Higher-Form Symmetries from String Theory

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2102.01693 , 2106.10265 with Lakshya Bhardwaj and Max Hübner

Motivation

Higher-form symmetries [Gaiotto, Kapustin, Seiberg, Willett, 2014] are by now very well established in QFT.

What's the added benefit of studying them in string theory?

Access to strongly-coupled QFTs, non-Lagrangians:

- Geometric engineering: 5d and 6d theories and SCFTs
- Strongly coupled theories via holography
- Non-Lagrangian theories, e.g. class S and generalizations

Motivation

Access to strongly-coupled QFTs, non-Lagrangians: some examples

- Geometric engineering of 5d theories and SCFTs: (all in the past year) Higher-form symmetries, 't Hooft anomalies, 2-group structures [Morrison, SSN, Willett][Albertini, del Zotto, Garcia Etxebarria, Hosseini][Closset, SSN, Yi-Nan Wang][Apruzzi, Dierigl, Lin][Benetti Genolini, Tizzano][Bhardwaj, SSN][Cvetic, Dierigl, Lin, Zhang][Apruzzi, Bhardwaj, Oh, SSN]
- Holography: class S, ABJM and revisiting Klebanov-Strassler confining cascade [Witten][Bah, Bonetti, Minasian][Hofman, Iqbal][Bergman, Tachikawa, Zafrir][Apruzzi, van Beest, Gould, SSN]
- Non-Lagrangian theories: from 6d such as class S, class R [Tachikawa][Cordova, Dumitrescu, Intriligator][Eckhard, Kim, SSN, Willett][Gukov, Pei, Hsin][Bharwaj, Hubner, SSN]^2.

4d $\mathcal{N} = 1$ SYM 1-form symmetry, is a diagnostic for confinement: Wilson lines with area law indicate confining vacua, where the 1-form symmetry is unbroken. Perimeter law implies deconfining phase and 1-form symmetry is spontaneously broken.

The goal of this talk is to generalize this insight to theories that may not have a simple underlying UV Lagrangian description, such as $\mathcal{N} = 1$ deformations of 4d $\mathcal{N} = 2$ Class S theories.

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For this, we first cast the standard case of SYM into this framework and show how to determine the line operators and 1-form symmetry in each vacuum.

Plan

1. Line operators, polarization and 1-form symmetry
2. 1-form Symmetry in 4d $\mathcal{N} = 2$ Class S
3. Confinement in $\mathcal{N} = 1$ deformations of class S
   (i) Super Yang-Mills
   (ii) Confinement index for theories with adjoint chirals
   (iii) Non-Lagrangian theories

1. Line Operators, Polarization and 1-form Symmetry

Pure gauge theory in 4d, gauge algebra $\mathfrak{g}$ and simply-connected group G.
The set of line operators are

$$\mathcal{L} = \Lambda_w/\Lambda_r \oplus \Lambda_{mw}/\Lambda_{cr} = Z_G \oplus Z_G \qquad Z_G\text{= center of } G.$$

Not all lines are mutually local (relative theory): Dirac pairing

$$L_\alpha L_\beta = L_\beta L_\alpha e^{2\pi i \langle L_\alpha , L_\beta \rangle} \, .$$

Polarization $\Lambda \subset \mathcal{L}$:
choice of a maximal set of mutually local line operators (absolute theory).

E.g. for $\mathfrak{su}(N)$: $\mathcal{L} = \mathbb{Z}_N \oplus \mathbb{Z}_N$ with

$$\langle W, H \rangle = \frac{1}{N} \, .$$

$\Lambda =< W >$: gauge group $G = SU(N)$
$\Lambda =< H >$: gauge group $G = PSU(N)$

1-Form Symmetry

$\Lambda \subset \mathcal{L}$ choice of polarization (absolute theory). Then the 1-form symmetry is the Pontryagin dual group to Λ

$$\Gamma^{(1)} = \widehat{\Lambda} = \mathrm{Hom}(\Lambda, U(1))\,.$$

E.g. $\mathfrak{su}(N)$: $\Lambda = \mathbb{Z}_N$ and $\Gamma^{(1)} = \mathbb{Z}_N$.

In a given vacuum r let

$$\Lambda_r = \{L \in \Lambda; L \text{ has perimeter law in vacuum } r\}$$

Then the 1-form symmetry that is preserved in this vacuum is

$$\Gamma_r^{(1)} = \frac{\widehat{\Lambda}}{\Lambda_r} \subset \Gamma^{(1)}\,.$$

If this is non-trivial, then the vacuum r is confining.

2. 1-form symmetry of 4d $\mathcal{N} = 2$ Class S

[Tachikawa][Bhardwaj, Hubner, Schafer-Nameki]

6d (2,0) of type $\mathfrak{g}$=ADE is a relative theory, with 2-form symmetry $\widehat{\mathcal{Z}}$.
Compactify on $\mathcal{C}_{g,n} \subset T^*\mathcal{C}$ to 4d $\mathcal{N} = 2$ class S [Gaiotto].

6d surface operators wrapped on $H_1(\mathcal{C}_{g,n}, \mathbb{Z})$ give line operators.

Consider $\mathcal{C}_{g,\emptyset}$:

$$\mathcal{L} = H_1(\mathcal{C}_g, \widehat{\mathcal{Z}}) \cong \widehat{\mathcal{Z}}_A \oplus \widehat{\mathcal{Z}}_B$$

from A-/B-cycles. There is a pairing on these line operators

$$a_i \otimes \alpha_i \in H_1(\mathcal{C}_g, \mathbb{Z}) \otimes \widehat{\mathcal{Z}} : \qquad \langle a_1 \otimes \alpha_1, a_2 \otimes \alpha_2 \rangle = (a_1 \cdot a_2) \langle \alpha_1, \alpha_2 \rangle \,.$$

$\Rightarrow$ relative theory

Canonical polarization choices:

$$\mathcal{L} \supset \Lambda = \widehat{\mathcal{Z}}_A \quad \text{or} \quad \widehat{\mathcal{Z}}_B \qquad \Rightarrow \qquad \Gamma^{(1)} = \widehat{\Lambda} \,.$$

Extra class S data:
twist lines and punctures i.e. $\mathfrak{g}$ with outer autos [Bhardwaj, Hubner, SSN, 2021].

Interlude: Geometric Engineering

Construct 4d $\mathcal{N} = 2$ theories from IIB on singular, non-compact CY3 X.
Line operators are D3s on non-compact 3-cycles, modulo screening by D3s on compact 3-cycles:

$$\mathcal{L} = \frac{H_3(X, \partial X, \mathbb{Z})}{H_3(X, \mathbb{Z})} \cong \{L \in H_2(\partial X, \mathbb{Z}); L \text{ extends trivially to } X\}$$

Computable for non-Lagrangian theories like Argyres-Douglas.
[Closset, SSN, Wang][Albertini, del Zotto, Garcia Etxebarria, Hosseini].

A subset of class S theories: $\mathbb{C}^2/\Gamma_{ADE} \to \mathcal{C}_g$.
ALE-fibration is governed by the Higgs field ϕ, which satisfies the Hitchin equations [Gaiotto, Moore, Neitzke]

$$\bar{D}\phi = 0\,, \qquad F_{z\bar{z}} + [\phi, \phi^*] = 0\,.$$

Spectral curve of this Higgs field is the Seiberg-Witten curve $\det(v - \phi) = 0$, which is an N-fold cover of the Gaiotto curve $\mathcal{C}$. Then

$$\mathcal{L} = \mathrm{Ab}(\Gamma_{ADE})^{2g}\,.$$

3. $\mathcal{N}=1$ Deformation

4d $\mathcal{N}=1$ from 6d (2,0) on $\mathcal{C}_{g,n}$ in $V_1 \oplus V_2 \to \mathcal{C}_{g,n}$, with sections (ϕ, ψ).

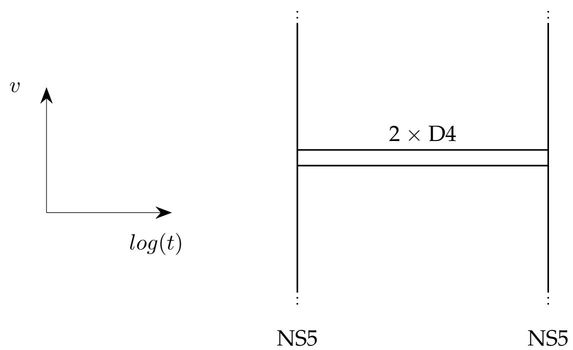
Simple class of configurations: $\phi = (1,0)$- and $\psi = (0,0)$-forms on $\mathcal{C}$, with BPS equations [Xie], corresponding to a generalized Hitchin system

$$\begin{aligned} \bar{D}\phi = \bar{D}\psi &= 0 \\ [\phi, \psi] &= 0 \\ F + [\psi, \psi^*] + [\phi, \phi^*] &= 0\,. \end{aligned}$$

(ϕ, ψ) each defines an N-sheeted covers of $\mathcal{C}$.

Strategy: start with $\mathcal{N}=2$ Higgs field ϕ, and then "rotate" to $\mathcal{N}=1$ [Barbon][Witten][Hori, Ooguri, Oz][Bonelli, Giacomelli,Maruyoshi,Tanzini] (related [Dijkgraaf, Vafa] curve).

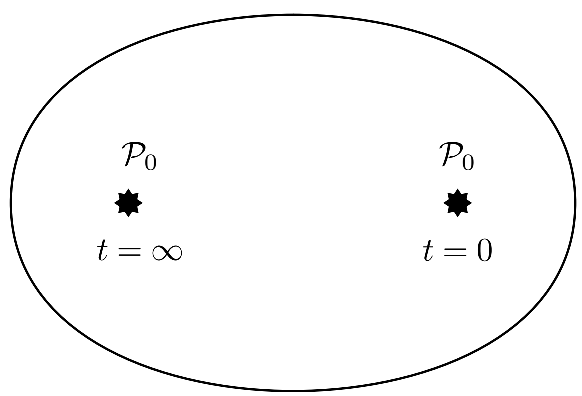
4d $\mathcal{N} = 2$ pure SYM from 6d



Focus at first on $\mathfrak{su}(2)$ for clarity.
The Seiberg-Witten curve is

$$v^2 = \frac{\Lambda^2}{t} + u + \Lambda^2 t\,.$$

t= coordinate on the Gaiotto curve $\mathcal{C} = S^2$.



Class S construction:
$g = 0$, $n = 2$ with two $\mathcal{P}_0$ irregular punctures:

$$\mathrm{Tr}\,\phi^2 \equiv \phi_2 = \frac{v^2}{t^2}dt^2 = \left(\frac{\Lambda^2}{t^3} + \frac{u}{t^2} + \frac{\Lambda^2}{t}\right)dt^2$$

ϕ_2 has poles of order 3 at $t = 0, \infty$.

Rotating to $\mathcal{N} = 1$ SYM

Turn on ψ, where we rotate to $\mathcal{N} = 1$ at $t = \infty$:

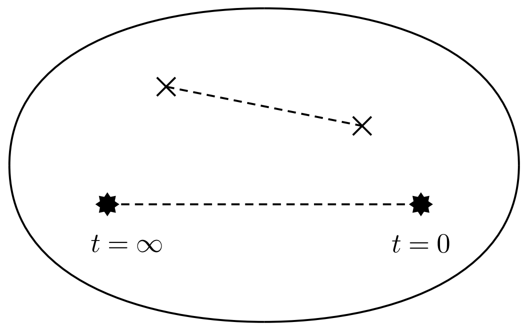
$$t \to \infty : \qquad \psi \to \mu \phi_\zeta \, , \qquad \phi_\zeta \frac{dt}{t} = \phi$$
$$t \to 0 : \qquad \psi \to c$$

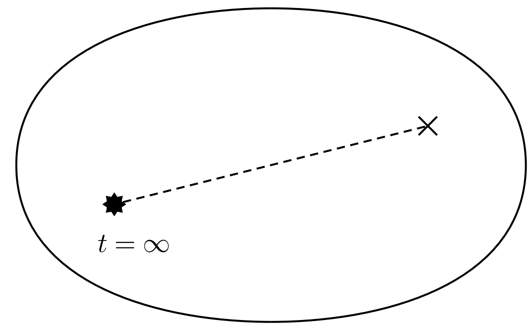
Furthermore for diagonalizable Higgs fields, the BPS equation $[\phi, \psi] = 0$ implies simultaneous diagonalizability. For generic eigenvalue spectrum, this implies that one is a function of the other, and thus the branch-cuts must match.

→ solving for the curve

→ topological matching of branch-cuts

The v-curve for ϕ and w-curve for ψ are:

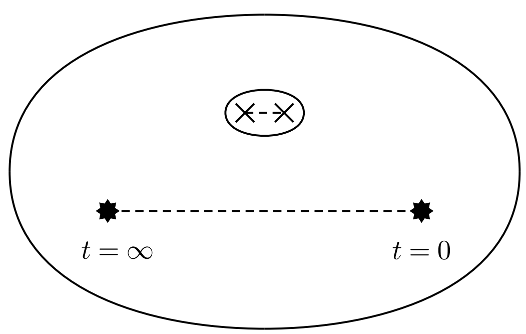


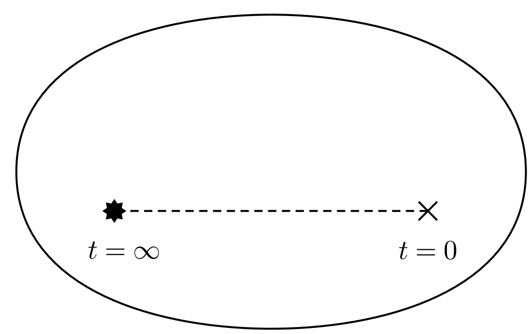


$v\ :\qquad v^2 = \frac{1}{t}(\Lambda^2 t^2 + ut + \Lambda^2)$

$w\ :\qquad w^2 = \mu^2\Lambda^2 t + c$

Tune CB moduli to combine into single cover, i.e. move branchpoints $\times$:





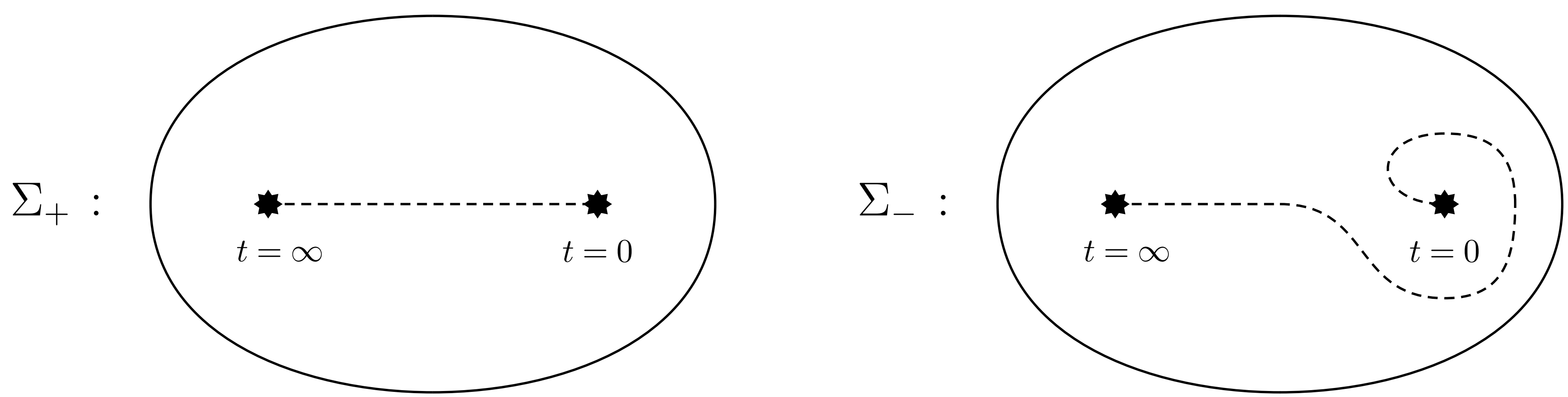
$$v^2 = \Lambda^2\left(\frac{1}{t} \pm 2 + t\right), \qquad w^2 = \mu^2\Lambda^2 t\,, \qquad vw = \mu\Lambda^2(t\pm 1)\,.$$

Curves describing the two vacua

Pure $\mathcal{N}=1$ SYM: take $\mu \to \infty$, while $\Lambda^3_{N=1} = \mu\Lambda^2$ fixed, and $\mu t = \tilde{t}$:

$$v^2 = \frac{\Lambda^3_{N=1}}{\tilde{t}}\,, \qquad w^2 = \Lambda^3_{N=1}\tilde{t}\,, \qquad vw = \pm\Lambda^3_{N=1}$$

The two vacua differ by asymptotics of w, which can be changed by encircling $t=0$:



Line Operators and 1-form Symmetry of the Vacua

$\mathcal{L}$ and Λ =lines and a polarization of the $\mathcal{N}=2$ class S theory.
Define for each $\mathcal{N}=1$ vacuum with curve Σ_r:

$$\mathcal{I}_r = \{\text{projections of 1-cycles on the } \mathcal{N}=1 \text{ curve } \Sigma_r \text{ onto } \mathcal{C}\} \subset H_1(\mathcal{C}, \widehat{\mathcal{Z}})\,.$$

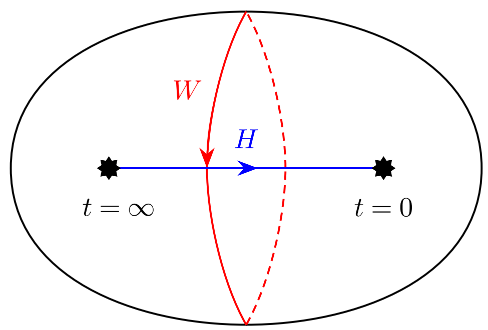
Then lines with perimeter law are $\Lambda_r = \Lambda \cap \mathcal{I}_r$ and the 1-form symmetry $\Gamma_r^{(1)}$ preserved in the vacuum r is

$$\Gamma_r^{(1)} = \widehat{\left(\frac{\Lambda}{\Lambda \cap \mathcal{I}_r}\right)} \subset \widehat{\Lambda}$$

If $\Gamma_r^{(1)} \neq \emptyset$ then the vacuum r is confining.

Why? Confining strings arise from from membranes on relative 1-cycles of the local CY3 and Σ_r[Witten].

Confinement for $\mathfrak{su}(2)$ SYM



$\mathfrak{su}(2)$: Line operators are W and H.
For both vacua $2W=0$ because of the branch-cut crossing (only going twice around the puncture gives a 1-cycle on the curves). The lines with perimeter law for each vacuum are:

$$\mathcal{I}_+ =< 2W, H >\,, \qquad \mathcal{I}_- =< 2W, H+W > \,.$$

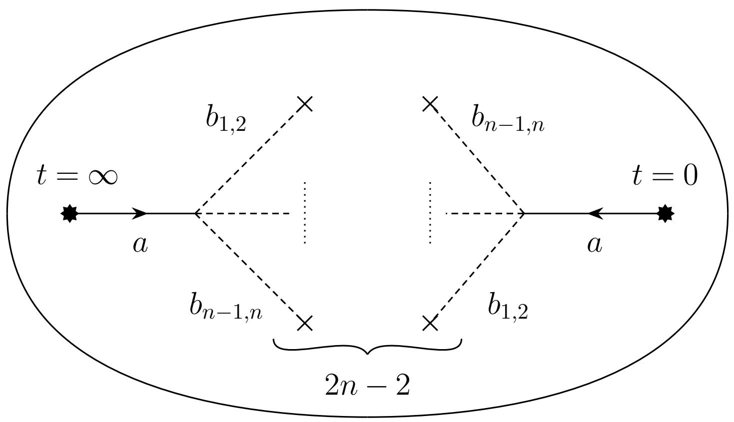
Preserved 1-form symmetry for each polarization:

Λ	G	$\Gamma^{(1)}_+$	$\Gamma^{(1)}_-$
$< W >$	$SU(2)$	$\mathbb{Z}_2$	$\mathbb{Z}_2$
$< H >$	$SO(3)_+$	$\emptyset$	$\mathbb{Z}_2$
$< H+W >$	$SO(3)_-$	$\mathbb{Z}_2$	$\emptyset$

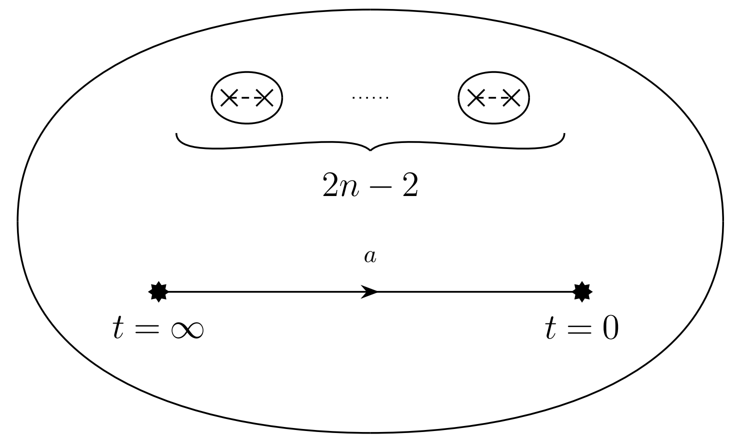
Of course well-known in general [Aharony, Seiberg, Tachikawa], but we derive it purely from the curve.

Generalization: Pure $\mathcal{N} = 1$ SYM $\mathfrak{su}(N)$

v :

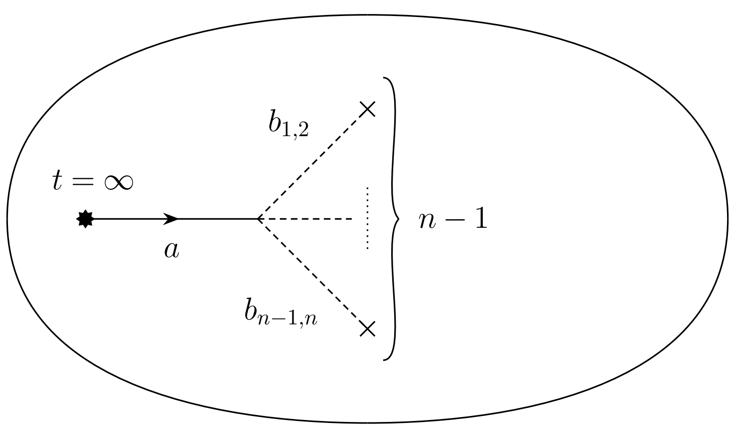


v :

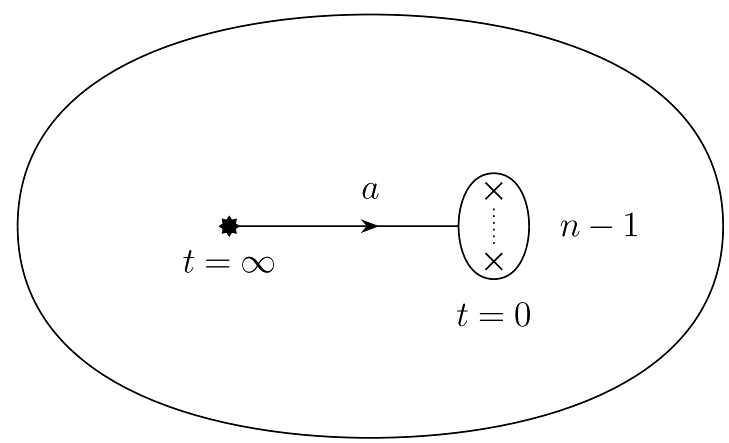


Match Branchcuts:

w :

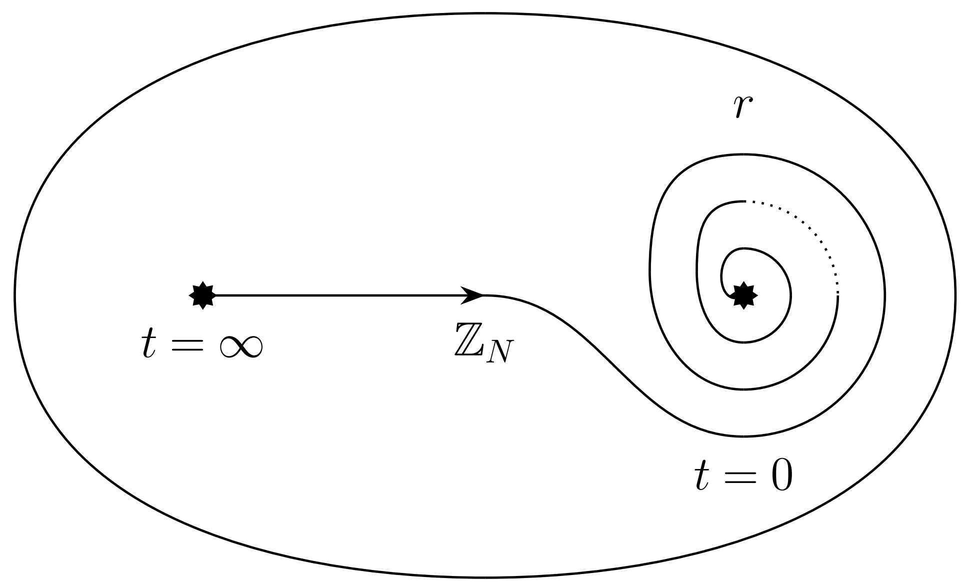


w :



The N vacua of the $\mathfrak{su}(N)$ SYM theories have curves

Σ_r :



Again choosing polarizations, we can determine the 1-form symmetries in each vacuum for all global forms.

Similarly we can determine the confinement index of $\mathfrak{su}(N)$ SYM with adjoint chirals with superpotential

[Elitzur, Forge,Giveon, Intriligator,Rabinovici][Cachazo,Seiberg,Witten]

$\rightarrow$ see [Bhardwaj, Hubner, SSN]

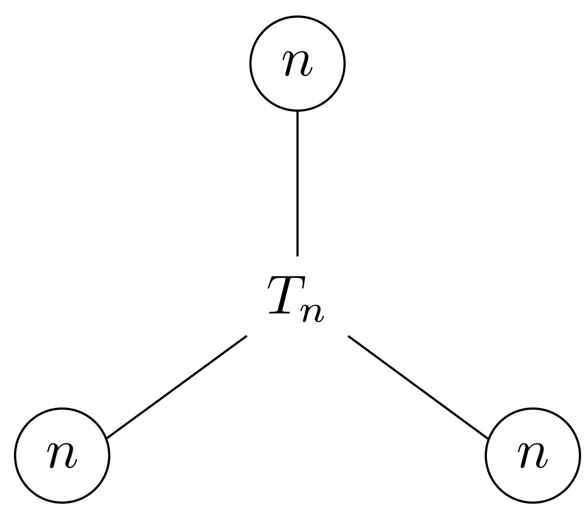
3. Confinement in Non-Lagrangian 4d $\mathcal{N} = 1$ Theories

The main reason for developing this class S based framework was to be able to apply it to theories without UV Lagrangians. In [Bhardwaj, Hubner, SSN] we determined a family of theories with no UV Lagrangian, which have confining vacua:

$$\mathfrak{P}_{n,\alpha} = \text{6d (2,0) } \mathfrak{su}(n) \text{ theory on a sphere with } \alpha \text{ irregular } \mathcal{P}_0 \text{ punctures}$$

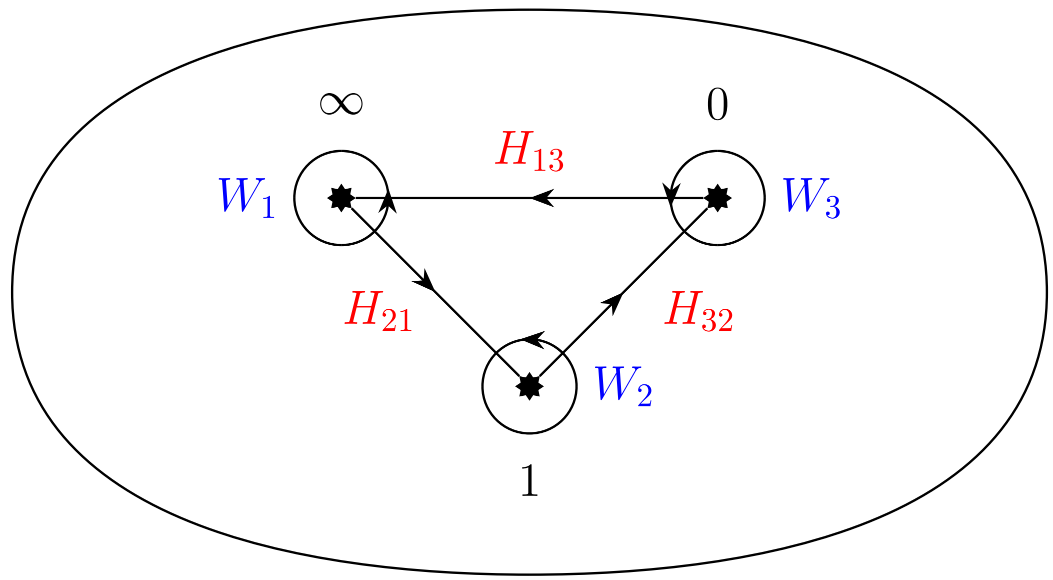
where $\mathcal{P}_0$ has pole of order $1 + 1/n$.
Simplest case $\alpha = 3$: T_n theory with $\mathfrak{su}(n)^3$ flavor gauged:

$\mathfrak{P}_{n,3}$:



Line operators:

v :



with

$$\mathcal{L} = \langle W_i, H_{jk} \rangle / \langle W_1 + W_2 + W_3 = 0;\, H_{21} + H_{32} + H_{13} = 0 \rangle \,.$$

The pairing on these line operators is

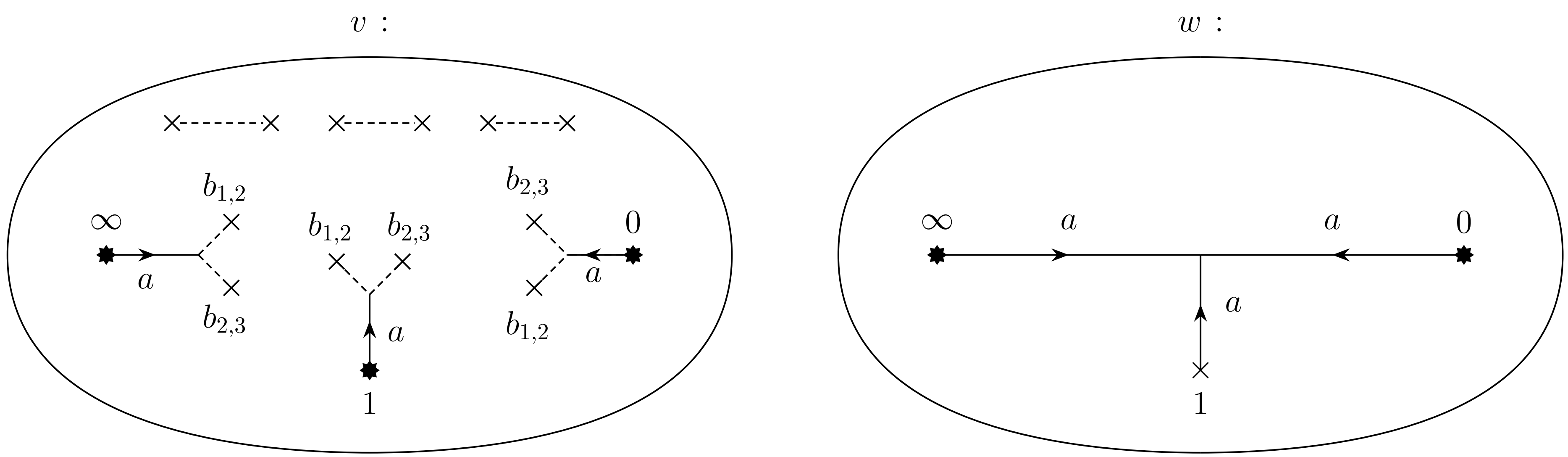
$$\langle W_i, H_{ij} \rangle = 1/n \,, \qquad \langle W_j, H_{ij} \rangle = -1/n$$

Again, many choices of polarization. A simple one is

$$\Lambda = \langle W_i \rangle / \langle W_1 + W_2 + W_3 = 0 \rangle \,.$$

$\mathcal{N}=1$ Curve for $\mathfrak{P}_{3,3}$

The $\mathcal{N}=1$ curve before collision of branch-points is, with punctures rotated at $t=0,\infty$:



The branch-cuts in v-curve can be collided to agree with the cut-structure on the w-curve.
In the electric polarization we find in this vacuum

$$\Gamma_{\Sigma}^{(1)} = \mathbb{Z}_3 \times \mathbb{Z}_3\,,$$

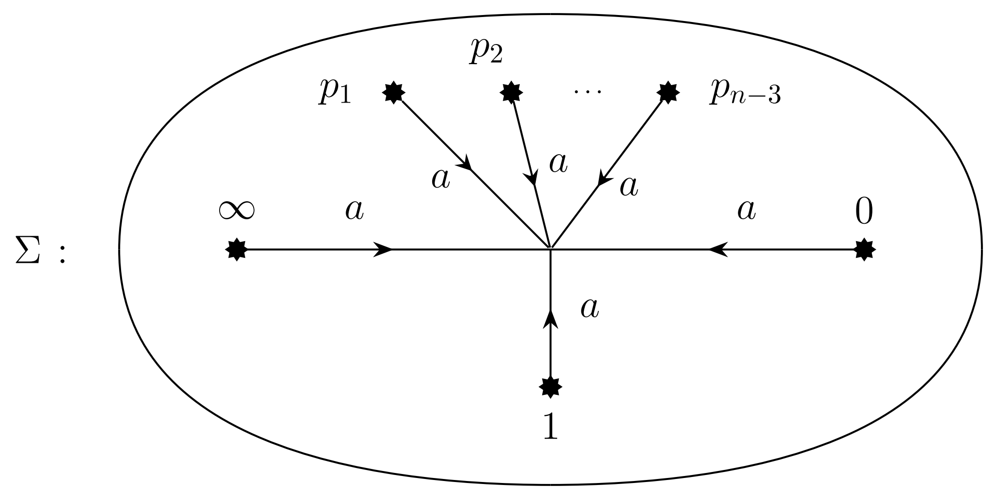
which means this vacuum is confining.

A Family of Confining Theories with no UV Lagrangian

$\mathfrak{P}_{n,n}$ = 6d (2,0) $\mathfrak{su}(n)$ on sphere with n $\mathcal{P}_0$s. Line ops $\mathcal{L} = \mathcal{L}_W \times \mathcal{L}_H$:

$$\mathcal{L}_W = \frac{\langle W_i \rangle}{\langle \sum W_i = 0 \rangle} \simeq \mathbb{Z}_n^{n-1}\,, \qquad \mathcal{L}_H = \frac{\langle H_{i+1,i}, H_{1,n} \rangle}{\langle \sum H_{i+1,i} + H_{1,n} = 0 \rangle} \simeq \mathbb{Z}_n^{n-1}$$

with pairing $\langle W_i, H_{ij} \rangle = \frac{1}{n}$, $\langle W_j, H_{ij} \rangle = -\frac{1}{n}$. Again, on special locus of CB match the branch-cut structure, resulting in the vacuum with curve:



With $\Lambda = \mathcal{L}_W$ polarization this vacuum preserves 1-form symmetry and is confining for n prime:

$$\Gamma^{(1)}_{\Sigma} = \mathbb{Z}_n^{n-1}\,.$$

Conclusion and Outlook

There's a strong case for studying higher-form symmetries in string theory constructions of QFTs.

Today we focused on 4d $\mathcal{N} = 1$ from class S, and determined the 1-form symmetry in known cases such as SYM, but we also predict confinement in theories without 4d UV Lagrangian.

Future directions: compute anomalies, and TQFTs governing the IR of the confining vacua from 6d.

In higher dimensional gauge theories/SCFTs: higher-form symmetries can provide important sets of 't Hooft anomalies in 5d and 6d theories [Benetti-Genolini, Tizzano], constrain the global, generalized and higher-group symmetry structure [Apruzzi, Bhardwaj, Oh, SSN].

Holography: using b.c. à la [Witten], the global forms of gauge groups can be determined e.g. in ABJM like theories in [Bergman, Tachikawa, Zafrir], and confinement in Klebanov-Strassler theories revisited and the IR TQFT derived from supergravity [Apruzzi, van Beest, Gould, SSN].