameterizing maps like this …

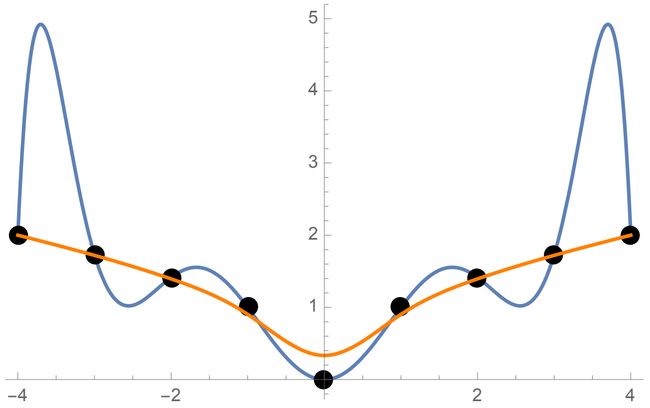
… is extremely efficient for pattern recognition



… is extremely efficient for language processing



… defies common lore in statistics (generalizes well, does not overfit)



Applications

Conjecture Generation

Try to learn a map between quantities with no previously known relation, formulate (and hopefully prove) conjecture

- Knot theory
  [Hughes `16; Jejjala,Kar,Parrikar `19; Gukov, Halverson,FR,Sulkowski `20; Craven,Jejjala,Kar `20]
- Toric geometry
  [Krefl,Seong`17;Carifio,Cunningham,Halverson, Krioukov,Long`17]
- Line bundle cohomology, Brill-Noether theory
  [FR `17; Klaewer,Schlechter `18; Brodie,Constantin, Deen,Lukas `18-20; Bies,Cvetič,Donagi,Lin,Liu,FR `20]

  [Mirjam's presentation]
- Many more… [especially He et.al.]

Optimization and Regression

Find solutions to a system of equations

- Searches for string vacua
  [FR`17; Wang,Zhang `18; Mutter,Parr,Vaudrevange `18; Halverson,Nelson,FR `19; Brodie,Constantin,Deen, Lukas`19; Larfors,Schneider `20; Deen,He,Lee,Lukas `20; Otsuka,Takemoto `20;Cabo Bizet,Damian,Loaiza-Brito,Mayorga,Montañez-Barrera `20, …]

  [Talk by Mariana for other DS technique]
- CY metrics
  [Ashmore,He,Ovrut `19; Anderson,Gray,Gerdes, Krippendorf,Raghuram,FR `20; Douglas, Lakshminarasimhan,Qi `20; Jejjala,Mayorga,Mishra `20]

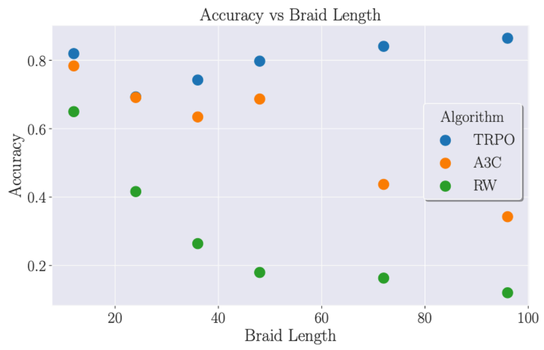
  [Talk by Lara]

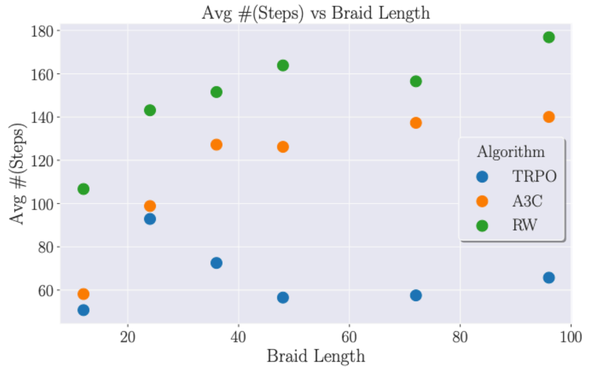
Example I - Pattern Recognition in Diophantine equations

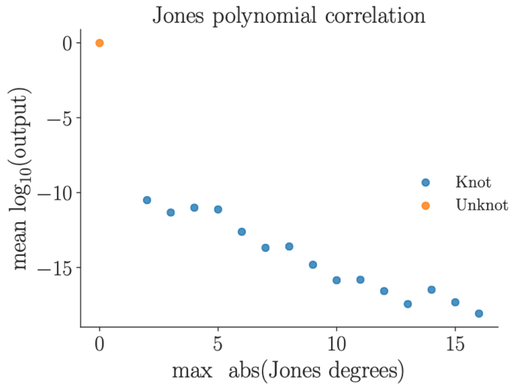
- **Background:**
  - Diophantine equations ubiquitous in ST (topological data, quantization conditions)
  - Asking whether an arbitrary Diophantine equation has a solution (let alone finding one) is an undecidable problem, i.e. as hard as it gets
  - However, Diophantine equations in string theory are not be arbitrary but inherit structure from consistency conditions, …
- **Study:** [Halverson, Nelson, FR `19]
  - Set up a “game” in RL to solve a particular set of coupled Diophantine equations related to flux vacua of type II orientifolds showed that the NN
    - … can rediscover human-derived solution strategy that leads to partial decoupling
    - … can find new, more efficient strategies

Example II - Conjecture generation and NLP in Knot Theory

- **Conjecture generation:** Train NNs on some knot representation/invariant. If they learn to predict something, hints at a (previously unknown) connection
  - Basic knot invariants $\Leftrightarrow$ quasi-positivity, slice genus, OS τ-invariance [Hughes `16]
  - Jones Polynomial $\rightarrow$ hyperbolic knot volume [Jejjala,Kar,Parrikar `19; Craven,Jejjala,Kar `20]
- **NLP & RL:** Get provably correct results by setting up a "game" (find the unknot) and have the NN device a solution strategy (sequence of Reidemeister moves)
  [Gukov,Halverson,FR,Sulkowski `21]







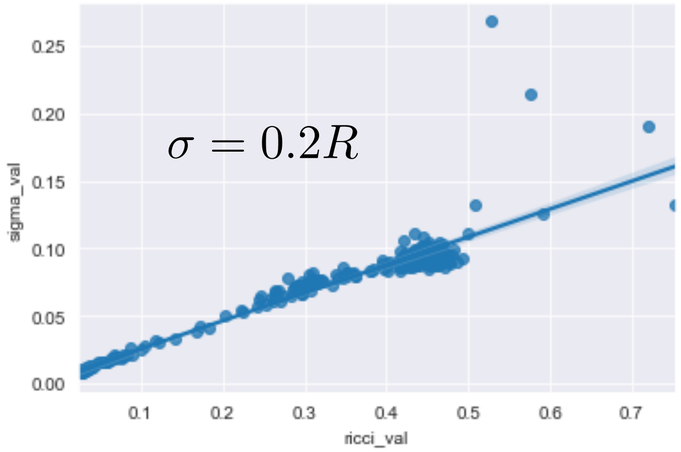
Example III - Interpolation and CY Metrics

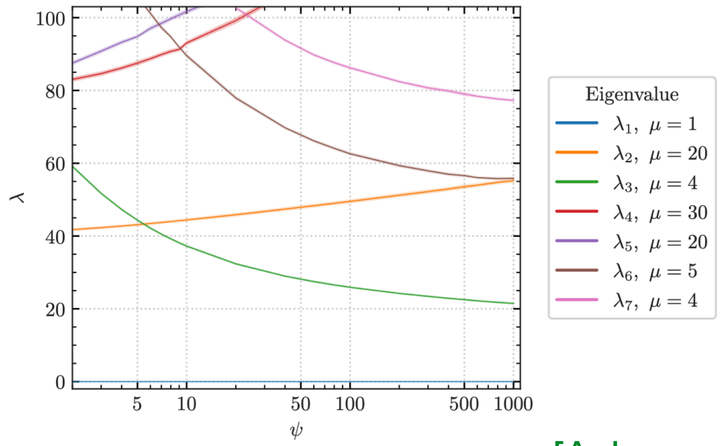
- Think of metric g_{CY} as a map $g_{CY} : \mathcal{M}_K \times \mathcal{M}_{CS} \times \mathbb{C}^d \to \mathbb{C}^{d\times d}$
- Parameterize as NN subject to conditions (complex, Kähler, Ricci-flat $\Leftrightarrow$ MA eqn)

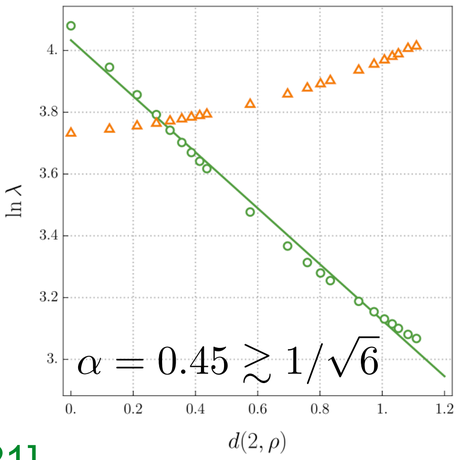
[NNs to solve PDEs: Talk by Gonzalo]

- Use the expression for the (moduli-dependent) metric to
  - Compute Yukawa couplings
  - Compute massive string spectrum and check swampland distance conjecture [Ooguri,Vafa `06]
  - Study SYZ conjecture [Strominger,Yau,Zaslow `96]

[Talk by Irene]







[Ashmore,FR `21]

Summary and Future Directions

- Search for consistent string solutions, use adversarial training to delineate swampland/landscape boundary
- Get insight from ML where symbolic methods fail / are unavailable
  - Solve PDEs / EOMs for other (reduced holonomy/special structure) manifolds and in cases where no existence proof exists
  - Massive spectrum and relation to degeneracy under symmetry group
  - Knot Theory (unknot, sliceness, SPC4)
- ML applications to RMT, SYK / JT Gravity (Graph NNs?) [Talk by Jeff] , Mera/Tensor Networks, quantum information, numerical bootstrap, integrability?

Open Questions and Speculations

- From the methods working/failing, can we learn something about the pattern underlying the problem at hand?
- Computational Complexity
  - Are approximate but fast results acceptable?
    - ✦ Can we check an answer once the algorithm proposes one?
    - ✦ Which problems can even be approximated fast?
    - ✦ If we find approximate solutions, what do we require in order to believe that they are actually solutions?
  - If consistent string theories are governed by Diophantine equations, which are in general undecidable, how did the Universe solve them? Does that teach us something about vacuum selection? [Denef,Douglas `07; Denef,Douglas,Greene,Zukowski `17; Halverson,FR `18; Halverson,Plesser,FR,Tian `19]
- If in the (very) distant future a very advanced AI proves a theorem such that no human can go through/understand the proof, do we accept it? Is it a proof?